THE PLANETARY EQUATORIUM

from Kāshī's rule, and as can be verified by inspection of the last column in the Almagest table of Mercury's equations ([43], ed. of Halma, vol. ii, p. 309).

33. The Planetary Sectors (f. 10r:2-11, 10v, 30r:2 - 31r:10)

The subject of Chapter II.10 in our text is treated in [28], which may be consulted for details by the reader. We content ourselves here with a minimum for understanding of the text, beginning with a restatement of the definitions with which the chapter begins.

For certain astrological purposes, it was customary to divide the deferent and the epicycle into four sectors each, called by our author apogee sectors (nitāqát-i aulī) and epicyclic sectors (nitāqát-i tādīrī) respectively. There were two categories of sectors, those computed according to distance, and those according to velocity (hārakah), or, as our author puts it, movement (sayr).

In all cases the beginning of the first sector was at the (deferent or epicyclic) apogee, the beginning of the third sector at the perigee.

The beginnings of the second and fourth distance sectors on either the deferent or the epicycle were defined as those points at which an object moving on the circle in question is at its mean distance from the center of the universe.

For the velocity sectors, the beginnings of the second and fourth are those positions of a point moving on the deferent or epicycle at which its angular velocity, as viewed from the center of the universe, attains its mean value.

The endpoint of each first sector coincides with the beginning of the second, and so on.

The table on f. 10v locates the sector boundaries for all categories. The entries have evidently been obtained from Kāshī's zīj ([23], f. 14lv). uniformly rounded off to minutes. We next address ourselves to the question of how these values were determined.

The problem is simplest for the sun, which has no epicycle, hence no epicyclic sectors, and no equant. If d and R are the deferent eccentricity and radius respectively, then the arc of the deferent from the beginning of the first sector to the beginning of the second, that is, the amplitude of the first solar distance sector is

\[ 90^\circ + \arcsin\frac{d}{2R}. \]

This disposes of all other solar sectors of this category, for, as always, the sectors are symmetrically located with respect to the line of apsides. Hence the amplitudes of the first and fourth sectors are always equal, while the first and second are supplementary.

The amplitude of the first solar velocity sector is

\[ 90^\circ + \arcsin\frac{d}{R}. \]

With the planets, the location of the initial point of the second deferent distance sector can also be obtained from (19) above. For use in the zījes, however, it was convenient to tabulate the angle subtended by the first sector at the equant rather than at the deferent center. For small d a good approximation to the former angle is

\[ 90^\circ + \arcsin\frac{3d}{2R}. \]

For a planetary deferent velocity sector the angle subtended by the first sector at the equant is approximately that given
by expression (20) above.

For the tabulated values of the epicycle distance and velocity sectors the expressions

\[
(22) \quad 90^\circ + \arcsin \frac{r}{2(R+d)},
\]

and

\[
(23) \quad 90^\circ + \arcsin \frac{r}{R+d}
\]

respectively have evidently been used by the original computer of the zij, where \( r \) is the epicycle radius.

In the zij itself the "equation" also is tabulated, that is, the variation in the results when \( R+d \) is replaced by \( R-d \). Something of the sort is necessary to take account of the variation in the distance from earth to epicycle center, a variation which causes small changes in the size of the sectors. For intermediate positions between the extremes of \( R+d \) and \( R-d \) an interpolation scheme was used which is not mentioned in our text. It is to this variation, however, which the author has reference in his statement on f. 30v:1.

Independent computations using Kāshī's parameters in the expressions above have resulted in a verification of all the entries in the text's table of sectors except for some involving Mercury and the moon, which, having special models, demand special treatment.

For Mercury's deferent distance sectors, the last column in the Almagest table of Mercury's equations ([43], ed. of Halma, vol.11, p.316) measures the variation in the maximum size of the equation due to the epicycle as compared with its mean value. Hence any point at which this function vanishes marks a position at which the epicycle is at mean distance, i.e. the initial point of the second deferent distance sector.

Interpolation in this table yields a zero when the argument is 67°13', a result reasonably close to our tabular value of 67°44'.

As for the deferent velocity sectors of Mercury, (19) above is applied as though it were valid for a non-circular as well as a circular deferent. Expressions (22) and (23) are used for the epicycle sectors of both Mercury and the moon.

Kāshī's definition of the initial point of the second lunar deferent distance sector is evidently taken to be the value of the double elongation at which the epicycle center will be at mean distance from the center of the universe. This is

\[
\arccos \frac{d}{2R} = \arccos \left( \frac{1019}{24941} \right) = 84°2^\circ,
\]

as required by the table.

In the Almagest table of lunar equations ([43], ed. of Halma, vol.11, p.316) the value of the double elongation which gives maximum displacement of the epicyclic apogee (Column 3 in the Almagest table) is 114°. This is our tabular entry for the moon's deferent velocity sector. In fact, however, the lunar epicycle travels on the deferent in such fashion that its angular velocity as viewed from the center of the universe is a constant. Hence the concept of deferent velocity sectors as defined above has no meaning. In some way Kāshī must have associated the 114° with the conditions which identify the mean angular velocity of the epicycle centers of the planets, but how he did so is not clear.

So much for the table. As for the instrument, since all deferents are marked on the plate, it is an easy matter to mark also the boundaries of the deferent sectors, as prescribed in the text (ff. 10r:2, 30v:2).

The epicycles as such do not appear on the instrument,
and to determine the epicycle sector of a planet at a given time recourse must be had to manipulations with the instrument. The author's directions in f. 30v:5 - 31r:2 have reference to a person standing at the periphery of the horizontally placed instrument, opposite the planet's epicycle center, and looking toward the center of the instrument. If the position of the planet is on his right this implies that the true longitude exceeds the adjusted mean longitude (i.e. the longitude of the epicycle center) and that the planet is in either the first or second epicyclic sector. If it is on the left it is in the third or fourth. Compare the distances of the planet and the epicycle center from the center of the universe. If the first exceeds the second the planet is in either the first or fourth epicyclic distance sector. If the reverse is the case it is in either the second or third. The combination of these two criteria suffices to locate the planet's epicyclic distance sector. The author gives no directions for the velocity sectors.

The terms "increasing" (or "increased") and "decreasing" (or "decreased") as used in the last part of this chapter (f. 31r3-9) are explained by Biruni in [4](p.203). "Increasing in computation" means that the equation is positive. The author is here referring to the epicycle sectors only. "Increased in magnitude" refers to the celestial object's apparent size, which varies inversely with its distance. When it is in the second or third epicyclic sector its apparent size will be increased over its mean value, hence the term.

34. Prediction of Lunar Eclipses (f. 31r:11 - 33r:3)

A lunar eclipse occurs whenever the moon enters the shadow cast by the earth. If for any time during the eclipse the moon is completely immersed in the shadow the eclipse is said to be total, otherwise it is partial. Our author seeks to describe eclipses to the extent of determining the times of (a) first contact of moon and shadow, (b) first totality, if the eclipse is total, (c) the middle of the eclipse, (d) the beginning of clearance, i.e. the end of totality, and (e) complete clearance. For a partial eclipse the magnitude also is desired.

Since the depth and duration of an eclipse is determined by how closely the broken line sun-earth-moon approaches straightness, it is clear that a necessary condition for a lunar eclipse is that sun and moon be in opposition, i.e. have longitudes differing by 180°. The condition is not sufficient, however, for the moon does not travel on the ecliptic, but along a second great circle which intersects the ecliptic at an angle of about five degrees. This circle rotates slowly westward so that the nodes, the points of intersection between the two circles, have a motion of about nineteen degrees per year. The moon's distance from the ecliptic, its latitude, is the element which determines whether or not an eclipse will occur at a given opposition.

Our author makes several simplifying assumptions. For one thing he regards the earth-moon distance as a constant, hence the apparent size of the lunar disk as viewed from the earth is also constant. The same sort of assumption for the earth-sun distance implies that the width of the shadow cone where it is cut by the moon is likewise a constant. These simplifications have the effect of making an essentially three-dimensional problem two dimensional, for now we can confine our attention to the surface of the sphere, concentric with the earth, in which the moon's center moves. As a final simplification, let Figure 16 represent to some scale a small
portion of this spherical surface, with 0 marking a point on the ecliptic AB at which moon and sun are in opposition. The path of the moon's center is MD and its apparent radius is MN. The intersection of the earth's shadow and the sphere at the time of opposition is represented by the circle ABC. OD is perpendicular to DM, and since the latter makes with the ecliptic an angle not greater than five degrees, the length of OD is a fair approximation to the moon's latitude at the time of conjunction. If the moon is tangent to the shadow circle at N, then, in the right triangle ODM, OM is the sum of the moon's apparent radius and the shadow radius, and DM is an approximation to the difference between the moon's elongation at first contact and at opposition. DM cannot be regarded as the difference in lunar longitudes between these times, for in the same period the sun itself, hence the earth's shadow, will have made some progress along the ecliptic.

When an eclipse is partial, as in the one illustrated in Figure 16, its magnitude is the length of EC measured in (eclipse) digits, twelfths of the apparent lunar diameter. Figure 17, on the other hand, shows a total eclipse. Here the beginning of totality is of interest, and this occurs when the moon is tangent internally to the shadow circle, as at H. In both cases the approximate middle of the eclipse is marked by the arrival of the moon's center at D, this corresponding to the time of opposition. The ends of totality and of contact are located on the eastward side of OC symmetrically with their beginnings.

35. Lunar Eclipse Limits

With this background we now examine the text of Chapter
II,11, beginning with f. 31v:1, a statement to the effect that a necessary and sufficient condition that an eclipse occur is that the lunar latitude at opposition ($\beta$) be less than the sum of the apparent radii of moon ($r_m$) and shadow ($r_s$). The next sentence, f. 31v:5, looks corrupt, and comparison with the corresponding passage in NS (p.280) confirms our suspicion. It says:

If the lunar latitude at the conjunction is more than sixty-three minutes, (then) undoubtedly its distance from the node will be more than twelve degrees, and so there will be no eclipse. But if it is less than that and greater than twenty-nine minutes, then part of it will be eclipsed.

Although not explicitly stated, it is clear from the figure that a necessary and sufficient condition that totality be attained is that

$$r_s - r_m > \beta. $$

The numbers in the Nuzhah lead to the equations

$$r_s + r_m = 63',
\quad r_s - r_m = 29',$$

whose solution is $r_s = 46'$ and $r_m = 17'$, both numbers being rounded-off Ptolemaic parameters ([43], ed. of Halma, vol.i, p.395).

That the latitude condition for a partial eclipse is equivalent to the statement made in our text about the ecliptic distance from node to opposition may be demonstrated as follows:

Consider the right spherical triangle (Figure 18) formed on the celestial sphere by the node, the moon's center, and the projection of the latter on the ecliptic. The relation

$$\sin \lambda \approx \frac{\beta}{50}$$

subsists, since $\beta$ and $50$ are both small. Putting $\beta = 63'$ we obtain

$$\lambda = \arcsin\left(\frac{63}{300}\right) \approx 12^\circ,$$

as demanded by the text.

We are now in a position to investigate the condition with which Chapter II,11 opens, namely that if the opposition occurs during daylight it must be less than 2$\frac{1}{2}$ hours after sunrise or before sunset. Violation of the condition implies that there will be no possibility of any part of even the longest eclipse taking place while the sun is below the horizon.

An eclipse of maximum duration is one for which first contact takes place when the elongation is sixty-three minutes of arc.
Since the mean rate of elongation is about 13;10,15° - 0;59,00 = 12;11,27° per day, half the length of the longest eclipse is about

\[
\frac{143(24)}{12;11,27} = 2;4 \text{ hours.}
\]

This is doubtless the origin of the number the author gives. It does not appear in the Nuzhah.

36. Lunar Eclipse Markings on the Ruler

A permanent mark (O in Figure 19) is placed on the ruler's edge at a distance of sixty-three parts (sixtieths of the plate radius) from the end. The mark of first totality (O") is put permanently on the ruler at a distance twenty-three parts from the same end. The segment O" will be thirty-four parts in length, equal to the apparent diameter of the moon in minutes. It is divided into twelve equal segments, each one an eclipse digit.

37. Determination of the Lunar Eclipse Times

Suppose now that \( \beta \) for the time \( t \) of the opposition has been computed and is sufficiently small to indicate an eclipse. Set the alidade point of Aries on the ring. Put a mark O on the plate such that the segment DO is of length \( \beta \) measured in divisions of the alidade, taking a sixtieth of the plate radius for each minute of arc. Rotate the alidade through an angle of ninety degrees, and place the ruler so that the lunar eclipse mark on its edge coincides with O, and so that its end touches the edge of the alidade (at \( M \)). The triangle DOM is now a mechanical construction of the triangle having the same letters in Figure 16. Hence the length of DM on the instrument gives the elongation in minutes of arc at the instant of first (or last) contact. Since the rate of elongation is, very crudely, a half degree per hour, "depressing" (cf. Section 3) the length
of DM once and doubling the result (i.e. dividing by a half) will yield the elapsed time from first contact to the middle of the eclipse. This is what the text prescribes. Add the result to t and subtract it from t to get the times of day at which last and first contact take place respectively.

If the eclipse is total, an exactly analogous use is made of the mark of first totality (0") instead of the lunar eclipse mark (O) to construct the triangle O'FD (in Figure 19) congruent with OFD (in Figure 17). From it is obtained DM, the elongation at first totality, which also is converted into time by doubling and depressing. Addition to and subtraction of the result from t gives the times of day of last and first totality respectively.

For a partial eclipse the magnitude is measured very simply by sliding the end (M) of the ruler along the alidade until the ruler occupies the position indicated by the pair of dotted lines on Figure 19. The position of the mark O with respect to the digit scale on the ruler then gives the magnitude in digits directly. The validity of this procedure can be seen from Figure 16, where it will be observed that the eclipse magnitude is, approximately

\[ EC = ED + DC = r_m + r_s - \beta = 63' - \beta, \]

which is the construction prescribed.

38. Parallax in the Altitude Circle (f. 35r:4 - 36r:5)

It is convenient here to break the order of the text and to insert appropriate comment on Chapter II,14, the substance of which is a preliminary to Section 39.

In Figure 20 assume the small circle with center C to be the terrestrial sphere. An observer at E takes the altitude of a celestial body at S, the latter being at a distance SC from the center of the earth. With respect to the observer's horizon, the tangent to the sphere at E, the altitude of S is angle SEF. As reckoned from the center of the earth, however, the altitude is SCD. The difference between these two angles is P, the parallax in the altitude circle. It is clear that P is a function of two variables, the earth-planet distance, and the altitude. For CS constant, P is maximum when the altitude is zero, taking a maximum value of, very nearly
arc \sin \frac{EC}{CD}.

The operation described in Chapter II, 14 consists essentially of making a scale representation of the situation set forth above, and our Figure 20 is sufficient supplement to the author's explanation.

It is of interest, however, to put down his parameters. For the three objects mentioned, horizontal parallax at mean distance will be

\[ \begin{align*}
C & \quad \text{arc } \sin 0;1,2 = 0;59,13^\circ \\
O & \quad \text{arc } \sin 0;0,2 = 0;1,55^\circ \\
\varphi & \quad \text{arc } \sin 0;0,4 = 0;3,50^\circ,
\end{align*} \]

where mean distance has been taken as 1,0;0 for the sun and moon and as 1,0;0/2 for Venus, as prescribed in f. 35r:13.

Comparison of these numbers with corresponding entries in a table of parallaxes on f. 185v of Kashi's zij [23] show as close a correlation as can be expected. Horizontal lunar parallax at mean distance is there given as 0;59,59°. For the sun it is 0;2,15°, and this accords with our result of 0;1,55°, since the author says vaguely (f. 35r:9) that for the sun CE should be "a little more" than the two minutes we have taken. For Venus the zij gives 0;4,8°, likewise reasonably close to our value.

For both the sun and Venus the constructions prescribed bear little relation to practical reality. For one thing, the measurement of angles as small as these on an instrument of this sort is out of the question. And for another, the distance of Venus from the earth is based on no observations at all, but on the assumption that the planetary system has been put together by nesting the successive Ptolemaic configurations as closely inside each other as will insure no planet's striking its neighbor.

39. The Table of Parallax Components (f. 33v)

In order that a solar eclipse be visible to an observer it is necessary that the sun and moon have, for the time of the eclipse, very nearly the same celestial coordinates. And since the observer is on the earth's surface, not at its center, it is clear that the coordinates used must be apparent ones and not true coordinates, that is, computed with respect to the observer rather than with respect to the center of the earth. In other words, it is necessary that a correction for parallax be applied to the true coordinates resulting from ordinary determinations.

The adjusted lunar parallax (ikhtilāf-i mangar-i muˈaddal-i gamar) is the difference between the parallax of the sun and moon at a time when both have the same apparent altitude. Our table breaks up this adjusted parallax in the altitude circle into its latitude and longitude components. Each pair of such components is for a conjunction occurring at an integer number of hours before, after, or at the local meridian, and at the initial point of one of the twelve zodiacal signs. Symmetry considerations permit the use of a single column for each pair of signs equidistant from the solstices; thus the second column from the left (on the transcription) serves for both Leo and Gemini, provided that the hours for the former are read from the column at the extreme right, and those for the latter from the left. The latitude components are given in minutes of arc; the longitude entries in minutes of time by which the effect of the parallax will delay or advance the time of apparent conjunction.
Since the rate of elongation is about twelve degrees per day, division of the longitude components by two gives an approximate conversion into minutes of arc.

The top row of entries in the table gives the length of daylight when the sun is in the beginning of the sign named at the head of each respective column.

Our author's statement (f. 14r:7) that the table is from Kāshī's zīj is confirmed by examination of the latter document. It is on f. 164v of [23], where the statement is made that the data have been computed for a place of terrestrial latitude 30°. In a special study [27] it has been shown that the numbers in the table are only crudely correct, and that they were probably lifted by Kāshī from some source unknown to us.

40. Solar Eclipses (f. 33r:5 - 34v:3)

The technique for computing solar eclipses with the aid of the instrument closely resembles that used for lunar eclipses, and can be described with reference to what has preceded. Of course it is now conjunctions, not oppositions, which present possibilities for solar eclipses. Just as before, a permanent mark is put on the edge of the ruler, this time at a distance from the end of thirty-three sixtieths of the plate radius. (Cf. f. 12r:7). This implies that the (apparent) lunar latitude at conjunction must be less than thirty-three minutes, a condition which the text gives explicitly in f. 34r:5. The same limit is given in the Almagest ([43] v1,5). The condition also implies that

\[ r_m + r_\varphi = 33' \]

where \( r_\varphi \) denotes the apparent radius of the solar disk. Since we have already taken \( r_m \) to be about seventeen minutes, it follows that the solar and lunar disks are assumed to be of about the same size. Hence even if a solar eclipse is total, the duration of totality will be very short, and in fact no mark of first totality is prescribed. The portion of the ruler edge from the solar eclipse mark to the end is to be divided into twelve equal parts for the solar eclipse digits (f. 13r:2).

One complication which does not enter in the case of a lunar eclipse is the fact that for a solar eclipse the geographical location of the observer is involved. Hence the true coordinates are to be corrected for parallax by use of the table on f. 33v. The effect of parallax being always to depress the apparent position beneath the true one, it follows that if the conjunction occurs in the forenoon the (longitudinal) correction will be subtracted from the time of true conjunction, whereas it is added in the case of an afternoon conjunction.

For the same reason the latitude correction is subtracted algebraically from the true latitude, north being taken as positive. Once the corrections have been made, the time from first contact to the middle of the eclipse and the magnitude of the eclipse are computed on the instrument just as they are for a lunar eclipse.

The parallax also acts to asymmetrize the necessary condition for a solar eclipse. Its effect is always to pull a celestial object down along a vertical circle from its true position. Hence when the true moon is north of the ecliptic, the allowable distance between node and conjunction is greater than when it is south. Thus the chapter on solar eclipses in the Nuzha (NS, p.281) begins with the statement that if the conjunction is after the ascending node and before the descending node, i.e. if the moon has north latitude, the critical distance is
sixteen degrees; if the moon has south latitude the distance is seven degrees.

The corresponding passage in our text, f. 33r:7-12, doubtless said the same thing in the original composition. However, it was garbled by the copyist who wrote twice the line and a half enclosed in braces in the translation. His mistake was made easier by the fact that the words for distance (bugd) and after (ba'ad) are written the same unless diacritical marks are used.

In his zīj ([23], f. 85r) Kāshī gives the same necessary condition as is found in our text and in the Nuzhah. He qualifies it by saying that it applies to localities in the third and fourth climates. A criterion which holds for all inhabited localities is, he says, that if the lunar latitude is north, the distance from node to conjunction shall be less than eighteen degrees; if south the distance shall be less than nine degrees. He does not derive this condition in the zīj, and it may very well be rounded off from Ptolemy's 17°41' and 8°22' arrived at in the Almagest ([43], vi, 5).

41. The Solar Mean Longitude at Equinox (f. 34v:4 - 35r:3)

In Islamic astrology the instant at which the sun crosses the vernal equinoctial point is called the year-transfer (Arabic tahwil al-sinah, or simply tahwil, cf. [4], p.150) and was considered to be of great significance. Even to the present in Iran the situation of the individual at the moment of transfer is supposed to affect his destiny throughout the coming year.

Chapter II,13 in our text explains a method for determining the time of transfer by use of the equatorium. The same problem is treated much more elaborately in the Khāgānī Zīj ([23], ff. 90r - 91r). It can be formulated as the inverse of a more common problem, the determination of a planetary true longitude (λ) as a function of its mean longitude (λ̄), the latter being a linear function of time. Now we have the solar true longitude given, at the equinoctial point (λ = 0°), and we seek the corresponding λ̄. Once this is determined the corresponding time can be inferred from the mean motion table.

The solutions in the zīj are set up in terms of the mean and true centers, λ̄ and λ, but the difference is trivial, since addition of the apsidal longitude to a center converts the latter into a longitude. (Cf. p.108.)

Two apparently alternative methods are given in the zīj for obtaining λ̄ from λ. The first says, find from the table of solar equations (e) the equation whose argument is the given λ. Add the result to λ to obtain a first approximation to λ̄, and repeat the process. That is, put

$$\lambdā_1 = e(\lambda) + \lambda$$,$$
\lambdā_2 = e(\lambdā_1) + \lambda$$,
$$\lambdā_3 = e(\lambdā_2) + \lambda$$,

Then, says Kāshī, λ̄₃ is the desired λ̄. This iterative method for inverting suitable types of functions is very old, and probably of Hindu origin (cf. [1] and [32]).

The second method consists of putting

$$\sin e(\lambda) = \sin e_{\text{max}} \cdot \sin \lambda$$,

where e_{max} = 240,29° is Kāshī's maximum solar equation. Then

$$\lambdā = e(\lambda) + \lambda = \arcsin(\sin e_{\text{max}} \cdot \sin \lambda) + \lambda$$.
THE PLANETARY EQUATORIUM

In another part of the zij, on f. 166v, is a numerical table of the function $L(\lambda_a)$, say,

$$L(\lambda_a) = 2\cdot 0.29 \sin \lambda_a + \lambda_a.$$  

In both cases, therefore, small angles, expressions (24) and (25) are approximately equivalent. Thus the table gives a third method for obtaining $\tilde{\lambda}_a$.

By contrast with these elaborate and approximate techniques, the solution with the instrument is direct and, at least theoretically, precise. One puts the ruler alongside the fictitious center and parallel to the line joining the plate center and Aries $0^\circ$. The point of intersection of the ruler edge with the graduations of the ring gives immediately the mean longitude at which $\lambda = 0^\circ$, the configuration being the same as that for determining $\lambda$ from $\tilde{\lambda}$.

The desired $\tilde{\lambda}$ having been determined, it is easy to find from the table of mean motions a date such that at the noon of that day the solar mean longitude $\tilde{\lambda}_n$ exceeds the equinoctial $\tilde{\lambda}$ just found, whereas on the preceding noon the solar mean longitude is exceeded by the equinoctial $\tilde{\lambda}$. The expression $(\tilde{\lambda}_n - \tilde{\lambda})/\tilde{\lambda}$ then gives the number of hours from the instant of transfer to the later noon, provided that $\tilde{\lambda}$ is the rate of change of $\tilde{\lambda}$ in degrees per hour.

For material on f. 35r:4 - 36r:5 see Section 38 above

42. The Equation of Time (f. 36r:6 - 37r:8)

Chapter II,15 of the text contains no application of the equatorium, and in fact makes no reference to either one of our instruments. The topic it discusses is, however, of at least theoretical interest in the major problem solvable with the equatorium, the determination of planetary true positions. The table of mean motions, on which such determinations are ultimately based, gives mean positions reckoned from some epoch. Local apparent time, reckoned from successive meridian passages of the true sun, differs from mean time because of two facts. These, as the author points out, are the variable angular velocity of the true sun in the ecliptic, and the unequal projections on the celestial equator of equal segments on the ecliptic. The difference is the equation of time.

The first part of the chapter is copied verbatim from the extensive equation of time material in the Khāqānī Zīj ([23], f. 93r). At about f. 36v:10 our author abandons this source, and from there on leans heavily on Appendix 8 (p.311) of NS. The NK version has nothing on the equation of time; the subject is another afterthought of Kāshī, but his presentation is much more satisfactory than that of our text, which slavishly copies his mistakes.

For instance, f. 37r:3-37r:6 in the text is on the conversion from degrees of arc to time, $360^\circ$ being equivalent to twenty-four hours. A degree of arc does correspond to four minutes of time, and a minute of arc to four seconds, but ten minutes of arc corresponds to forty seconds of time, not to a minute, as both the NS and our text have it.

In order that this chapter be useful the author should have provided the user with specific means of determining the equation of time. One way would be to give a numerical table such as that in Kāshī's own Khāqānī Zīj ([23], ff. 126v, 127r). Another way would be to give explicit directions for computing both components of the equation of time. The determination of one component, the solar equation, has already been described,
in Chapter II,6. Nothing has been done in the text, however, about the computation of the right ascensions which make up the second component. In NS (p.311) Kāshī describes a technique with the instrument for solving this spherical trigonometric problem. The methods resemble those used for the determination of the lunar latitude and commented on in Section 25 above.

To clarify the passage f. 36r:11 - 37r:2 we remark that the equation of time \( E \) at any instant \( t \) is the difference between the change of mean solar longitude \( \Delta \xi(t) \) from epoch \( t_0 \) to time \( t \), and the change in right ascension \( \Delta a(t) \) of the true sun from \( t_0 \) to \( t \). Symbolically this is

\[
E(t) = \Delta \xi(t) - \Delta a(t) \\
= \left[ \xi(t) - \xi(t_0) \right] - \left[ a(t) - a(t_0) \right] \\
= \xi(t) - a(t) - \xi(t_0) + a(t_0).
\]

The rule in the text says

\[
E(t) = (\xi(t) + 3457,30^o) - a(t).
\]

Comparison of the two expressions shows that we must have

\[
a(t_0) - \xi(t_0) = 3457,30^o,
\]

that is, the constant to be added to the mean longitude is a number which depends on the epoch of the tables and on the base longitude for which they have been computed. This 3457,30^o, however, has been taken over from NS without change, whereas the mean motion tables of the text have both a different epoch and a different base longitude than those of NS.

43. The Plate of Conjunctions (f. 37r:9 - 38r:11)

The construction of the second instrument is easily understood from the description on ff. 15r-17r, especially when considered in conjunction with the drawing on f. 17v, its modern counterpart on the opposite page, and Figure 21, reproduced from page 287 of NS.

Figure 21. The Plate of Conjunctions, from NS

Of the metrological units mentioned in this passage, the size of the cubit has been discussed in Section 2 above. The
digit is usually taken as one twenty-fourth of a cubit.

As for the operation of the instrument, the device has been designed to predict the time of day at which a conjunction between two planets will occur, given the noon longitudes of the planets for the given day and for the following day. It is further assumed, of course, that their longitudes show one planet in the lead at the first noon, and the other at the second. Regard the variations in longitude as linear throughout the course of the day. Then if \( t \) is the time in hours from the first noon until the conjunction takes place,

\[
t = \frac{24d}{b},
\]

where \( d \) is the difference between the longitudes of the two planets (Kāshī's past distance), and \( b \) is the difference between the daily rates of the two planets (Kāshī's daily motion) for the day in question. Incidentally, the Persian-Arabic term *buht*, standard for the longitudinal speed of a planet, is from the Sanscrit word *bhukti* (cf. [4], p.105).

The turning ruler (f. 17r:12, see also Figure 2) is set so that its edge crosses the "divisions of travel" scale at distance \( b \) from the beginning of the scale. Find the point on the same scale corresponding to \( d \), from there project horizontally to the edge of the turning ruler, thence vertically down to the base of the triangle. It is clear that the resulting point is at the required distance \( t \) from the left vertex of the triangle.

The three sliding scales at the bottom are to convert \( t \) from time measured from noon to time measured from local sunrise or sunset, depending on whether the event occurs during the day or the night respectively. To do this, set the day ruler so that the pivot on the turning ruler is opposite the point on the day ruler corresponding to half the length of daylight for the date and locality in question. Set the head of the night ruler opposite the point on the day ruler corresponding to the number of hours of daylight. Put the head of the next-day ruler opposite the point on the night ruler corresponding to the number of hours of darkness. Then, as the author remarks, the right angle of the triangle will fall opposite the point of the next-day scale marking the hour of noon. Now the place where the vertical line distant \( t \) units from the pivot intersects one of the three slides at the base gives directly the desired hour of day or night.

In all of the late medieval Persian zījes inspected by the editor, many pages are given over to a double-entry table of the function defining \( t \) above (cf. [29], p.162). It is evident that Kāshī invented this simple device to obviate the need for such tables. As such it fulfilled a practical purpose, yielding results of sufficient precision for the problem at hand. The instrument's most serious drawback follows from the fact that the usual daily motion of the planets is of the order of a degree. That of the moon is much larger, averaging over thirteen degrees. This implies that if the conjunction does not involve the moon, the turning ruler would be elevated by so small an angle that the result would be considerably affected by small inaccuracies in the construction.
<table>
<thead>
<tr>
<th>English</th>
<th>Persian</th>
</tr>
</thead>
<tbody>
<tr>
<td>clearance (of an eclipse)</td>
<td>أطلاع</td>
</tr>
<tr>
<td>obliquity</td>
<td>انتقال</td>
</tr>
<tr>
<td>solstices</td>
<td>انتقال</td>
</tr>
<tr>
<td>apogee(s)</td>
<td>اورجات</td>
</tr>
<tr>
<td>mean (motions and positions)</td>
<td>اختلاف</td>
</tr>
<tr>
<td>substitute</td>
<td>بدل</td>
</tr>
<tr>
<td>sign(s) (zodiacal)</td>
<td>اورج (اصطلاحات) اصطلحات</td>
</tr>
<tr>
<td>slow</td>
<td>نزدیک</td>
</tr>
<tr>
<td>distance(s)</td>
<td>نزدیک</td>
</tr>
<tr>
<td>doubled distance</td>
<td>نزدیک (اصطلاحات) اصطلحات</td>
</tr>
<tr>
<td>rate</td>
<td>دیجیت</td>
</tr>
<tr>
<td>compass</td>
<td>اقلام (اصطلاحات) اصطلحات</td>
</tr>
</tbody>
</table>

(Readers who desire to locate in the text one of the words listed below can do so by looking up its English equivalent in the index.)
south
node, lunar
sine
acute (angle)
product
ring
motion(s)
computation
argument
perigee
depression, or
trough
true
ring
Aries
quotient
eccentric
anomaly
compound anomaly
lunar eclipse
wood

calendar or date
complete (of years)
transfer
epicycle
doubling
equation
equation of time
differences
subtraction
intersection
true (celestial)
longitude
halving
succession of the signs
hole, drill hole
table(s)
part(s), or
division(s)
addition

line(s)
angle
right angle
increasing
projection,
excess, lug
tongue
Saturn
Venus
astronomical
handbook
year
complete (or
elapsed) year
speedy, fast
plane
the two inferior
planets, Mercury
and Venus
immersion (of an
eclipse)
immersion (of an
eclipse)
quadrant(s)
retrograde

line of apsides
latitude lines
thread
circle(s)
latitude circle
degree(s)
rotation, or
revolution
cubit
epicyclic apogee
reduced
degree
retrogradation,
or retrograde
elevate (a
sexagesimal), to
numeral(s),
or mark(s)
difference mark
string
alidade
Mercury
magnitude (of a star)
node
mark
mathematics
astronomy
superior (planets)
extravtity, extreme (value), maximum
Persian
excess, or difference
ecliptic, or zodiac
base
perpendicular
gibla, the direction of Mecca
disk
Constantinople

plumbline
day, nychthemeron
yellow copper
sun
brass plate
multiplication
plate of the heavens
longitude, usually terrestrial longitude, but in the text (f.3r5) celestial longitude.
longitudinal
shadow
universe
latitude(s)
bride, (theorem of the)

sine (astrolabe or quadrant)
preserved
axis
circumference
orbit, on f.8r11, but in an astronomical context the word usually denotes any circle of the celestial sphere whose pole is the north pole.

split (of an eclipse)
center(s), or planet(s)
sights
apparent pointer
Mars
fictitious
inclinat or inclined
explicit (years)
triangle
sum
Libra inclination, or declination

THE PLANETARY EQUATORIUM

ruler
peg
path(s) (pl.) مسیرات
Jupiter ascensions, or risings
adjusted equant

equator, celestial divisions
station(s) مقام
true position
stationary, stance
duration (of an eclipse), first totality
tangent
parecliptic
see: مسا مطلع

depress (a sexagesimal), to
heaven(s), مطلقاط (pl.) مقام

deferent(s)

parallel position

Yazdigerd

LIBRARY

DEPARTMENT

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18. Al-Kāshī, Nuzhat al-Hadā'iq, India Office MS. 210 in [48], and referred to in this book as NK.

19. Al-Kāshī, Nuzhat al-Hadā'iq, the Samarqand recension referred to in this book as NS, lithographed in Tehran in 1889 in a single volume with the Miftāh [17].

20. Al-Kāshī, Risālah dar sharh-i ā lat-i rasād. The only extant copy is bound with Leyden Cod. 945 (Warner).

21. Al-Kāshī, Al-Risālat al-muhīṭīyah. Published versions are [36] and [47].

22. Al-Kāshī, Sullam al-samā' = Al-Risālat al-kamāliyah, extant in many manuscript copies, and in a Tehran lithograph edition of 1306 A.H.

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INDEX

References to the text and translation give folio and line, separated by a colon. The corresponding numbers appear along the left-hand edge of each page of the translation. Numbers preceded by an asterisk (*) are references to pages of the commentary.

Acute angle, 16v:11,17r:1,5. 182,231,233.
Addition, 3r:12.
Adjusted, 22r:6-11,22v:8,13. 13r:6,19v:9,12,20r:10,13, 21v:2,9,10,21r:1,4,6,13, 21v:5,12,13,22v:1,3,23r:1, 4,5,7,8,23v:1,3,8,24r:12, 13,24v:1,8-10,13,25v:6,10, 13,26r:8,9,12,13,26v:1,2, 26v:1,3,30v:6,8,31v:12, 32r:2,3,7-10,34v:6,8,10,11, 35r:6,8,12,35v:3,6,7,9,12, 36r:1,3,4; * 11,175,160-184, 190,192,193,195,199,200,204, 208,209,228,230. 38v:9-I3r3órr5.
Agha, M.; * 188.
Albion; * 182.
Alidade, 11v:1,3,7,12r:5,7, 13r:6,19v:9,12,20r:10,13, 20v:2,9,10,21r:1,4,6,13, 21v:5,12,13,22v:1,3,23r:1, 4,5,7,8,23v:1,3,8,24r:12, 13,24v:1,8-10,13,25v:6,10, 13,26r:8,9,12,13,26v:1,2, 26v:1,3,30v:6,8,31v:12, 32r:2,3,7-10,34v:6,8,10,11, 35r:6,8,12,35v:3,6,7,9,12, 36r:1,3,4; * 11,175,160-184, 190,192,193,195,199,200,204, 208,209,228,230. 38v:9-I3r3órr5.
Almagest; * 4,9,169,170,179, 184,200,203,206,211,213, 217,218,220,221,234,236. Altitude, 13r:6,35r:4-6, 35v:2-4,9-13,36r:5; * 13, 256

257

Astrolabe, 5v:12,6r:3,11v:3,8; * 4,180,182. 257
Astronomy, 3r:4.
Axis, 17r:9,11.
Badr al-Dîn; * 5. 257
Base, 16r:1,9,11.
al-Battânî; * 170,186,216,217.
Bayazid, 4r:8; * 9,185.
al-Bîrûnî; * 4,170,222.
Brass, 5v:11,11v:2,15r:3, 16r:7.
Bride (theorem of the), 15r (marginal note).
Calendar, 18r:11.
Center(s), 6r:8,13,7r:2,10, 11,20r:2,4,7,9,11,12,20v:9, 10,12,21r:6,12,21v:4,6,9,12, 22r:8,22v:8,13,23r:1,25r:1, 3,5,7,13,25v:11-13,26r:3,8, 26v:3,13,27r:1-5,28r:10,11, 13,28v:3,30r:7,12,13,30v:1, 6,7,9-11.31r:1,32r:9, 34v:8,35r:7,13,35v:1, center (longitude reckoned from apogee); * 188,196,202,237.
Chain, 13r:4,5.
Chaucer; * 165,166,200.
Chord, 15r (marginal note).
Circle(s), 6r:9,10,12v:1-3.
Circumference, 6r:5. 257
Clearance (of an eclipse), 32v:8,9; * 223.
Climate(s), 14r:7.
Color(s), 6v:7,15v:8,10.
Compass, 12v:6,15r (marginal note), 24r:6,10.
Complete (= elapsed) years, 18v:2,4,6.
Compound anomaly, 13v:1. 257
Conjunction(s), 14r:5,6, 33r:5,7,13,34r:3-5,34v:3, 37r:9,36r:2,5; * 233-236, 241-243.
Computation, 3r:11,3v:8, 31r:4,6; * 13.
Consonance(s), 14r:5,6, 33r:5,7,13,34r:3-5,34v:3, 37r:9,36r:2,5; * 233-236, 241-243.
Constantinople, 18v:10; * 9, 184120012A3120612II, 213, 22vtIr2r5.
Argument, 22v:1,2,5.
Aries, 20v:12,21v:7.
Ascension (right), 36r:11,36v:1, 37r:2.
Decreasing, decrease, 31r:5-7; * 222.

Deferent(s), 7r:2,7v:2,4,10, 11, 8r:12,13, 8v:1,10,10r:2, 20r:10, 21r:5, 24v:5, 27r:9, 27v:3,10,26v:3,6,30v:3; * 11, 167-173,175,179,180, 184,192,195,200-204,206,207, 209-214,216-221. Deferent, oval; * 13,214.

Degree(s), 6v:3,9r:12.

Depress, to, 32r:11; * 166, 229,230.

Depression, 12r:1.

Descriptive geometry; * 207. Deviation; * 202,204.

Diameter, 5v:13, 6r:10,11, diameter, first; * 201,202, 205,207,210, diameter, second; * 201,202,205,207.

Difference, 26v:9,11.

Difference circle(s), 12v:3.

Difference mark(s), 12r:9,11, 23v:10,24r:1; * 11,184,193, 195,208.

Digit (of a cubit); * 242.

Digit(s) (eclipse), 12v:13, 13r:2,3,15v:13,33r:2,34v:2, 3; * 225,228,230,235.

Direct (motion), direct (station), 28r:5,7,29v:7, 12,13,29v:1-3,6-8,13.

Disk, 5v:9,10,6r:8,13,6v:10, 13; * 164,167,170.

Distance(s), 9r:3,4,9, 19v:5, 20r:3,7,8,26v:13,27r:2-5,8, 10,27v:11,12,28r:10-13, 28v:2,4,5,7,11-13,29r:1,2, 5,30r:6,12,30v:1,3,10,13, 31r:1,9,31v:6,33r:8,11, 34r:3,35r:2,12,13,35v:8, 36r:4,37r:10,37v:1,9,38r:4, 7,8.

Distances, planetary, 26v:13; * 12,214,231.

Division(s), 3r:12,3v:8, 7r:13,6v:3,11,14r:1,10r:13, 15v:1,3,5,7,17r:2,18r:4, 19v:6,9,12,13,20r:3,4,9, 20v:1,11,21r:2,22r:1,22v:2, 23r:2,10,23v:10,24r:1,6, 24v:1,2,25r:1,25v:9,10, 26r:1,9,27r:5,6,8,10,12,13, 27v:1,28r:10,13,26v:8,32r:1, 10,33r:1,2,34v:7,11,35r:7, 8,36v:1,37v:1,6-11,38r:3.

Doubled distance = double elongation, 20r:8.

Doubling, 3r:12.

Drill, 6v:10,17r:6,10.

Duration (of an eclipse), see totality, first, 12v:10.

Earth, 3r:9; * 12.

Eccentric, 8r:1,27v:9,10, 30r:7.

Eccentricity; * 168-171,173, 175,179,180,190,211,219.

Eclipse marks; * 11.

Eclipse(s), see lunar eclipse and solar eclipse; * 184.

Ecliptic; * 12,179,198,200- 206,210,223-227,239.

Elevate, elevated, 3lv:12, 32r:1,32v:13; * 166,176.

Elliptical (?), 8r:1.

Elongation, 2Ov:12.

Elongation; * 192,221,224, 227-230,234.

Epicycle, 8r:11,9r (marginal note),10v:2,12r (marginal note),24v:4,8,11,27r:4, 28r:10,13,30r:3,7,10-12, 30v:1; * 169,173,175,180, 184,192,193,195-197,200- 202,204-213,215-222.

Epicyclic apogee, 23r:10,11.

Equant(s), 7r:3,8v:7,21r:3; * 171,173,175,194,196,219.

Equation(s), 3r:13,19v:5, 20r:2,21v:8,10,22r:1,2,4, 25r:12,25v:1,2,3; * 12, 190,195-197,210,218,220-222, 237,239.

Equation of time, 36r:6, 36v:10,11,37r:7; * 13,238- 240.

Equator, 36r:10,12,36v:9; * 239.

Equatorium, * 8,165,173,193, 196,200,236,238,239.

Equatorium, types of; * 11, 13,175,182.

Equinox; * 236-238.

Excess of revolution; * 13.

Excess, 20v:12.

Explicit (years), 18r:12.

Extremity, 24v:5.

Fast, 36r:13.

Fictitious center, 8r:3, 19v:8,20r:2,34v:8; * 190, 238.

Fractions, 6v:3.

Forward (motion), 28r:2,3.

Ghīyāth, 3v:3; * 7,9.

Gnomon; * 4.

Habash al-Ḥāsib; * 170.

Halving, 3r:12.

Abū Hanīfa; * 4.

Heaven(s), 7v:10,6r:2,18r:1.

Hindu; * 237.

Hipparchus; * 170.
INDEX

Hole(s), 6v:9, 11, 13, 17r:6, 7, 10, 18r:6.

Hypotenuse, 17r:8.

Īlkhanī (Ḳīj, or observations); * 168, 170, 184, 188.

Immersion, 32r:12, 32v:3, 6.

Inclination, 23r:9, 11, 23v:3, 5, 11, 24r:2, 7, 24v:3, 5, 7, 11; * 202, 205, 206.

Inclining, or inclined, inclination, immersion, hypotenuse, Ι.

Inferior (planets), l3v:1.

Intercession, (planet(s),) * 193.

Intersection, 26r:13.

Interpolation, * x.

Inversion, 13v:18.

Jummal; * 166.

Jupiter, 7v:9, 9r:1; * 179, 182.

Kamal al-Dīn Mahmūd; * 1.

Kāshān; * 1-3, 10, 11.


Kāshī, death of; * 7.

Khāqānī Ḳīj, 14r:7; * 1, 6, 7, 167, 169, 170, 184, 186, 188, 232, 236, 239.

al-Khwārizmī; * 186.

Latitude point(s), or marks, latitude circle(s), or marks, the two; * 170.


Lunar eclipse(s), 4r:12, 12v:7, 9, 13, 31r:11, 31v:3, 4, 6, 7, 9-11, 12, 15, 16, 32r:5, 13, 32v:7-9, 12, 13, 33r:3, 34r:12, 34v:2; * 1, 12, 188, 222-230.


Lunar eclipse(s), 4r:12, 12v:7, 9, 13, 31r:11, 31v:3, 4, 6, 7, 9-11, 12, 15, 16, 32r:5, 13, 32v:7-9, 12, 13, 33r:3, 34r:12, 34v:2; * 1, 12, 188, 222-230.

Magnitude, 31r:7, 9; * 223, 225, 230, 235.

Library, Tehran; * 6, 7.

Masdūd Canon; * 4.

Mathematics, 3r:3.

Mean (motions and positions), 13r:9, 12, 13, 13v:3, 14r:12, 18r:9, 10, 16v:9, 19r:3, 7, 11-13, 19v:1, 2, 5, 7-9, 13, 20r:2, 3, 6, 21r:2, 7, 12, 21v:11, 13, 22r:6, 7, 9, 10, 13, 34v:4, 11-13, 35r:1, 36v:4, 8, 9, 12, 13, 37r:6; * 11-13, 184, 186-190, 192, 193, 195-197, 222, 236-240.

Mercury, 6r:4, 11, 12, 6v:5, 9r:2, 10r:2, 25r:10, 11, 25v:3, 26r:6, 7, 26v:6, 28v:5; * 13, 170-173, 175, 176, 180, 184, 187, 195, 204, 212, 217, 218, 220, 221.

Meshed Shrine Library; * 1, 6, 7.

Miftāḥ al-Asbāb fī 'Ilm al-Zīj; * 6.
INDEX

Miftāḥ (al-Hisāb); * 2,6,7,10.
Model; * 171,173.
Moon, 7r:13,7v:9,8v:2,7,13r:12, 20r:6,8,11,20v:8,12,22r:2, 13,22v:4,27r:10,31r:9,31v:1, 2,4,6,12,35r:7,13; * 12,13, 167,170,171,173,176,182,184, 188,192,193,197-199,202,214, 220-225,227,228,232,233,243.
Motions, planetary,3r:5, 19r:1.
Movement, 3Ov:8,13.
Muʿīn al-Dīn; * 3.
Multiplication, 3r:12,3v:8.
Musā, bani; * 120.

Naṣīr al-Dīn al-Tūsī; * 168.
Negative numbers; * 209.
Next-day ruler; * 242.
Night ruler; * 243.
NK; * 10,11,176,188,239.
Nodal lines; * 179.
Nodes, lunar, 13r:13,22r:13, 22v:9,10,31v:7,33r:8,9,11; * 198,223,235.

Moon, 18v:10,19r:3,5-7,9,10, 36r:8,37r:10,12,37v:4, 38r:4,6.
Norm; * 169-171,175,179,184 191,214.

North, 25r:9,11.
NS; * 10,11,170,171,176,177, 180-183,189,226,235,239, 240.
Numerals, 6v:2; * 166.
Nuzahā; * 2,6,11,165,169,170, 175,180,183,186,188,200, 215,226,228,235,236.
Obliquity, 25v:5,6,8; * 202, 209,210-213.
Observatory; * 5,7,10,168.
Opposite, 21r:12,21v:3, 26v:3.
Opposite point, 7r:3,8v:7, 20r:12; * 173,192,197.
Orbit, 8r:11; * 173,174.
Oval; * 171-173,217.
π; * 5.
Parallax, 14r:5,35r:5,11, 35r:5,10,36r:3; * 13,230- 232,235, parallax, adjusted; * 233.
Parameters; * 168-170,175,186-188,226, parameters,
INDEX

206, 210-213, 215, 216, 226, 233, 236.

Qadızadah; * 4-7.
Qara Yusuf; * 2.
Qiblah (the direction of Mecca), 3r:8; * 7.
Quadrant(s), 24v:6.
Quotient, 29r:15,13.
Qur'an; * 3,164.

Rate, 37r:9,37v:7.
Ratio, 27v:1,2.
Retrogradation, retrograde, 13v:10,11,13,28r:1,3,5,7, 8,29r:7,8,10,11,29v:2,3, 5,7,9,12; * 12,214.
Revolution, 29r:11.
Richard of Wallingford; * 182.

Right (angle), 15r (marginal note), 16r:4, right ascension; * 240.
Ring, 5v:9,6r:2,4,6,9,6v:10, 13,7r:1,9r:13,11v:5,14r:1, 18r:4,19v:6,12,13,20r:9, 20v:1,11,21r:2,21v:11,13, 22r:1,22v:2,23r:2,24v:2, 25r:1,26r:1,9,34v:7,11; * 164,165,167,176-178,181, 183,189,190,192-196,199, 200,208,209,228,238.

Risālat 'amal al-darb...; * 7.
Risālah dar Sākht-i Aṣṭurlāb; * 6.
Risālah fī īstikrāj jaib...; * 6.
Risālah fī mārifat samt al-qiblah; * 7.
al-Risālat al-iqlīminah; * 7.
al-Risālat al-muhītiyāh; * 5.
Risālat al-watar w'al-jaib; * 6.
Rotation, 36r:10,36v:8.
Ruler(s), 6r:7,9r:13,11v:2, 12r:1-3,5,6,12v:3,5,10,13, 13r:4,16r:7,9,10,12,13, 16v:1-3,5,7,8,10,17r:9,11, 12,19v:7,9-11,20r:3,11,13, 20v:8,21r:3,5,21v:4,23r:5, 6,23v:4,9,12,24r:6,10,11, 13,27r:6,27v:12,32r:4-6,8, 33r:1,3,34v:7,9,35v:5,7, 36r:2,4,37r:11,12,37v:1-7, 10,12,38r:1; * 13,180-183, 189-195,217,228,230,234, 235,238.
Ruler, parallel; * 13,183.
Samarqand; * 2-4,7,8,10,11.
Sanskrit; * 242.
Saturn, 7v:9,9r:1; * 179,182, 186.
Sector(s), 3r:7,8v:9,10r:4, 8,10v:1,30r:3-5,10,30v:2, 4,5,11,31r:2-4,7; * 11,12, 21-222.
Semicircle, 9r:10,9v:2,5,8, 11v:10.
Semi-circle, 22r:1,22v:1, 23r:2,24v:2, 25r:1,26r:1,9,34v:7,11; * 164,165,167,176-178,181, 183,189,190,192-196,199, 200,208,209,228,238.

INDEX

Solstices, 32r:2.
South, 25r:5,10.
Speedy, 30r:9.
Square(s), 15r (marginal note). "Squeezing" parameters; * 186, 187.
Stance, 29v:6,7.
Station(s), 3r:6,13v:9,10,13, 14r:2,28r:2,3,7,9,29r:3,7, 8,10-13,29v:1-7,10,30r:1; * 214-217.
Stationary, 28r:6.
String, 13r:7.
Sublime Porte, the, 4v:1.
Substitute, 24r:7-9,12; * 209.
Subtraction, 3r:11.
Succession of the signs, 6r:12,7r:6,20v:2.
Sullam al-Samā'; * 1.
Sultan, 4r:2,8,9.
Sun, 26v:9,11.

Sun, 7r:12,8r:1-3,12r:8,13r:12, 18r:3-5,19r:12,19v:1,5, 20r:3,6,21r:7,12,22r:1,6,9, 27r:8,27v:9,10,30r:3,6,8,9, 31r:9,34v:5,6,35r:9,12, 36r:11,12,36v:9,12,13, 37r:2; * 12,167,170,173, 175,190,191,193,195,196, 219,223-225,227,232,233, 236.
Superior (planets), 8v:1,
INDEX

9r:8,13r:12,19r:13,21r:8, 11,22r:9,22v:7,9,10,23v:3, 24v:4,25r:4,29r:1; * 178, 
179,193,195,196,202,206,213.
Syzygy; * 227.

Tabataba'i; * 1,3,7.
Table(s), 10r:7,9,10,10v:1, 13r:9,10,13v:7,12,29r:4, 
34r:1; * 11,167,168,175,179, 184-189,192,196,206,213,218, 
Tamerlane; * 2.
Tangent, 9v:9,12,10r:1,31v:4.
Text, history of the; * 9.
Thread, 17r:7.
Time, equation of; * 238-240.
Tongue, 6r:5,6,6v:10; * 165, 170,192.
Total (eclipse), 32r:13, 
32v:10; * 223,225,226,228, 
230,235.
Totality (first) see also 
duration, 12v:10,32r:7, 
32v:1,2,4,5,7.
Transfer, year, 34v:4,6,12, 
35r:1,2; * 12,236-238.
Triangle, 15r:5,7-9,15v:11, 
16r:1,4,9,11,16v:7,11,13, 
17r:8.
Trough, 15v:12,16r:1-3,5,6,6, 
16v:1,2,4,12,37r:13.
True, 14r:5,20r:1,6,20v:13, 21r:1,21v:7,9,22r:13, 
35r:4,5,35v:3,9,11,13, 
36r:7,10,36v:2,10,12, 
37r:7.
True position, 19v:13,20r:1, 
2,20v:11,13,21v:6,7,22r:5.
Turning center, 8r:4,5,8v:6; 
* 171,173.
Turning ruler, 17r:12; * 242, 243.
al-Tūsī, see Nasīr al-Dīn 
al-Tūsī.
Ulguh Beg; * 2-7,10,216,217.
Universe, 19v:5,26v:13, 
27r:3,5,26r:10,30r:7,12, 
30v:2,35r:13,35v:1.
Venus, 7v:9,8v:1,9r:2,9v:13, 25r:9,10,25v:2,26r:6, 
26v:5,35r:10,13; * 169, 
170,173,178,180,195,204, 
206,212,232.
Abū al-Wafā'; * 170.
Wood, 5v:12,11v:3,15r:2, 16r:7.
Yazdferd, 13v:3; * 167,165, 
188,189.
Year(s), 13v:3,18r:12,13,

18v:1-6,8,9,36v:6.
Yellow copper, 5v:12.

Hāginī al-Tashīlāt; al-Zīj, 6v:3.

188,196,219,220,236-239,243.
Zīj-ī Ilkhānī, see Ilkhānī 
Zīj.
Zīj-i Khāqānī, see Khāqānī 
Zīj.
Zīj al-Tashīlāt; * 6.
Zodiac, 6v:3.