A Note on two Books of Biruni on Chords

a. Istikhraj al-awtar fi l-da'ira

b. Jam` al-turuq al-sa'ira fi ma`rifat awtar al-da'ira

Abbriviation used in this note :

(C) Cairo, Dar al-Kutub, Mustafa Fadil Riyada 41m, 74b-81a

(L) Leiden Or. 513, 108-129

(M) Istanbul, Murat Molla 1396/4 (52b-72b)

(B) Patna (India), Bankipore 2468, 282b-298b

(R) Rasa'il al-Biruni, Hyderabad (1948)

(H) Hogendijk (4), rearranged version of (R)

(Q) Qorbani (2), Tahrir-e Estekhraj al-Awtar

Abstract[1]

Abu'l-Qasim Qorbani (1290-1380 S.H.[2]/1912-2001), was an author of mathematical textbooks and a historian of Islamic mathematics, in 1976 he wrote a detailed study on Biruni's books on Chords, on basis of (L) & (R) and a version by Ahmed Said al-Dimirdash, published in Cairo 1965 (I shall not refer to Cairo edition, even though Qorbani compared it with other two[3]), he came to the conclusion that (L) & (R) are two different texts on same subject and both written by Biruni, his detailed argument in (Q) has hardly left anything out, except that in the end, he made a wrong opinion as to which is Istikhraj and which is Jam` al-turuq, considering that (L) is " Jam` al-turuq al-sa'ira fi ma`rifat awtar al-da'ira " and (R) is " Istikhraj al-awtar fi l-da'ira ", even though in his argument he states that (R) is written by more mature Biruni and is the more elaborate of the two, but in the end,

despite all of the correct argument he makes, he fails to see that its actually the other way around[4]. Anyway, anyone that wishes to prepare a critical edition of these texts, one day, is going to need (Q), since in his commentary, he has given all the differences between (L) & (R) in his critical apparatus, and I am not going to repeat him. Am just going to prove Qorbani — may God rest him in peace — was right, except for the titles of the works.

1. All the dates in this note is given as A.H./A.D. unless stated otherwise. To prevent any confusion, I have restrictively used a Persian version of Ferdinand Wüstenfeld and Eduard Mahler comparative tables (Wüstenfeld-Mahler'she Vergleichungs-Tabellen der Mohammedanishen und Christlichen Zeitrechnung).

2. S.H. = Solar Hijri calendar, is the Persian civil calendar.

https://en.wikipedia.org/wiki/Solar_Hijri_calendar

3. Qorbani (2) Tahrir (pp. 5-6), knew that this edition is defected like (R), I assume the only reason he used it, was the fact that Dimirdash (p.18) says he had used (M) as one of the manuscript for his edition, and Qorbani hadn't seen (M).

4. Qorbani (2) Tahrir (pp. 1, 6-10), accepts the date 418/1027 at the end of (R) as the year that Istikhraj al-awtar fi l-da'ira was finished, and as a consequence of that acceptance, explain the highlighted part of this sentence :

" وقد سلكت فى استخراج وتر الجزء الواحد فى شرحى لعلل زيج حبش طريقاً آخر. ثم جمعت ذلك الى ما القدماء والمحدثين فى كتاب عملته لحصر الطرق السائرة فى استخراج اوتار الدائرة."

As Biruni referring to his other work, l'Hasar al-turuq al-sa'ira fi ma`rifat awtar al-da'ira (i.e. Jam` al-turuq al-sa'ira fi ma`rifat awtar al-da'ira). And in his last study on Biruni, published in 1995, still held the same opinion (Qorbani (1) Tahqiq, pp. 97-102).

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1. While comparing all the available MSS, that has been made accessible by Dr. Hogendijk (1) — unfortunately no copy of (C) was available, which can actually be an important one[5] — it became obvious to me that (L) & (M) are

identical, neither have colophon[6] and both end alike, the misreading of mathematicians names and other mistakes they make is similar to one another, the titles of all " Sections " in both MSS are identical, (L) is written more clearly and the geometrical figures are better than (M) which is in different handwriting and toward its end (p. 31 onward, according to my page numbering), the titles and parts of the text tend to fade away, whether one is written from another or both have used a same copy is unclear to me and needs closer inspection. These MSS can be divided into : Introduction and three " Chapters " and each chapter divided into several " Sections ".

(L) Facsimile in : A. Qorbani, Tahrir-e Estekhraj al-Awtar, Tehran (1355 S.H./1976) 14-55, Qorbani has also given the MS alphanumeric of one to forty-two, I shall only refer to the page numbers (i.e. 14-55).

(M) PDF provided by Dr. Hogendijk (1), each PDF page contains two folios, I numbered these folios in sequence 1,2,3,4,5 ... and will refer to them as such.

Note : The numbers assigned to each " Section " are added by me, titles between square brackets added by me, three **...** means word(s) is/are illegible to me, no intention have been made to correct the titles or names, they are given as they appear in MSS, the numbers under (L) & (M) are the page numbers that those sections start and end in.

5. King (p.48, 6.2.24), says " *an edition by Mustafa Sidqi* ", dated 1153 H.[/1740]* by editor, then king says :

 " *Both* [i.e. (C) and a photo of (L) kept in that library]* *of these differ from the text of the Bankipore manuscript published in Hyderabad* ".

From Dr. King's remarks we can conclude that (C) is another copy of Istikhraj al-awtar fi l-da'ira, and from the word " Edition " I get it that Sidqi probably had more than one copy at his disposal. So did he used (L) & (M) or had he other copies? That is why I think (C) can be important. My other reason is what Dr. King (p. 112 : D81, 6.2.24) says about Sidqi and his works, which gives me the impression that even if he only had one copy, still his edition could be better than the manuscripts (L) & (M).

6. King (p.48, 6.2.24), date (L) ca. 900 H.[/1494]*

(M)	(L)	Titles
1	15-14	**[مقدمة]**
		[قسمت يكم]
2-1	15	**الدعوي**
2	15	**1.** البرهان عليه لارشميدس فيير
2	16-15	**2.** برهان لابى سعيد محمد بن على الضرير الجرجانى
3-2	16	**3.** برهان ثانى لارشميدس فى الدواير
3	17-16	**4.** برهان لابى الحسن اذرخور ابن اشتاد جشنس
4-3	17	**5.** برهان ثالث لارشميدس فى

ير و وجه بعينه فى

مسائل لايونانين لانقه ان يكون لابلونيوس ترجمة يوحنا بن يوسف

واتفق مثله بعينه

لابى سعيد الشجزى و

ابى على البصرى

بطريقه واحده

[قسمت سوم]

مفروضتين محيطان بزاوية
مفروضةٍ وتساوي مجموعهما

الوتر الى نصف دايرة ادا كان

مجموع قطر الدايرة و وتر التمام معلوماً و كل واحد منهما بانفراده مجهولا لي

تمامها الى نصف الدايرة المعلومة

القطر اذا كان الوتران بمجموعها معلومين ... مجهولين لي

قوسين معلوماتى الوتر من قبلوتر

مجموعهما و معرفة وتر مجموعها من قبل وتر... لابى نصر بن عراق

التفاضل كل واحد ... بذات من غير ان يحتاج احدهما الى

الآخر لي

المجموع كل واحد منهما بذات

		لابى نصر بن عراق فى كتابه
		الموسوم بالمجسطى الشاهي
39	52	**65.** معرفة وتر التفاضل و وتر
		المجموع كل واحد منهما بذات
		بطريق لى شبيه بالذى تقدم
		لابى نصر منصور بن على بن عراق
40-39	54-52	**66.** معرفة وتر المجموع و وتر
		التفاضل لي فى علل زيج حبش
41	54	**67.** استخراج وتر العشر
41	55-54	**[خاتمه]**

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2. The fact that these two MSS are a copy of " Istikhraj al-awtar fi l-da'ira ", and their differences from (B). For convenient I give the sentences from (R), but I have compared them with (B).

I. The first and foremost difference can be found in the " Introduction ", although most of this introduction is repeated in (B), but there are also some important differences; in (L) & (M), Biruni writes :

" اخرالامر ما هو العمدة فى علم الهية اعنى استخراج الاوتار فى الدائرة و نسبة مقاديرها الى مقدار القطر و اجزاء المفروضة له "

This is not mentioned in (B), instead Biruni says :

" و ذلك ان يفعل اذا لم يقنع فى المطلوب بالطريق الموصل اليه دون تضييع

الزمان فى طلب طرق اخر اليه ثم لم يسفر فى آخر الامر عن نتائج هى عمدة علم الـهيئة. فـامّا كـثرة الـطرق فسبب جـمعى ايـاهـا تـدريب المتعلم بـتنوعـهّا ثم اتحادها ...“

Again in the last " Chapter " of (B), which is devoted to the " Chords of Circle ", he starts the introduction of this chapter with these words :

وما لاخفاء بـه ان مـعرفة اوتار قسى الدائرة لـعلم الـهيئة قائمة مقام الطور من المادة فيها تخرج من القوة الـى الفعل و خـواص هذا الشكل يتسوى فى اكثرها سريـان الروح فـى البدن و لنشر الـى ذلك فنقول انه لا بد من ان تكون من اوتار الدائرة واحد معلوما لنستنبط سائرها منه و ننسب مقاديرها اليه.

And ends this introduction with these words :

وقد استبان ان وتر السدس مساوي لنصف القطر فهو اول وتر عرفناه فى الدائرة وهو المنطق من بين سائره.

These sentences are not in (L) & (M).

II. As it has been stated in (Q), the most important part that is an obvious proof that (L) & (M) are not the same text as (B), and (B) is definitely a later writing by Biruni, is the last section. In (L) & (M) Biruni writes :

” فاما وتر الجزء الواحد ... ذلك الى استنباط صنوف الحيل لاستخراجه بالتقريب لا بالتحقيق“

while in (B) Biruni writes :

” وقد سلكت فى استخراج وتر الجزء الواحد فى شرحى لـعلل زيـج حبش طريقاً آخر.“

Biruni refers to the same book (لـعلل زيـج حـبش) in (L) & (M), sections 23 and 66, these two sections are not to be found in (B) the subject of these two sections are discussed in different manner there (see (R) pp. 13-16, 94-95)

and he does not refer to the same book[7]. On the other hand, Biruni (2) Fehrest (no. 4), which he wrote in 427/1036, says that one third of this book (i.e. لــعلل زيــج حــبش) is in 250 folios, so it was incomplete, and he doses not list it among the books he intended to finish, so was the " Proof " in this one third or this can mean he did finish it ? We already know that Habash's works are one of the major sources of Islamic trigonometry[8], so Biruni, probably, had found a solution while studying Habash.

7. In page 15 of (R), also in (P) (fol. 2v), Biruni refers to his book : تــعليلى لــزيــج حــبش , and Qorbani (2) Tahrir (104-107), consider this as لــعلل زيــج حــبش (which is in (L), that Qorbani used as the basis of his commentry). For full title of these books see **VIII.**
8. S. Tekeli, " Habash ", in DSB, Persian translation by Ahmad Birashk.

III. The third and most significant difference between (L) & (M) and (R)[9] is that the former two are written in three " Chapters ", while the later is divided into five " Chapters ".

(M) & (L) : I give the page numbers of (L) and you can find the pages in (M) from the chart I provided.

Chapter One : Introduction (pp. 14-15), The First Theorem (p. 15), and 23 Sections (pp. 15-26).

Chapter Two : The Second Theorem (p. 26), and 9 Sections (pp. 26-30).

Chapter Three : The Last Statement[10] (p. 31), and 33 Sections (pp. 31-54)

(R) : The page numbers are according to the rearranged version in (H).

Chapter One : Introduction (pp. 3-4), The First Theorem (pp.4-6), and 14 Sections (pp. 6-25).

Chapter Two : The Second Theorem (pp.25-27), and 10 Sections (pp. 27-39).

Chapter Three : Completion of the Second Theorem and also the Third Theorem[11] (pp. 39-40), and 6 Sections (pp. 40-45).

Section Four : The Forth Theorem (pp. 45-47), and 25 Sections (pp. 47-84).

Chapter Five : Chords of Circle, in 38 Sections (pp. 84-108 and 224-226).

As it is noted in (Q), the theorems and their proofs in (M) & (L) have been incorporated in (B), with corrections, omissions and additions, what differs most, is that (B) is much better organized, you can hardly find First Proof, Second Proof, Third Proof ... by this or that mathematician or by Me or from Me, instead Biruni has separated these proofs, accordingly to the Chapters devoted to each theorem and the Chapter on chords, and given them in the appropriate Chapter, with extra sources added and more elaborate proofs by himself, without mentioning Me, in most cases.

9. I have used the printed version, since the Sections in (B) differs somewhat and I don't want to cause any confusion.

10. This is shorter version of Chapter 5 in (B), with few proofs and solving of several problems in algebra, astronomy and geometry, which are to be found, mainly, in Chapter 4 of (B). The only proof that Biruni mentions in Chapter 5 of (B) and can also be found in Chapter 3 of (M) & (L), is the proof by his mentor Abu Nasr b. Iraq. The rest of the proofs in Chapter 5 of (B) are all by Biruni.

11. **اتمام هذه الدعوى الثانيه بقسمها الثانى حتىى ثالثه.**

IV. The forth differences is that the الــدعــوي[12] in (L) & (M) is only 4 lines, but in (B) is in 15 lines (see (R), pp.4-6, in 21 lines) with two geometrical figures complementing the text. In general the introduction (despite the part that he repeats from (L) & (M), see **V**) and the الــدعــوي shows a mature and confident man, a much more knowledgeable man than the one who wrote (L) & (M) and this is evident throughout the entire (B) as is stated in (Q).

12. The introduction of the text shows hostility toward Biruni, and the word الــدعــوي can have twofold meaning in Biruni's text (Theorem/Alligation) and I don't know an English equivalent for it, that is why I just gave the Arabic term.

V. Hostility toward Biruni in the introduction of (L) & (M) is obvious — and he repeats this part in (B) maybe to remind himself of the bitter memory of the past — maybe that is why he discreetly uses (فــضول الــهندسـه), which cannot be understood properly[13], he does not mention anything else about the term because he didn't want to bring more trouble upon himself. But in (B) he names Muhammad ibn Zakariyya Razi as the source of the term (فـضول الـهندسـه)[14]. Which again shows his confident and maturity when he wrote (B).

13. H. Suter translation of (L) :

" *so daß du mir wegen der Beschäftigung mit diesen Kapiteln der Geometrie Vorwürfe gemacht hast (?)* ".

14. As it is stated in (Q), on the basis of M. Mohaghegh study, Biruni is referring to Razi's Kitab al-Sirah al-Falsafiyyah, Razi says :

" *With respect to mathematics*[*] *, I acknowledge that I have not consumed my time in trying to master them is deliberate on my part and not due to an incapacity for them. For those who so wish, I have set forth my excuses to the effect that what I have done is correct and not what those designated as philosophers do who consume their lives* **busying themselves with the details of geometry** (i.e. فضول الـهندسه)[**] ".

In the same book Razi name one of his other works as :

فــی عـذل مـن اشـتغل بـفضول الــهندسـة مـن المـوسـومـين بـالـفلسفة (On Blaming Those *Characterized as Philosophers Who Occupy Themselves with What Is Superfluous in Geometry)* "[***].

* So by geometry, Razi means mathematics in general.
** English translation by Butterworth (no. 32); Razi, al-Sirah al-Falsafiyyah (p. 101, Persian translation p. 102); between parentheses added by me.
*** Butterworth (p. 229); Razi, al-Sirah al-Falsafiyyah (p.109, Persian translation p. 92); Mohaghegh, Filsuf-i-Rayy (p. 232).

VI. While comparing (L) & (M), I happen to notice that Biruni does not use the term " Jayb / **جيب** ", at all, in this text (which I find peculiar), but he uses

it frequently in (B) and even uses it in Maqāla fi'l-taḥlīl wa'l-taqṭīʿ le'l-taʿdīl, which Biruni definitely wrote before Istikhraj al-awtar, since he gives proof from it in this book. I double checked (L) & (M), but could not see the term.

VII. As for the Title of these two books, (L) & (M) is listed by Biruni (2) Fehrest (no. 64) of his works and in the introduction of the book the name is mentioned, in-between the sentence. But at the end of (B) the name of the book is referred to as " *l'Hasar al-turuq al-sa'ira fi ma`rifat awtar al-da'ira* ", as I showed in **2-I** Biruni uses the terms of the title of his book, in his sentences to describe as to why it is named as such, beside the term Hasar, not only don't suit the book, is not even used in the entire book, and last Biruni (2) Fehrest (no. 108), list the book among the works he intended to finish and named it " *Jam` al-turuq al-sa'ira fi ma`rifat awtar al-da'ira* ", so Hasar is just an error by one of the scribes who copied the book.

VIII. List of Biruni's own works, that he has given proof from them in (L) & (M), and (B), Biruni does not refer to them explicitly, I give the full name based on his own list (Biruni **(2)** Fehrest), even though he doesn't always give the full name in this one either and one has to find it, in-between his sentences. And these are the ones that I have noticed.

Biruni **(2)** Fehrest)no. 1), (L) p. 25 :

المَسائِلَ المُفيدةَ و الجواباتِ السَّدِيدَة فى عِلَلَ لزيجِ الخوارزمى

Biruni **(2)** Fehrest)no. 2), (L) p. 45 :

إبطالَ البُهتانِ بإِيرادِ البُرهانِ على أَعمالِ الخوارزمى فِى زيجِهِ

Biruni **(2)** Fehrest)no. 4), (L) p. 25 :

تكميلِ زيجِ حَبَش بالعِلَلِ و تهذيبِ أعماله مِن الزَّلَلِ

Biruni **(2)** Fehrest)no. 11), (L) p. 41 :

مقالة فى التحليل و التقطيعِ للتَّعديلِ الشَّمْس فى زيجِ حبش

Biruni **(2)** Fehrest)no. 56), (L) p. 44 :

مَقالةً فِى دَلالةِ الآثارِ العلويّةِ على الأحداثِ السُفليَّةِ

List of Biruni's own works, that he has given proof from them in (B), I give the page numbers from (R).

Biruni **(2)** Fehrest)no. 1), (R) p. 21 :

المَسائلَ المُفيدةَ و الجواباتِ السَّدِيدَة فى عِلَلَ لزيجِ الخُوارزمى

Biruni **(2)** Fehrest)no. 2), (R) p. 78 :

إبطالَ البُهتانِ بإيرادِ البُرهانِ على أَعمالِ الخُوارزمى فِى زيجِهِ

Biruni **(2)** Fehrest)no. 4), (R) p. 226 :

تكميلِ زيجِ حَبَش بالعِلَلِ و تهذيبِ أَعماله مِن الزَّلَلِ

Biruni **(2)** Fehrest)no. 11), (R) p. 15 :

مقالة فى التحليل و التقطيعِ للتَّعديلِ الشَّمْس فى زيجِ حبش

Biruni **(2)** Fehrest)no. 21), (R) p. 9 :

كتابَ تِصْحِيفِ المَنْقُول مِنَ العَرْضِ و الطُّول

Biruni **(2)** Fehrest)no. 56), (R) p. 79 :

مَقالةً فِى دَلالةِ الآثارِ العلويّةِ على الأحداثِ السُفليَّةِ

Biruni **(2)** Fehrest)no. 88), (R) p. 14 :

كتاب تَحصِيلَ الرَاحَةِ بِتصحيحِ المَسَاحةِ

Biruni **(2)** Fehrest)no. 101), (R) p. 69 :

التَّطبيقُ الى تَحقيقِ حَرَكَةِ الشَّمسِ

Biruni might have used one or more of these works, more than once, but I only gave one page as sample and as it is obvious, three of the works in (B) are not in (L) & (M).

3. Approximate dating of the Books.

I. *Istikhraj al-awtar fi l-da'ira* : The book has no dedication, in this book one is faced with young Biruni that has been attacked by some elder scientist, who accuses Biruni of writing on unworthy subjects with unnecessary details and calls him فضول الهندسه (see **2-V**) and this book is Biruni's reply to that person and writing the book in exile reminds him of his colleagues and soothe his soul. In 385/995, as Biruni puts it :

" *disturbances broke out between the two lords of Khwarizm, and the work had to be stopped because I had to take shelter, and I then sought safety by emigrating from the fatherland. After I had barely settled down for few years ...*"[15].

Lamented Prof. Kennedy says Biruni went to Khurasan and Ray[16], during his forced eviction from his homeland, but was back in Khwarazm in 387/997[17], than he must have written the book between these dates. The other documents that support this idea is that Biruni[18] states that Khujandi had orally informed him of a discrepancy in the observation he made in 384/994, in Ray, obviously Khujandi must have found out later, and there is no evidence that Biruni ever visited Ray later in his life. Also, Biruni must have gathered his information for his other book Maqālīd — which, in my opinion, was written shortly after Biruni was rescued from these devastating time[19] — when he visited Khujandi and Kushyar. Another passage that Sachau refers too and I find relevant to Istikhraj, is what Biruni says :

" *For when I was separated from the court of His Highness, and was bereft of the happiness of the royal service, I met a man in Rai who was counted among the learned astronomers [i.e. astrologers]*[20] *... But then the man became very haughty, and treated me slightingly, though he was inferior to me in all his knowledge. He told me my theory was a lie, and behaved very rudely to me, being very lengthy about the differences between us in wealth and poverty, which changes subjects for glory into subject for blame. For at that time I was in a miserable condition, tried (troubled) on all sides;*

afterwards, however, when my troubles had subsided (ceased) to some extant, he chose to behave in a friendly way toward me. ”[21] ,

One can easily find the similarity of this “ *miserable condition* ” in his introduction to *Istikhraj al-awtar*, which makes me think that this is the person who was harassing Biruni. The last thing I noticed is that Biruni does not use the honorific title for Abu Nasr b. `Iraq, in this book, which he uses in most of his work, maybe Biruni was angry with him (due to his “ *miserable condition* ”, as he states), blaming Abu Nasr b. `Iraq's royal family for the situation he was left in.

Conclusion : I don't think this book is a result of a pre-planned study, it is a spontaneous work, written out of rage and frustration, due to the devastating time he was going through[22] and then finding himself   alongside some nobody that calls him and his works worthless, made him pen this paper. Biruni must have written it very early at his arrival in Ray, since in Chronology he says after a while that his situation got better[23], that guy chose to behave in a friendly manner, that is the reason that makes me consider the fact that he has written it before meeting Khujandi and Kushyar. So I reckon Biruni stayed in Ray less than two years, and wrote *Istikhraj* in or around 386/996[24], before meeting Khujandi and Kushyar, while at exile in Ray. I can conclude that this book is the oldest surviving “ complete ”[25] text from Biruni, that we possess now.

15. Kennedy (1), pers. trans. by H. Masoumi-Hamedani; Biruni (6) Taḥdīd (109; eng. trans. 78; pers. trans. 83)
16. Dr. Azkaei (18-19; eng. trans. 3), says 385-388/995-998; but since Biruni give detail of his observation in 387 at Kath, and there is no record of Biruni ever visiting Ray for the rest of his life, I personally agree with Kennedy (1).
17. Kennedy (1); Biruni (6) Taḥdīd (250; eng. trans. 215; pers. trans. 218), Biruni says Khwarazm (i.e. Kath).
18. Abdollahzadeh and Namatov (58); Kennedy (1); Biruni (6) Taḥdīd (101 ff.; eng. trans. 77 ff.; pers. trans. 76 ff.).
19. Biruni (4) Maqālīd, dedicated this work to Marzobān b. Rostam b. Sharvin, who was

the ruler of small independent principality of Perīm or Ferīm, in Ṭabarestān (see Casanova, passim); Debarnot (Biruni (4) Maqālīd, fr. trans. p. 88, note 2) hesitantly says it was written before 387/997; Qorbani (1) Tahqiq (p. 199), says the book was written between 385-390/995-99; Kennedy (2) writes :

" *it was written a year or more after 384/994* " or " *So the Maqalid was produced within the decade beginning with 384/994* ";

EIr. (Madelung) :

" *Marzobān ibn Rostam ibn Sharvin was in power when Bīrūnī visited him ca. 389/999* ";

Madelung source is unclear to me. According to Clifford E. Bosworth (p. 83), Marzobān b. Rostam b. Sharvin died in 396/1005. In my opinion Maqālīd, was written between 387-390/997-1000, and it was prince Marzobān who helped Biruni and brought him back to his noble court, where Biruni truly belonged (see note 22, below).

20. Between the square brackets, added by me.

21. Sachau (Biruni (1) Ātār, XXIII., ed. and eng. trans. 338, pers. trans. 542), though Sachua can't conclude what Biruni exactly meant; see also Kennedy (1).

22. Forouzanfar (p. 188); Biruni (4) Maqālīd (Persian translation of Biruni's introduction in : Qorbani (1) Tahqiq, p. 204). Biruni's short introduction to this book, is not an orthodox dedication of a book to a royal patron, it is a very sincere and humble introduction, full of passionate gratitude of a man, with wounded soul, to another man, that has rescued him from his misery. The humanitarian gesture of prince Marzobān had such an immense impact on Biruni, that made Biruni feel eternally indebted to prince Marzobān. This introduction also tells us how devastating these few years must have been for Biruni. That is what make me consider that prince Marzobān is the person who saved Biruni from his miserable condition. But how prince Marzobān found Biruni is obscure, according to the historians :

" *Faḵr-al-Dawla reigned in Ray till his death in Šaʿbān 387/October-November 997* ".

(EIr. "BUYIDS"), Ray was subsequently under the rule of Bavandids or Buyids and prince Marzobān might have visited Ray after Faḵr-al-Dawla death, due to political reasons, since Faḵr-al-Dawla successor was his 4 years old son, prince Marzobān was also related to Faḵr-al-Dawla's wife, Sayyeda Khatun (EIr. "ĀL-E BĀVAND"), but Sayyeda seized the reins of government and remained the actual ruler of the principality until her death in 405/1014 (EIr. "BUYIDS"). Casanova (p. 123) states that Marzobān was Sayyeda brother. So prince Marzobān had several reasons to visit Ray in 387/997 and maybe that is how he found Biruni.

23. Abdollahzadeh and Namatov (pp. 33-34), refer to a passage in Biruni's Mineralogy, which is probably when Biruni's troubles had subsided to some extant :

" *I had a friend in Rayy who bailed from Isfahan. He invited me to his house. I was surprised to see that all the utensils in his house were made from porcelain. I was amazed at his magnificent taste. The cups, saucers, pickle jars, salt-cellars, plates, goblets, pots, glasses, even water-spouts, crays, basins of the bathroom, chandelliers, and articles of light, in fact, all the articles of this class, were made of porcelain.*"

Biruni (3) Jamāher (370, eng. trans. 195). But unfortunately he does not give any date, and obviously, someone that pays attention to such things is not in such a miserable condition.

24. Bulgakov has written a detailed study on the life and works of Biruni (P.G. Bulgakov, Zhizn u trudy Beruni, Tashkent 1973), which I am incapable to use, but Abdollahzadeh and Namatov (p. 33) say that Bulgakov, based on good grounds, proves that Biruni was at Ray in 386/996.

25. Since in Section 45 of (L) & (M), Biruni refers to his Maqāla fi'l-taḥlīl wa'l-taqṭīʿ le'l-taʿdīl, which we posses most part of it. See Qorbani (2) Tahrir (p. 212) and Hogendijk (3) Rearranging (pp. 148, 150-152).

II. *Jam` al-turuq al-sa'ira fi ma`rifat awtar al-da'ira* : I have no answer for the date 418/1027 at the end of (R), perhaps those who will edit these two text critically one day will find an answer by inspecting (B) in person[25]. So I just list the facts, that I know, till that day :

a. I proved in **2-I** that Biruni has used the terms of his book's title in the text, so there is no doubt about the title.

b. Biruni (2) Fehrest (no. 108), himself in 427/1035 list it among works in progress.

c. The scribe at the beginning of the text, give the title as " *Istikhraj al-awtar fi l-da'ira* ", but at the end " *l'Hasar al-turuq al-sa'ira fi ma`rifat awtar al-da'ira* ", which incidentally, both are wrong.

d. The ending of the text makes no sense, Biruni says :

" وقد سلكت فى استخراج وتر الجزء الواحد فى شرحى لعلل زيج حبش طريقاً اَخر.“

But unlike the rest of the text, does not provide any proof, Biruni knew, better than anyone else, that this is an importent discovery and would have never end his book like that.[26]

e. None of Biruni's work, that I have studied, give any date at the end, as to when Biruni had finished the work, so the date at the end of this text is an exception.

f. The collection that contain this text, has fallen apart, at least once, and have been rearranged more than once[27], so its very probable that some of the leaves are lost for good or are still undiscovered.[28]

g. Qorbani — he knew the correct page ordering of (R) — who studied this text critically, believed that the printed text of *Istikhraj al-awtar fi l-da'ira* still has lacuna.

25. Lorch and Folkerts (p.62) : " *Yet another source al-Biruni's Istikhraj al-awtar, which has recently been studied by Dr. Abdul-Latiff of Amman, who concludes that there are two versions of the text, long and short, both by Biruni.*[*]

* Private communication from Dr. Abdul-Latif, quoting his 1987 publication[**] ”.

** Lorch and Folkerts (p.78) *: “Ali I. Abdul-Latif, Muadalat Heron ibra l-usur, Journal of the Institute of Arabic Manuscript, 31 (1987) 59-145 ”.* I have not seen Dr. Abdul-Latiff article, but he might have reached a conclusion by now, or even edited one or both works?

26. Fortunately, we do possess Biruni's proof in his other work, see : Biruni (5) Qānūn (pp. 297-302); C. Schoy, Die trigonometrischen (26-30); Qorbani (1) Tahqiq (pp. 307-316).

27. Hogendijk (3) Rearranging. Perhaps if Dr. Hogendijk had the chance to see the collection in person, we would have had better understanding of the collection.

28. In page 108 of (R), the publisher says in the margin :

" من هنا الى عدة صفحات اغتشاش فى اوراق الكتاب كما يظهر »من بيان كاتب اصل النسخة« فتأَمل. "

And taking into consideration Dr. Hogendijk's remarks (Hogendijk (3) Rearranging) about the " Gatherings " and how the number of each treatise and their order of copying, don't actually match, the treatises 1 to 30 were copied in 632/1234, and 31 to 40 were copied between 631-632/1233-1234, in no particular order. I tried to make a logic out of the ordering of these treatises, but couldn't find one in whole, they are not ordered by subject or authors (except for Abu Nasr b. Iraq's treatises) properly, and all of this makes me think that the scribe had a disorganized collection at his disposal and tried his best to reorganize them.

Conclusion : Since after 427/1035, that Biruni wrote a list of his own work, till the end of his life, he stayed at Ghaznavids Court, and there is no dedication in the book, than he might have written it in later stages of his life, just like Ketāb al-ṣaydana. But there are no means of determination.

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(B) Patna (India), Bankipore 2468, 282b-298b, scan provided by Dr Hogendijk, available at Hogendijk (1).

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(H) See Hogendijk (4).

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