A UNIQUE AND UNKNOWN BOOK OF AL-BERUNI

Ghurrat-uz-Zijat OR *Karana Tilaka*

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IT would be a great surprise to the learned world, especially to the orientalists, if I say that I am going to reveal to them the contents of a book which was considered to have been lost long ago. From the writings of Abu Rayhan Muhammad al-Beruni it was known that a very valuable book on Indian Astronomy, named *Karana Tilaka*, existed at the beginning of the eleventh century A.D. and that it was very popular throughout India in those days. Beruni gave numerous quotations from this book in his various works, but nowhere did he specifically mention that he had completed an Arabic translation of the whole book. Shankara Balkrishna Dixit had also declared in his well-known book on the History of Indian Astronomy that *Karana Tilaka* was no longer in existence anywhere in the world. But now we find that a marvellous Arabic translation of this book made by Beruni himself exists in the library of Dargah of Pir Muhammad Shah, Ahmadabad under the name of *Ghurrat-uz-Zijat*. I consider myself most fortunate that I am placing this important book before the public for the first time.

Beruni has mentioned the name of *Karana Tilaka* or *Ghurrat-uz-Zijat* at many places and has given ample quotations from this work. For instance, the following references may be seen:–

(1) كتاب تحقيق ما للهند Hyd. Ed., p. 121 (1.7), p. 266 (1.16), p. 289 (1.10), p. 346 (1.18), p. 384 (1.14), p. 392 (1.17), p. 410 (1.12) p. 419 (1.10), p. 420 (1.2), p. 511 (1.1 & 19), p. 513 (1.20)

(2) القانون المسعودى Hyd. Ed., p. 973 (1.7), p. 1313 (1.15 & 20).

(3) تمهيد المستقر Hyd. Ed., p. 27 (1.18), p. 32 (1.4).

(4) افراد المقال Hyd. Ed., p. 107 (1.1), p. 136 (1.8), 152 (1.7)

So far as the Arabic manuscript of *Ghurrat-uz-Zijat* is concerned, I think it was first referred to by Dr. Muhammad Nazim in his well-known work "The Life and Times of Sultan Mahmud of Ghazna" published in 1931 by the Cambridge University Press. On page 55 of this book is mentioned the date on which Sultan Mahmud and Khan Yousuf Qadir Khan met near Samarkand, and a footnote says, "This date is given by al-Beruni in his unique and hitherto known work named *Ghurrat-uz-Zijat*" Again, in appendix O at page 239 of the same book a further clarification is given,

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Ghurrat-uz-Zijat being a translation into Arabic of the Sanskrit *Karana Tilaka* of Vijaya Nanda, son of Jayananda of Benares. (Pir Muhammad Shah's Dargah Library, Ahamadabad).

From the above-mentioned references it is clear that the manuscript of *Ghurrat-uz-Zijat* kept in Pir Muhammad Shah's Dargah Library is a very authentic and valuable document, especially because its original Sanskrit book *Karana Tilaka* has definitely been lost, and I have not been able to trace it anywhere. The real importance of *Karana Tilaka* lies in the fact that it is a missing link between *Karana Khandakhadyaka* of Brahmagupta and *Karana Kuthala* of Bhaskaracharya. Shankara Balkrishna Dixit was very anxious to trace out a Karana based wholly on modern Suryasiddhanta so that its antiquity could be proved, but unfortunately he could not find it out. Had he been alive today, he would have been extremely happy to see that his dream has taken a real shape in the form of the Arabic translation of *Karana Tilaka*, which is entirely based on modern *Suryasiddhanta*. And now we can say without any doubt that the so called modern *Suryasiddhanta* is not "modern" at all, it is much more ancient than the 10th century A.D.

Another important and interesting fact about *Karana Tilaka* is that, according to it, the revolutions of Chandrochcha and Rahu in a Chaturyuga are, respectively, 488211 and 232234, while these revolutions, according to the old Suryasiddhanta, are 488219 and 232226 and, according to the modern Suryasiddhanta, are 488203 and 232238. By comparing the above-mentioned three different versions, we see clearly that *Karana Tilaka* is a connecting link between the old and the modern Suryasiddhantas. It will not be out of place to mention here that Makaranda, the author of the famous *Makaranda Sarani* had further modified these revolutions to 488199 and 232242, respectively, in order to bring the calculated values closer to the observed conditions of solar and lunar eclipses. It will be seen that the Chandrochcha revolutions are being gradually decreased while the Rahu revolutions are being gradually increased. All this is being done to enable one to calculate the accurate time of a solar and lunar eclipse, but unfortunately the error has not been eliminated upto this time. As a matter of fact the main reason for this error does not lie in the number of revolutions of Chandrochcha and Rahu, but it lies in the basic assumption that at the beginning of Kaliyuga all the mean planets were on the first point of Aries. Actually the mean planets were not exactly on the first point of Aries at the beginning of Kaliyuga; they were at various distant places somewhere near the first point of Aries. So if we want to rectify our calculations, we should modify our basic assumption and not go on changing the number of revolutions.

Apart from the above-mentioned facts, there are many other peculiarities of *Karana Tilaka* which will be described side by side with the translation of that book. The original Arabic manuscript of *Ghurrat-uz-Zijat* consists of single articles arranged serially, but I have divided the whole text into fourteen chapters and have also suggested an appropriate heading for each chapter. I have not changed the order of the original text except at one or two places where it was considered most essential. It appears that the original Sanskrit manuscript possessed by Beruni was full of mistakes of the scribe and the same mistakes have consequently crept into its Arabic translation. The

Arabic scribe of the existing manuscript has made many mistakes in his turn. Also there are many mistakes in the solution of the numerical examples which are due to careless calculations most probably made by one of his disciples, because very often he used to get such tasks done by his pupils. I have been able to correct all these and other mistakes in the text and have explained such corrections fully in the commentary on each article.

The following is the list of contents of *Ghurrat-uz-Zijat* or *Karana Tilaka* and its first two articles with translation and commentary.

بسم الله الرحمن الرحيم

وبه نستعين

زيج بجيانند البانارسى الذى سمّاه كرَنَ تلكَ و معناه غُرّة الزيجات

فهرس الموضوعات

الباب الثانی عشر ـ فی معرفة اجتماع الکواکب بالثوابت .
الباب الثالث عشر ـ فی معرفة رؤیة الهلال (۱) معرفة رؤیة الهلال (۲) معرفة ما مضی من القمر .
الباب الرابع عشر ـ فی معرفة بتیپات و بیدهرت .

الباب الاول

فی استخراج الأصل من التواریخ المختلفة .

(۱) سبب تألیف الکتاب : ـ

زیج بجانند الپاتارسی الذی سماه کرن تلك و معناه غرة الزیجات ـ قال الاستاذ ابو الریحان محمد بن احمد البیرونی کنت وجدت فی الهند زیجا مختصرا صغیرا عمله بجبانذ بن جیانند و هو احد المفسرین ببلد الپاتارسی المقصود عندهم بالتعظیم من جهة الخلة و سماه غرة الازیاج و احب بعض اصدقائنا نقله الی العربی حرصا ؟ منه علی العلوم فسابقته الی بث الخیر فی اهله و ترجمته علی وجهه الی ان ألحق بعض علله فاطلع علیه انوار البرهان و الی ان تکون ذلك فانی لم ازد فیه غیر الامثلة لتسهل المعرفة و الله المعین علی تحصیل المطالب بمنه ـ .

(۲) استخراج الأصل من التاریخ شك کال : ـ .

و هذا نص الکتاب علی نظام ـ هذا کتاب صغیرا لحجم غزیر الفوائد قریب من الفهم بنیر خواطر الفضلاء المبرزین و یسهّل الطرق علی المبتدئین ، عمله بجانذ عند تمام ۸۸۸ سنة التاریخ الشق الذی یستعمله الهند ـ فمن اراد تقویم الکواکب جامعا فیه الصحة الی السرعة یبسط تاریخ الکتاب کله أیاماً فلیضع شك کال و هو سنو التاریخ المذکور تامة و ینقص منها تاریخ الکتاب و هو ۸۸۸ و یضرب الباقی فی ۱۲ و یزید علی ما اجتمع ما مضی من السنة من الشهور التامة التی اولها چیتر و یضع المبلغ فی مکانین و یضرب اسفلها فی ۹۰۰ و یزید علی المجتمع ٦٦۱ و یقسم الجملة علی ۲۹۲۸۲ فیخرج شهور الکبائس التامة یزیدها علی ما فی الاعلی و ما بقی فهو الماضی من مدة الکبیسة الناقصه ثم یضرب المجتمع فی المکان الاعلی فی ۳۰ و یزید علی ما اجتمع الایام الماضیة من الشهر و یضع الجملة فی مکانین و یضرب اسفلها فی ۳۳۰۰ و یزید علی ما بلغ ٦٤۱۰٦ و یقسم الجملة علی ۲۱۰۹۰۲ فما خرج فهو محفوظ اول و ما بقی فهو محفوظ ثانی لما بعد ـ فأما الاول فیقصه من الاعلی فینبنی ایام التاریخ مفتتحة بنصف اللیل الذی بعد النهار المحسوب و هی الأصل ـ

Contents of *Ghurrat-uz-Zijat*

4. To find true places of the five planets.
5. To find true daily motions of the planets.
6. To find retrogression and direction of the planets.
7. Nodes of the planets.
8. Equation of the Nodes of the planets.
9. To find the latitude of the planets.

CHAPTER VI– Relating to the three Problems of Diurnal Motion.

1. To find the latitude of the Moon.
2. To find the true declination of the Moon.
3. To find the duration of a day.
4. To find the meridian line.
5. To find the latitude from the shadow at noon.
6. To find the latitude from equinoctial shadow.
7. To find the equinoctial shadow from the latitude.
8. To find the co-altitude of the Sun at noon.
9. To find the versed-sine of the day.
10. To find the past period of the day from the shadow at any time.
11. To find the length of shadow at any time from the past period of the day.
12. To find the duration of ascendants at any place.
13. To find the ascendant from the past period of the day.
14. To find the past period of the day from the ascendant.

CHAPTER VII— Relating to the Lunar eclipse.

1. To find the measures of the Moon and the Sun.
2. To find the measure of the shadow.
3. Detailed description of the lunar eclipse.

CHAPTER VIII Relating to the solar eclipse.

CHAPTER IX— Relating to the shapes of the lunar and solar eclipse.

CHAPTER X — Relating to the heliacal rising and setting of the planets.

CHAPTER XI— Relating to the conjunctions of the planets.

1. To find the measures of the planets.
2. To find the conjunctions of the planets.

CHAPTER XII— Relating to the conjunctions of the planets and stars.

CHAPTER XIII– Relating to the visibility of the Moon

1. To find the visibility of the Moon.
2. To find the lighted portion of the Moon.

CHAPTER XIV– Relating to the Vyatipata and Vaidharita.

Chapter I

Relating to Ahargana

1. Karana Book of Vijaya Nandi of Varanasi the name of which is Karana Tilaka and its (Arabic) translation is Ghurrat-uz-Zijat:–

Master Abu Rayhan Muhammad ibn Ahmad-al-Beruni said:–

"In India I came across a Karana book which was very concise and was written (in Sanskrit) by Vijaya Nandi, son of Jaya Nandi, one of the commentators of the city of Varanasi. The Hindus have a high regard for it from the religious point of view and the author has named it as *Karana Tilaka*. Some of my friends, who are fond of knowledge, have expressed their desire to get it converted into Arabic. So I hasten to do this work in order to spread knowledge among those who deserve it. I am translating it in such a way that its reasonings are expressed clearly, and with this aim in view I have not added anything from my own side except the examples which are meant to make the understanding of the subject easy. And it is God Who, through His kindness, helps in achieving the objects."

Commentary

(a) Correction of the Manuscripts:–

No correction is required.

(b) Translation of the technical terms:–

(i) *Ahargana:–* It is a Sanskrit term which means "number of civil days". Beruni has translated this term into Arabic as "Asl" meaning root or base, but I have preferred to use the original Sanskrit term in the English translation.

(ii) *Karana Book:–* It is the translation of the Arabic term "Zij". In English I could not find a suitable equivalent. A Karana book is generally a book which is based on some astronomical canon known as Siddhanta, but it gives much easier methods of calculations as compared with those of Siddhanta. As will be seen later, this Karana book is based on modern Surya Siddhanta.

(iii) *Vijaya Nandi:–* In the Arabic manuscript this name is written as "Bijaya Nand" but its correct pronunciation is the same as I have given in the translation. This author flourishedin Vāranasi in the middle of the 10th century A. D. Another Indian astronomer of this name flourished in the 5th century A. D. or even earlier than that, Dr. Edward C. Sachau seems to have got confused, because he thinks both these astronomers identical, although there is a time gap of more than five centuries between these two personalities (Vide *Alberuni's India*, 1910, Vol. II, page 306 Annotations).

(iv) *Varanasi:–* In the Arabic manuscript this name is written as Banarasi, but I have preferred to write it as Varanasi, because it is more correct and at present also the city is known by this name. This is a famous city of India situated on the banks of the river Ganges and its religious status is still maintained.

(v) *Ghurrat-uz-Zijat:–* This is the Arabic translation of the Sanskrit name "Karana Tilaka," and Beruni has himself mentioned this name in his Indica (vide *Alberuni's India*, 1910, vol. II, page 90). Dr. Edward C. Sachau has made a serious mistake in

assuming this book to be identical with the *Kitab-algurrah* (vide *Alberuni's India* 1910, Vol. II, page 388 Annotations), although at another place he clearly states that the title of Vijaya Nandi's book *Karana Tilaka* would be in Arabic Gurraht-uz-Zijat (vide *Alberuni's India*. 1910, Vol II page 345 Annotations). This name is given by Beruni to his Arabic translation of *Karana Tilaka*.

(vi) *Beruni*:- He was the most learned Muslim author of the early 11th century A.D. He wrote more than 150 books on various subjects and translated many Sanskrit books into Arabic. The present book is perhaps the only Arabic translation of a Sanskrit book and probably the only manuscript available, but has hitherto been unknown to the world. Beruni had made this translation before writing his Indica, but he could not revise and rectify the manuscript afterwards; that is why he did not mention anywhere that he had made a translation of *Karana Tilaka*. He quoted only those passages of this translation in his *Indica*, which he had verified from *Pulisa Siddhanta* available with him. He did not quote those passages of this translation which were based on *Surya Siddhanta* because the *Surya Siddhanta* was not available with him and hence he could not verify the correctness of those passages. Due to these unverified passages, he did not think it proper to mention the name of this translation as a complete and authentic work. He, however, had given an indication of it in his letter to one of his friends nearly 13 years before his death, where he had mentioned that there were various incomplete translations of Sanskrit books with him at that time. Beruni was born in 973 A.D. and he died in 1048 A.D. As stated in the *Encylopaedia Britannica* and judged from his writings, Beruni was a Shia by faith and believed in the twelve Imams. His complete life has been given in the Urdu book named "Alberuni" written by Syed Hasan Barni, who died on 16 October, 1959.

(vii) *Commentator*:- This is a literal translation of the Sanskrit term "Tikakara", for which Beruni has used the Arabic word "Mufassir." In Sanskrit an acharya is one who writes a Siddhanta on astronomical principles, but a Tikakara is one who merely follows the Siddhantas and, on the basis of these Siddhantas, writes a Karana book introducing easy calculations.

(c) *Elaboration of the principle*:-
No elaboration of any principle is required here.

(d) *Explanation of the text*:-
In this article Beruni has expressed his views about the Sanskrit book *Karana Tilaka* written by Vijay Nandi, son of Jaya Nandi of Varanasi. From his statement it is clear that this book was very popular among the Hindus throughout the whole country. It appears that

in those days, that is at the beginning of the 11th century A.D., it was the only Karana book which was considered a standard book by the Indian astronomers and astrologers. Most probably the main reason for its popularity was that its calculations were based on the modern *Surya Siddhanta* which is considered by the Hindus as a revelation from Sun God to Maya Danava. It will be observed later on that even the ahargana of this book is based on the modern *Surya Siddhanta* and not on *Pulisa Siddhanta* as believed by Dr. Sachau and Dr. Schram (vide *Alberuni's India*, 1910, vol. II, page 375, Annotations). The methods of calculations given in this book are so accurate that they can be adopted even today without any appreciable error, although the usual range of a Karana book is supposed to be only two or three centuries. The most important fact about this book is that its text has proved the antiquity of the modern *Surya Siddhanta*, because it is based on it and now we can say without any doubt that the time of the so called modern *Surya Siddhanta* is much earlier than the 10th century A. D.

Beruni has not given a literal translation of the text of *Karana Tilaka*; he has given only a summary of this book in his own words. By adopting this method of reproduction he has made a few mistakes because he had some wrong notions in his mind about certain technical terms of the Hindus such as *tithi*, *Karana*, *Yoga*, etc. Moreover, the Sanskrit manuscript possessed by Beruni seems to be carelessly written and Beruni has copied the same figures in his translation without any verification. Such mistakes have been rectified by me in this English translation and full justification of this rectification has also been given.

The most unfortunate part of it is that Beruni could not get hold of a copy of the modern *Surya Siddhanta* as stated by him in his *India* (vide *Alberuni's India*, 1910, vol. I, page 154), and therefore he could not verify many formulae mentioned in *Karana Tilaka* He could not even know that *Karana Tilaka* was actually based on the modern *Surya Siddhanta*, otherwise he would have stated this fact somewhere. The solved examples given by Beruni from his own side are very useful but are not free from petty mistakes of calculation; these mistakes have been rectified in the English translation. At certain places such solved examples are not given by Beruni, and so I have tried to give some additional examples in the commentary.

(e) *Additional examples*:-
No examples are required.

2. *The substance of the book and its arrangement*:-

This book is small in bulk, important for use and easy to understand. It enlightens the brains of big scholars and smoothes the path for beginners. It is written by Vijaya Nandi at the end of the year 888 Shaka-kala which is in use amongst the Hindus.

Any one who wants to compute the true places of the planets completely, correctly and quickly, should find out the total number of civil days since the date of this book (upto the selected date). Take the complete years of Shaka-kala of the selected date and deduct from these the years of this book, i. e. 888. Multiply the remainder by 12 and add to the product the complete (Synodic lunar) months of the current year of Shaka-kala beginning from (Shukla Pratipadu of) Chaitra. Put the sum at two places and multiply the lower one by 900; add to the product 661 and divide the sum by 29282; the quotient represents the number of complete adhimasas which are to be added to the number already kept at the upper place; the remainder represents the past period of the incomplete adhimasas (which is to be ignored, because only complete adhimasas are taken into account). Now multiply the sum at the upper place by 30 and add to the product the past tithis of the current month (at Lanka upto the midnight which follows the day you have selected). Put this sum at two places and multiply the lower one by 3300; add to the product 64106 and divide the sum by 210902. The quotient represents the first fixed number and the remainder represents the second fixed number. Both these fixed numbers should be kept in memory, because these will be used later on (for computation of the Moon). Now deduct the first fixed number from the number already kept at the upper place; the remainder will represent the total number of civil days (passed since the date of this book at Lanka, upto the midnight which follows the selected day. This is the ahargana.

COMMENTARY

(a) Correction of the Manuscript:–

No correction of the manuscript is required; only a few phrases have been added within brackets to make the text more intelligible.

(b) Translation of the technical terms:–

(i) *Shaka-kala*:– This is an era which depends on solar years and is generally 78 years less than the Christian era, so that the year 1957 A. D. corresponds to 1879 Shaka-kala. When we write 1957 A. D., it means that 1956 years of the Christian era have passed and the 1957th year is current; but when we write 1879 Shaka-kala, it means that 1879 years of this era have passed and the 1880th year is current. In other words, we always write the number of past years to indicate the Shaka-kala, but to indicate the Christian era we write the number of the current year. This is just a convention. Shaka-kala may be used either with solar months or synodic lunar months. When used with solar months, the first month of the year begins when the sun passes the first point of Aries (without any regard to degrees of precision). and nowadays it generaly happens on 14 April; the second month begins when the sun passes the first point of Taurus and it generally happens on 15 May, and so on. But when this era is used with synodic lunar months, the first month of the year corresponds to the first month of the year of Vikrami Samvat. Vikrami Samvat also is indicated by the number of past years and it is 135 years longer than the Shaka-kala, so that 1879 Shaka-kala corresponds to 2014 Vikrami Samvat. In *Karana Tilaka* the Shaka-kala is used with synodic lunar months

and luni-solar years. In this luni-solar system normally each year consists of 12 synodic lunar months, but generally after every two years the third year consists of 13 synodic lunar months in order to keep the luni-solar years in step with the solar years If the synodic lunar month is reckoned from full moon to full moon, the arrangement is known as purnimanta; if the month is reckoned from new moon to new moon, this arrangement is known as amanta. The purnimanta arrangement is used all over North India, and the amanta arrangement is generally used in South India. Each month of purnimanta is divided into two parts, dark half and light half, but in the amanta arrangement each month is divided into light half and dark half. From new moon to full moon it is light half and from full moon to new moon it is dark half. In both purnimanta and amanta arrangements the light half of each month is identical, but in the purnimanta arrangement this light half is considered the second half of the month, the preceding black half being the first half, and in the amanta arrangement this light half is considered the first half of the month, the succeeding black half being the second half. Thus, if we take the purnimanta arrangement, then the month of Chaitra begins just after the full moon, generally occurring between the middle of March and the middle of April; but if we take the amanta arrangement, then the month of Chaitra begins just after the new moon, generally occurring in the month of March. In this way the months of purnimanta arrangement begin nearly 15 days earlier than those of amanta arrangement, but the new year always begins in both the arrangements on the same day, i. e., with the beginning of the light half of Chaitra. Hence it will be seen that, according to the purnimanta arrangement, the new year begins from the middle of Chaitra and, according to the amanta arrangement, the new year begins from the beginning of Chaitra. There are very elaborate methods of calculations for this luni-solar system and these will be explained at some other proper place. The names of the synodic lunar months are given below in the correct order:–

1. Chaitra	7. Ashwina
2. Vaishakha	8. Kartika
3. Jyeshtha	9. Margashirsha
4. Ashadha	10. Pausha
5. Shravana	11. Magha
6. Bhadrapada	12. Phalgana

(ii) *Tithi*:– Each of the two halves of a synodic lunar month is divided into 15 parts and each 15th part is known as synodic lunar day or simply a *tithi*. The light half is known as Shukla Paksha and the dark half is known as Krishna Prakasha. The last tithi of the light half is known as purnima and the last tithi of the dark half is known as amavasaya. The names of the remain-

ing 14 tithis are identical for both the halves and are given below:–

1. Pratipada.	8. Ashtami
2. Dutiya.	9. Naumi
3. Tritiya.	10. Dashami
4. Chaturthi	11. Ekadashi
5. Panchami.	12. Dwadashi
6. Shashti.	13. Trodashi
7. Saptami.	14. Chaturdashi

When the sun and the moon meet together at one point of the Zodiac, it is the end of amavasya and beginning of Shukla Pratipada; and when the sun and the moon come exactly opposite to one another at a distance of 180 degree of the Zodiac, it is the end of purnima and beginning of Krishna pratipada. In fact the time during which the moon moves 12 degrees away from the sun is one tithi, and so in one synodic lunar month there are 30 tithis equivalent to one complete synodic revolution of the moon comprising 360 degrees. A tithi may begin or end at any moment during day and night, but a civil day is generally named according to the tithi which is present at the time of sunrise of that day. In the Hindu system a civil day is reckoned from sunrise to sunrise.

(iii) *Adhimasa*:– As mentioned above, after every two years the third year consists of 13 synodic lunar months, instead of 12. The additional synodic lunar month is known as adhimasa and it may fall at any part of the year. It is supposed that during each synodic lunar month the sun should transit one sign of Zodiac. But every third year it so happens that the transit of the sun does not take place during any one synodic lunar month; such a synodic lunar month is known as adhimasa and so the next month is given the same name. In this way two months of the same name are reckoned in one year and the actual number of months becomes 13, instead of 12. It is important to note that for the sake of finding out the transit of the sun during a month, the amanta arrangement is always used, and a synodic lunar month for this purpose is reckoned from new moon to new moon and not from full moon to full moon. For example, if there is no transit of the sun in the Shukla Paksha of Chaitra, and its succeeding Krishna Paksha, then we will say that this Chaitra is an adhimasa and so its next month will also be named as Chaitra. The first Chaitra will be considered inauspicious for all religious festivals, etc., but the second Chaitra will be considered just a normal month. Very rarely after a very long time it may also happen that during one synodic lunar month there occur two consecutive transits of the sun. In such cases the particular synodic lunar month is dropped from the serial order

and is known as kshaya-masa. Whenever a kshaya-masa occurs, there must occur two adhimasas in that year. Such kshaya-masas can only occur in the months of Kartika, Margashirsha, and Pausha, because the daily motion of the sun becomes maximum during these months.

(c) *Elaboration of the Principle*:– The first and the main problem in computing the planetary positions is to find out the total number of civil days passed after a particular day upto the selected date. In the Christian calendar this problem is very simple, but in the Hindu calendar of the luni-solar system this problem becomes a little complicated. In this article a method is given by which the synodic lunar days or the tithis are converted into civil days and after making an allowance of tithis of adhimasas, the correct number of civil days passed is found out. First the number of past solar years is found out and it is converted into solar months by multiplying it by 12; then the past synodic lunar months are also added to this solar month, and with the help of this sum the number of past complete adhimasas is calculated ignoring the remainder which represents the incomplete adhimasa; then these adhimasas are also added to the above-mentioned sum and then it is converted into a sort of mixed number of solar days and synodic lunar days by multiplying it by 30, and to this product the past tithis of the current month are added; then with the help of this sum the number of tithis in excess of the corresponding number of civil days is calculated ignoring the remainder which represents excess tithis; then these excess tithis are deducted from the above-mentiond sum and the remainder represents the number of civil days passed after a particular day fixed by the author of the book. This number is known as ahargana.

It will be observed that in the above-mentioned method at one place we add synodic lunar months to solar months, and at another place we add synodic lunar days to a sort of mixed number of lunar and solar days. This is apparently a wrong process; but as neither of the two remainders in the two divisions is taken into account in calculating the ahargana, this irregularity does not affect the final result.

(d) *Explanation of the Text*:– The starting point for calculating the ahargana as fixed by Vijaya Nandi, the author of *Karana Tilaka*, is the midnight at Lanka at the beginning of Chaitra Shukla Pratipada 888 Shaka-kala. From this day the ahargana, or the number of civil days that have passed upto any selected day, is calculated to compute the planetary positions on the Zodiac. This fixed moment corresponds to the midnight at Lanka between 23rd and 24th of March, 966 A.D. In future references we shall always mention various dates preceding the midnight under consideration and accordingly we may say that the above-mentioned fixed day corresponds to Saturday, the 23rd of March, 966 A.D., 30 Isfindar-muz-Mah 334 Yazdjird, 28 Rabi-ul-Awwal, 335 A.H. 23 Azar 1277, Roman era,

and the Amavasya preceding Chaitra Shukla Pratipada 888 Shaka-kala. On this particular day the ahargana of *Karana Tilaka* was zero; it was " 1 " on the next day, i.e., on Sunday, the 24th of March, 966 A.D., 1 Farwardin-Mah 335 Yazdjird, 29 Rabi-ul-Awwal 355 A.H., 24 Azar 1277 Roman era, and Chaitra Shukla Pratipada 888 Shaka-kala (or 1023 Vikrami Samvat). In this way the ahargana increases one by one from midnight to midnight at Lanka; if the ahargana is 1, it is Sunday, if it is 2 it is Monday if it is 3, it is Tuesday and so on.

The location of Lanka is supposed to be at the equator of the earth and at 75° 45′ E longitude with reference to Greenwich; so if we suppose the beginning of each day of the Julian period at the Greenwich mean noon, the above-mentioned fixed midnight corresponds to 2073972.28958 day of the Julian period. Similarly, the mean noon of the next day at Lanka when the ahargana of *Karana Tilaka* was " 1 " corresponds to 2073972.78958 day of the Julian period. If we assume the number of days of the Julian period as a whole number considering the decimal fraction as one, we may say that the first day of the ahargana of Karana Tilaka corresponds to the day 2073973 of the Julian period.

According to the modern Surya Siddhanta (*vide* English translation of Surya Siddhanta, 1935, edited by E. Burgess, formerly Missionary of the A.B.C. F.M. in India), the number of civil days in a Chaturyuga is 1577917828, and within this period the number of complete cycles for the sun is 4320000 and for the moon it is 57753336. A Chaturyaga is subdivided into four smaller yugas, e.g., Satya-yuga Treta-yuga, Dwapara-yuga and Kali-yuga. The duration of these smaller Yugas in terms of solar years is 1728000, 1296000, 864000 and 432000, respectively. The first three yugas of the present Chaturyuga have already passed and the fourth yuga, i.e., the Kali-yuga, has started since B. C. 3102 or 3179 before Shaka-kala. At the beginning of Kali-yuga the mean places of all the planets were at the first point of Aries. If we take more accurate figures, then the number of civil days that passed since the first midnight after which the Kali-yuga started up to the above-mentioned fixed midnight is 1485507; in other words, the Kali-yuga ahargana on 23 March, 966 A.D. is 1485507. At the beginning of Kali-yuga the mean places of all the planets were at the first point of Aries, and there was no incomplete adhimasa.

From the above-mentioned figures it is clear that in a Chaturyuga there are $4320000 \times 12 = 51840000$ solar months and $57753336 - 4320000 = 53433336$ synodic lunar months. Therefore, the number of excess synodic lunar months or the adhimasas is $53433336 - 51840000 = 1593336$. So the number of adhimasas in each solar month is $\frac{1593336}{51840000}$.

In order to simplify this fraction we can select by inspection an easy whole number 900 as numerator, and its corresponding denominator (say D) can be found out in the following manner:–

$$\frac{900}{D} = \frac{1593336}{51840000}$$

$$\text{or } D = \frac{900 \times 51840000}{1593336} = 29281\ \frac{1528584}{1593336}$$

The fraction being equal to more than half can be taken as one, so that D = 29282 and therefore the fraction is $\frac{900}{29282}$

Now, the number of solar years that elapsed up to the beginning of 888 Shaka-kala since the beginning of Kali-yuga is 888 + 3179 = 4067; it is equal to $4067 \times 12 = 48804$ solar months. Hence the number of adhimasas that passed since the beginning of Kaliyuga up to the beginning of 888 Shaka-kala is

$$\frac{1593336}{51840000} \times 48804 = 1500\ \frac{1170144}{51840000}$$

It shows that 1500 complete adhimasas had already passed and been accounted for, and only a fraction of 1501 st. adhimasa had been accumulated. This fraction is yet to be accounted for in all the calculations to be made since the beginning of 888 Shaka-kala.

We have already seen that the approximate value of adhimasa in one solar month is $\frac{900}{29282}$, and if we adopt the same denominator for the fraction $\frac{1170144}{51840000}$, the numerator (say N) can be found out in the following way:—

$$\frac{N}{29282} = \frac{117044}{51840000}$$

$$\text{or } N = \frac{29282 \times 117044}{51840000} = 660\ \frac{49756608}{51840000}$$

The fraction being equal to more than half can be taken as one, so that D = 661, and the fraction = $\frac{661}{29282}$

Hence if the number of solar months that elapsed since the beginning of 88 Shaka-kala is "S", then the number of adhimasas that accumulated during "S" solar months is given by the expression $\frac{900 \times S + 661}{29282}$.

We have seen that in a Chaturyuga the total number of synodic lunar months is 53433336, so the total number of tithis is $53433336 \times 30 = 1603000080$.

But the number of civil days in a Chaturyuga is 1577917828, so the excess synodic lunar days or the unaratra days are 1603000080–1577917828=25082252. Therefore, the number of unaratra days in one tithis is $\frac{25082252}{1603000080}$. In order to simplify the fraction select the numerator by inspection as 3300, and as such the corresponding denominator (say d) can be found out in the following way:–

$$\frac{3300}{d} = \frac{22082252}{1603000080}$$

$$\text{or } d = \frac{3300 \times 1603000080}{25082252} = 210902\ \frac{2936696}{25082252}$$

The fraction being equal to less than half may be ignored, so that d = 210902, and the fraction is $\frac{3300}{210902}$

Now, at the beginning of 88 Shaka-kala the total number of solar days that elapsed since the beginning of kaliyuga is 48804 × 30=1464120: the number of tithis in 1500 synodic lunar months is 45000; the sum of these two is 1509120. The number of unaratra days during this period is

$$\frac{1509120 \times 2508225}{1603000080} = 23613\ \frac{48724920}{1603000080}$$

The complete number of these unaratra days is deducted from 1509120 and the remainder 1485507 is the number of civil days or the ahargana, but the fraction $\frac{48724920}{1603000080}$ remains unaccounted for, which represents the portion of incomplete unaratra day. In order to account for this fraction we adopt the same denominator and the numerator (say n) can be found out as follows:–

$$\frac{n}{210902} = \frac{48724920}{1603000080}$$

$$\text{or } n = \frac{210902 \times 48724920}{1603000080} = 64105\ \frac{1510650000}{1603000080}$$

The fraction being equal to more than half can be taken one, so that n = 64106, and the fraction is $\frac{64106}{210902}$

Therefore for "L" synodic lunar days since the beginning of 888 Shaka-kala, the unaratra days can be found out by the expression

$$\frac{3300 \times L + 64106}{210902}$$

From this expression whatever we get as the quotient is the first fixed number and whatever we get as the remainder, is the second number; the first fixed number represents the complete unaratra days and the second fixed number represents the incomplete unaratraday. Both of these fixed numbers will be utilised later on for the computation of the moon's position in the Zodiac.

Dr. Schram could not find out the actual basis of the above mentioned expression and so he had erroneously supposed that the method employed was based on the Pulisa's theory in which the number of civil days in a Chaturyuga is 1577917800; but actually it is based on the theory of the modern Surya Siddhanta, in which the number of civil days in a Chaturyuga is 1577917828. Due to this misunderstanding, Dr. Schram, after giving the detailed calculations in his support, had wrongly concluded that "The fraction of unaratra days $\frac{232}{703}$ is equal to $\frac{69600\ \frac{464}{703}}{210902}$ or nearly to $\frac{69601}{210902}$ Therefore we must add 69601 before dividing by 21092. Alberuni has, instead of this number 69601, the number 64106, 4 instead of 9, and the last three numbers reversed" (vide *Alberuni's India*, 1910, vol. II page 376, Annotations).

From the above reasoning it is quite clear that the supposition of Dr. Schram is totally baseless and the figures given by Beruni are absolutely correct.

(d) *Additional examples:*– It will be seen that from this method we can find out the ahargana only for the period after 888 Shaka-kala. But from similar reasoning it is quite possible to evolve an additional method to find out the negative ahargana for any date before 888 Shaka-kala. Such a negative ahargana will be counted backward from the end of 887 Shaka-kala and separate methods can also be evolved for every planet to compute its position by means of this negative ahargana. This method of finding the negative ahargana along with completely solved examples is given in Appendix 'A' of this English translation.

It will not be out of place to give a simple method here for finding out the ahargana of Karana Tilaka from the dates of the Christian calendar. This method can be employed for any date after 1601 A. D.

"Deduct 1601 from the current Christian Era and multiply the remaining number by 365. Keep this product at one place. Now divide the same remaining number by 4 and add the quotient to the above-mentioned product kept at one place. Then divide the above-mentioned quotient by 25 and deduct this second quotient from the above-mentioned product. Then divide this second quotient by 4 and add this third quotient to the same above-mentioned product. To this sum add the number of days of the current year with effect from 1st January, including the current day. Finally, add 231841 to this total number; then you will get the ahargana of Karana Tilaka which begins with Sunday." Actually 231841 is the ahargana of Karana Tilaka on 31st December, 1600 A. D. which was a Sunday.

For example,let us calculate the ahargana on 10 May, 1957 A. D., when the 100th anniversary of the Freedom Struggle of India was celebrated all over Indo-Pak sub-continent. We deducted 1601 from 1957 and got 356 as remainder; we multiplied this remainder by 365 and got the product 129940. We kept this product at one place. Then we divided the same remaining years 356 by 4 and got the quotient 89; we added it to the above-mentioned product andgot 130029; then we divided 89 by 25 and got 3 as the quotient, which was deducted from the above-mentioned product and we got 130026 as remainder. When we divided 3 by 4 we got zero as quotient, so we ignored it. To this we added 130 days of the current year and got 130156; to this we added 231841 and got the total sum 361997. This is the ahargana of Karana Tilaka for 10 May, 1957, which is a Friday, because after dividing this ahargana by 7 we get 6 as remainder, which gives us Friday by counting it from Sunday.