

## AL-BĪRŪNĪ'S MASUDIC CANON

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Of the one hundred and forty-six writings estimated as having been composed by Abū Rayḥān al-Bīrūnī (b. 973), the great scientist of Central Asia, a hundred and twenty-three seem to have disappeared. Of the few which are extant, seven can be regarded as major works, and of these, four have been edited in the original and translated into a European language. Two more have been published, but in Arabic only. One of the latter is *al-Qānūn al-Maṣūḍī* (listed under *Canon* in the bibliography) an exhaustive astronomical treatise written after the manner of Ptolemy's *Almagest*. It is a prime source for the history of the medieval exact sciences, and portions of it have been translated (or paraphrased) into German, while other sections have been exploited for particular studies. The translation of the document as a whole has repeatedly been called for, and this is underway in the U.S.S.R. pending the completion of this, and since there are many historians of science who read neither Arabic nor Russian, the present paper is offered as a temporary stopgap.

It is essentially a detailed table of contents of the excellent Hyderabad edition, and at least makes known the topics discussed in the *Qānūn*. The relevant literature, to the extent it is known to the author, has been listed in the bibliography at the end. References to the entries are made by using italicised abbreviations. The only detailed description of the *Canon* thus far to appear is *R & R*, but it is in Russian.

The title page and dedication are not paginated, nor are the two pages listing the seven MSS used in the edition. Abbreviations employed in the critical apparatus are also listed.

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To "the Indians" (al-Hind) Bīrūnī attributes in 788,958,225,000 days, 26,716,650,000 lunations, 28,87[6,6]50,000 lunar revolutions, and 28,632,597,071 lunar anomalistic revolutions.	727
These are the numbers of the events indicated occurring in half a kalpa of the Brāhmasphuṭasiddhānta. Following is a set associated by Bīrūnī with "Pulisa (or Paulisa) the Greek." In 889, 0[4]0 days are 30,105 lunations, 32,5[3]9 lunar revolutions, 32,265 lunar anomalistic revolutions, and 2,434 years.	727
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Find  $f_1(\eta)$  and  $f_2(\eta)$

Then put  $\gamma + f_1(\eta) = \gamma'$ , the modified anomaly,

and find  $f_3(\gamma')$  and  $f_4(\gamma')$ .

Compute  $\Delta\lambda = f_3(\gamma) + f_4(\gamma) \cdot f_2(\eta)$ .

Then the desired longitude is

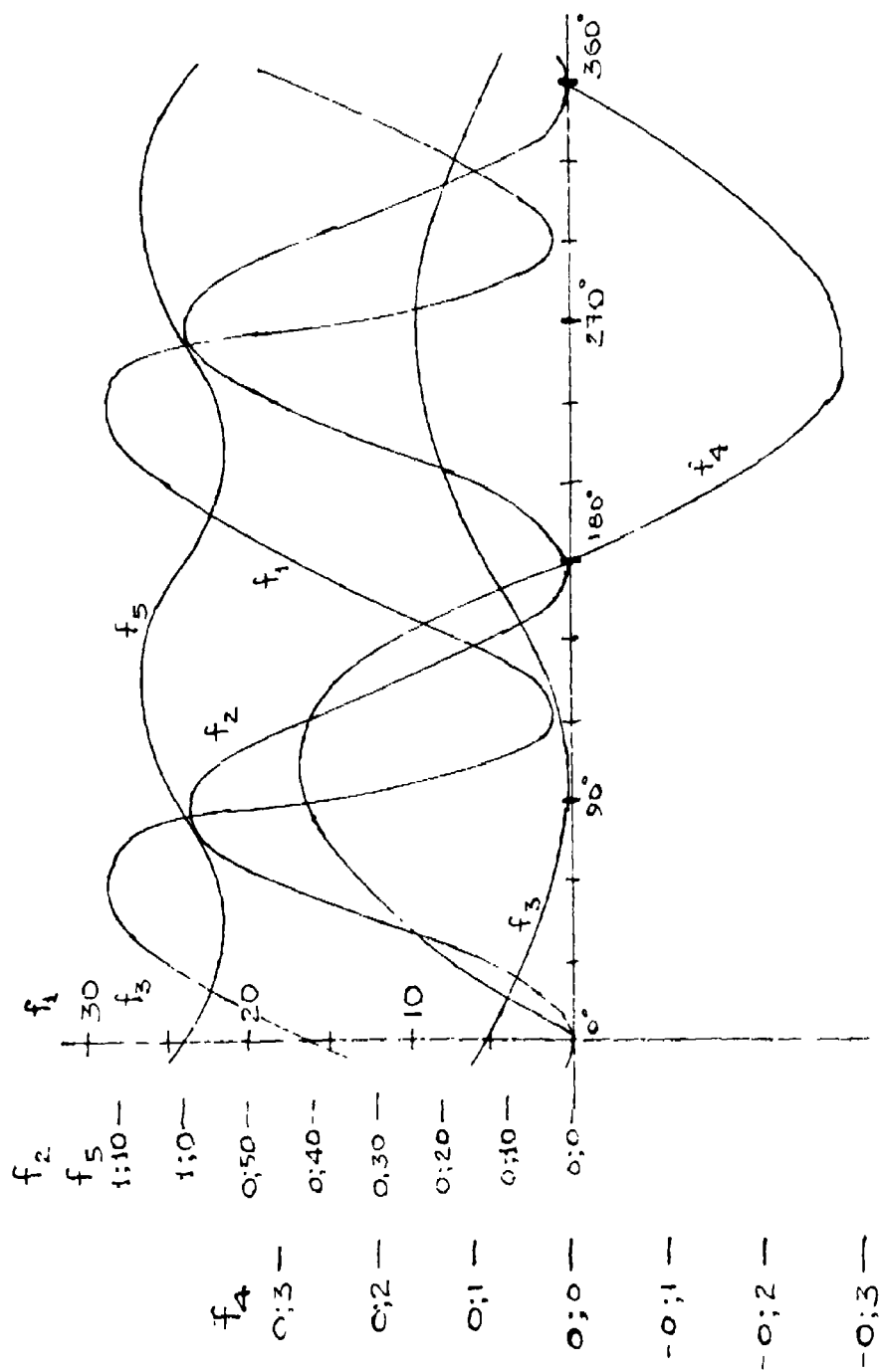


Figure 1 Lunar equations in the Canon

$$\lambda = \kappa + \Delta\lambda - i + f_1(K_n),$$

where  $K_n$  is the argument of the lunar latitude

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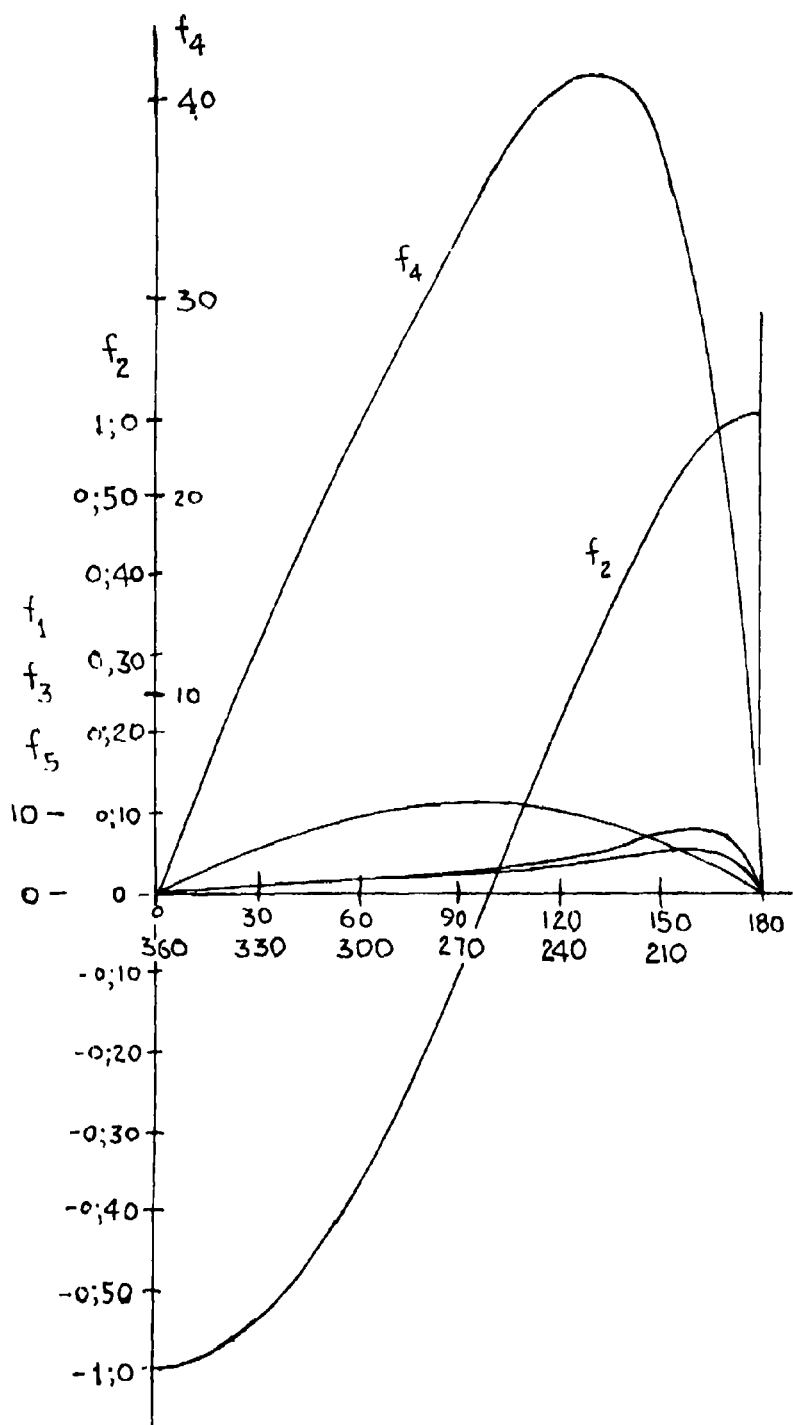


Figure 2 The equations of Mars in the Canon

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	Saturn    Jupiter    Mars    Venus    Mercury
Pulisa the Greek	2            8            1            16            4
Khaṇḍakhādyaaka = Karaṇṭilaka	0;2,30    0;3,30    0;2            0;4            0;3
Ghurraṭ al-Zijāt	5            7            4            8            6

He says that in the *Mustakhraj Zīj* the values are different. He reports the following length (*R*) for the radius of the defining circle of the trigonometric functions

<i>Pulisa</i>	3438'
<i>Brāhama</i> (?)	3270'
<i>Khaṇḍ</i>	150'
<i>Ghurraṭ al-Zījāt</i>	200'
<i>Al-Zīj al-Mustakhraj</i>	300'

apparent diameters given above are mean values, he says. In order to find the apparent diameter at a particular time, multiply the mean value by *R* and divide the product by the distance from the earth to the planet at the time in question, the mean distance being taken as *R* 1310

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Mercury	0.2,8,57"
Venus	0.3,18,26
Mars	0.1,36,43"
Jupiter	0.2,41,12"
Saturn	0.1,47,28"
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*Index* (in Arabic) of places and peoples

Beginning from the other end of the volume is material in English: title page, dedication, list of MSS used in the edition, and

*General Introduction* of eighteen pages, the last five of which describe five of the MSS used.

*The place of the Qānūn-i-Mas'ūdi in the History Science*, by H. J. Winter, in fifteen pages, the pagination beginning anew.

*Al-Birūnī and His Magnum Opus, Al-Qanun al-Mas'udi* by S. H. Barani, in seventy-five pages, again paginated anew, this time in numerals, consisting of a useful and interesting biographical sketch of Birūnī and abstract of the *Qanūn*.

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