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A NEWLY DISCOVERED BOOK OF AL-BIRUNI
GHURRAT-UZ-ZIJAT AND AL-BIRUNI'S
•MEASUREMENTS OF EARTH DIMENSIONS

by

SYED SAMAD HUSSAIN RIZVI

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A NEWLY DISCOVERED BOOK OF AL-BIRUNI"GHURRAT-UZ-ZIJAT" AND AL-BIRUNI's MEASUREMENTS OF EARTH'S DIMENTIONS

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SAIYID SAMAD HUSAIN RIZVI
C. & G. (London); B.Sc. (Eng.) (Alig.)
Executive Engineer, M.E.S., (Pakistan)

AL-BIRUNI'S"GHURRAT-UZ-ZIJAT"

1. INTRODUCTORY NOTE:-

The Arabic Manuscript of al-Biruni's Ghurrat-uz-Zijat was discovered by me in the year 1959 A.D. and I had obtained its photo-copy from India with great difficulties; its clue was found out by me in the year 1953 A.D. from Dr. Muhammad Nazim's famous book named Life and Times of Mahmud of Ghazna published in the year 1931 A.D. by Cambridge University Press, on pages 55 and 239. This book pertains to Indian Astronomy and is based on a Sanskrit book named Karana Tilaka written by Vijaya Nandi son of Jaya Nandi of Varanasi (India); the original Sanskrit book is now lost and is not traceable anywhere in the world as confirmed by the famous Indian scholar Shankara Balakrishna Dixit in his book on History of Indian Astronomy. This unique and unknown book of al-Biruni, alongwith an English translation and a comprehensive commentary, was first published by me in the Quarterly Islamic Culture of Hyderabad Deccan (India) in eight instalments beginning from April 1963. The original Manuscript of Ghurrat-uz-Zijat, consisting of 67 pages and one title page, is kept in the library of Dargah of Pir Muhammad Shah at Ahmadabad (India), and according to my information it is the only Manuscript in existence in the whole world.

Ghurrat-uz-Zijat is a sort of Astronomical Handbook for calculating the planetary positions at a particular time, especially for the preparation of Horoscopes etc; other useful subjects have also been included in the text as will be be seen from the contents of this book. It is a very important book in many respects, especially it is a missing link between Karana Khandakhadyaka of Brahmagupta and Karana Kutuhala of Bhaskaracharya: it can fill a vast gap in the History of Indian Astronomy. Most probably the Sanskrit Manuscript of Karana Tilaka, on which al-Biruni has based his Ghurrat-uz-Zijat, was obtained by al-Biruni at Multan from the local astronomer Durlabha whose name has been mentioned by him in his Kitab-ul-Hind while describing his stay at that city, (vide Alberuni's India 1910, Vol. II pages 9,10,54).

Al-Biruni has based his solved examples on the latitude and longitude of Multan which shows that Ghurrat-uz-Zijat was written by him at Multan where he had stayed for a considerable period. He has also added much information from his own side in the beginning of this book about various calendars and their inter-relations, in order to make this book useful for all countries of the world. This book contains brief accounts of nearly all those subjects which have been dealt with by al-Biruni in his Qanun-ul-Masudi in details; so it will be more appropriate if we say that Ghurrat-uz-Zijat is a miniature Qanun-ul-Masudi. In other words, after understanding the contents of Ghurrat-uz-Zijat thoroughly, it will be very easy for the scholars to understand the contents of Qanun-ul-Masudi.

2. ARRANGEMENT OF THE TEXT:-

Al-Biruni has mentioned the name of Ghurrat-uz-Zijat or Ghurrat-ul-Azyaj as a source of various quotations given by him in many of his books such as kitab-ul-Hind, Qanun-ul-Masudi, Tamheed-ul-Mustaqar, and Afrad-ul-Miqal etc., but no-where he has mentioned its name as a complete book written by himself. It shows that at that time this book was in the form of his un-checked "First Draft", that is why it was in the shape of independent items which were not arranged under suitable chapters. But now I have arranged all these items in the following fourteen chapters without disturbing the serial order of these items; the captions of these chapters have also been given by me on the basis of similar books of this kind.

Chapters of Ghurrat-uz-Zijat.

Chapter I	Relating to Ahargana
Chapter II	Relating to the Sun and the Moon
Chapter III	Relating to Panchanga
Chapter IV	Relating to the Mean Places of the Planets
Chapter V	Relating to the True places of the Planets
Chapter VI	Relating to the Three Problems of Diurnal Motion
Chapter VII	Relating to the Lunar Eclipse
Chapter VIII	Relating to the Solar Eclipse
Chapter IX	Relating to the Shapes of Lunar and Solar Eclipse

Chapter X	Relating to the Heliacal Rising and Setting of the Planets
Chapter XI	Relating to the Conjunctions of the Planets
Chapter XII	Relating to the Conjunctions of the Planets and Stars
Chapter XIII	Relating to the Visibility of Moon
Chapter XIV	Relating to the Vyatipata and Vaidharita

3. CORRECTION OF THE MANUSCRIPT:-

There were many serious mistakes in the Arabic Manuscript, especially in the formulas for calculating Ahargana or Day-Number, due to which the whole book had become useless for practical purposes; few remarks by unknown persons were also given on the margin of this Manuscript indicating that certain formulas of this book did not give correct results, but no-body was able to find out the actual mistakes. Now I have found out the real source of errors and have corrected the text accordingly. Some examples of such corrections made by me in Ghurrat-uz-Zijat are given in the following paragraphs.

The most serious mistake committed by al-Biruni is the wrong calculation of Ahargana or Day-Number of the selected date 29 Naisan 1336 Roman era corresponding to 29 April 1025 A.D. (Thursday). For this date he had wrongly calculated the Indian date as 27th tithi of Jeshtha 948 Shakakala instead of 29th tithi of Vaishakh 947 Shakakala (as per Amanta system of reckoning) or 14th tithi of Jeshtha 947 Shakakala (as per Purnimanta system of reckoning), because he was confused in Amanta and Purnimanta systems of reckoning which are very complicated and difficult to understand; similar confusion can also be noticed in his Kitab-ul-Hind especially where he has described the dates of the Indian Festivals.

Due to this mistake he had wrongly calculated the Ahargana (or the Day-Number since the reference date of Ghurrat-uz-Zijat which is 23 March 966 A.D. Saturday) as 21614 instead of the correct Ahargana 21586, thus making a mistake of 28 days; and due to this mistake of 28 days all the formulas established by al-Biruni on the basis of this wrong Ahargana, especially for finding out the dates of various calendars, had gone absolutely wrong. The date of Islamic calendar corresponding to the above mentioned Roman date was also wrongly given by al-Biruni as 25 Safar 416 A.H. instead of 27 Safar 416 A.H. as per visibility of new Moon; this mistake was also due to some mis-understanding or due to lack of time to check the whole text again. Now I have removed all these mistakes from the main text as well as from the solved examples and have compiled a corrected text of Ghurrat-uz-Zijat. Actually the reference date of Ghurrat-uz-Zijat corresponds to 2073972.2895 Julian Day-Number, therefore if we deduct this figure from the Julian Day-Number of any date, we get the Ahargana or the Day-Number of Ghurrat-uz-Zijat. As per the basic conception of Ghurrat-uz-Zijat 1955884067 solar years or 714403782134 civil days had passed since the creation of the whole--

-Universe upto the reference date of Ghurrat-uz-Zijat i.e. upto 23 March 966 A.D. (Saturday).

As can be explained by analysing the basic formula given in Ghurrat-uz-Zijat, its Ahargana is based on Surya Siddhanta and not on Pulisa Siddhanta as wrongly supposed by Dr. Sachau and Dr. Schram (vide Alberuni's India 1910, Vol.II page 375, Annotations); consequently the figure 69601 as suggested by them is wrong and the figure 64106 as given by al-Biruni is correct.

Another mistake committed by al-Biruni was that he had supposed the measure of a yojana equal to 32000 Arabian cubits instead of 16000 Arabian cubits; similar mistake has been made by him in his Kitab-ul-Hind (vide Alberuni's India, 1910, Vol.I, page 167) In fact the yojana used by common people of India at that time was equal to 32000 cubits, but the yojana used by the Astronomers was equal to 16000 cubits (one cubit being equal to 1.61825 feet), but al-Biruni could not make a distinction between the two types of yojanas due to some confusion. In Ghurrat-uz-Zijat the diameter of Earth is said to be 1600 yojanas and the circumference is said to be 5028 yojanas which are equal to 7840 miles and 24637 miles respectively (one yojana being equal to 4.9 miles); the modern values of mean diameter and mean circumference of Earth as per reference Ellipsoid 1967 are 7918 miles and 24874 miles respectively; this subject will be described in more details in the second part of this Paper under the caption al-Biruni's Measurements of Earth's Dimensions.

Another correction made by me in Ghurrat-uz-Zijat is that I have deleted the wrong Table of 28 Yogas given by al-Biruni in one of his solved examples and have substituted the correct Table of 27 Yogas in place of the wrong Table; these two types of Yogas are altogether different from one another and this mistake of al-Biruni is just a slip of pen due to the hurry in which he has written this book, because he was quite conversant with these two types of Yogas as mentioned by him in his Kitab-ul-Hind (vide Alberuni's India 1910, Vol II page 210).

I have also corrected the mistake of al-Biruni in the calculations of Karanas where he has wrongly supposed that tithi Amavasya comes in the beginning of Shukla Paksha, but in actual fact tithi Amavasya comes in the end of Krishna Paksha. Exactly the same mistake has been made by al-Biruni in his Kitab-ul-Hind (vide Alberuni's India 1910 vol II pages 197, 198 and 200). Here again he was confused due to the complicated determination of various Karanas which actually are half-portions of a tithi. Even Dr. Gorukh Prasad of Allahabad University U.P. (India) has recently made a similar mistake (vide his Hindi book on History of Indian Astronomy, chapter 18, page 266).

4. ACCURACY OF CALCULATIONS :-

The formulas and calculation mentioned in Ghurrat-uz-Zijat by al-Biruni are so accurate that this book can be safely adopted for ordinary practical purposes even today after the lapse of a thousand years. Just to give an idea of the accuracy of this book as compared with the modern results I am giving the following Table which shows the comparative values (in terms of civil days) of the periods of different planets (i.e. the time of one complete revolution of a Planet).

Periods of Planets in Civil Days.

Name of the Planets	Periods as per Ghurrat-uz-Zijat (in civil days).	Period as per modern Astronomers. (in civil days).
Sun (or Earth)	365.25875	365.2563612
Moon	27.32167	27.32166148
Mercury	87.9697	87.96925
Venus	224.69792	224.7007869
Mars	686.9975	686.9796458
Jupiter	4332.3206	4332.58482
Saturn	10765.773	10759.2198174

5. AL-BIRUN'S PREDICTION ABOUT HIS DATE OF DEATH:-

Al-Biruni was not only a great Astronomer and Astrologer as mentioned above, but he was a good spiritualist too, and it will not be out of place here to say something about this aspect also. A glimpse of his spiritual vision can be seen from an event in which he had predicted his date of death while writing a letter to one of his friends. Now-a-days there is no controversy about his date of birth which is the Morning of Thursday, the 4th of September 973 A.D. corresponding to 3 Zilhijja 362 A.H., but there is a controversy about his date of death, and I think this problem can be solved on the basis of his own prediction as described below.

Probably in October 1036 A.D. corresponding to Zilhijja 427 A.H. he had written a letter in Arabic to his friend from which the English translation of few quotations is given below. This letter is preserved in the Library of Leiden.

"At present my lunar age is 65 years and solar age is 63 years...... when my age entered the 61st year, I saw a dream one night that I am trying to ascertain the places of visibility of Crescent and disappearance of Moon, but I am not able to visualize it. In the same state some invisible personality told me_______GIVE IT UP; YOU ARE ITS SON FOR ONE HUNDRED AND NINTY TIMES_______. After that when I awoke, I converted these lunar 15 years 10 months into solar years and months by deducting 5½ months.............."

Although al-Biruni has not mentioned the exact date of this dream in his letter, yet from the other data given by al-Biruni himself I can clearly visualize that this dream was seen by him at the night between 21 and 22 July 1034 A.D. (i.e. between Sunday and Monday) corresponding to 2 and 3 Ramzan 425 A.H. At that time his age was in the 61st solar year as mentioned in his letter, i.e. 60 years 10 months 17 days (solar) and 62 years 8 months 29 days (lunar), that was the night when he was informed by some spiritual power that his remaining age was 190 lunar months or 15 years 10 months (lunar).

In order to convert these 15 years 10 months (lunar) into solar years and months we deduct 5½ months (lunar) and get 15 years 4 months 15 days (solar), because the difference between the lunar and solar periods can be obtained approximately if we multiply the lunar months by 11/380. More accurately, one lunar month consists of 29.5305888 days, so that 190 lunar months consist of 5610.81182 days. Similarly one solar year consists of 365.242195 days, so that the total solar period comes to 15 years 132 days. If we add this solar period to 21 July 1034 A.D. (which is the solar date of his dream), we get 30 November 1049 A.D., which must be the probable solar date of his death. Similarly if we add 15 years 10 months (lunar) to 2 Ramzan 425 A.H. (which is the lunar date of his dream), we get 2 Rajab 441 A.H. which must be the probable lunar date of his death.

In view of the above mentioned facts, I can definitely conclude that the correct date of death of al-Biruni, as predicted by himself about $15^1/_3$ years earlier, is the Night of 30 November 1049 A.D., corresponding to 2 Rajab 441 A.H. (i.e. the night of 3 Rajab), Thursday (i.e. the night of Friday). It can also be the night of 2 Rajab if the crescent was sighted one day later at the horizon of Ghazna due to dust or clouds. It should be borne in mind that according to Islamic Calendar the dates and days always change with effect from the time of local Sun-Set. This is the date which satisfies nearly all the statements given by various authorities, such as the facts of "2 Rajab" and "Friday-night" etc. The commonly believed date of death i.e. 2 Rajab 440 A.H. corresponding to 11 December 1048 A.D. appears to be incorrect, because it does not give Friday-night; it rather gives Monday-night which is not supported by any contemporary statement. In this way the total length of his life comes to 76 years 2 months 26 days (solar) and 78 year 6 months 29 days (lunar).

AL-BIRUNI'S MEASUREMENTS OF EARTH'S DIMENSIONS

1. INTRODUCTORY NOTE:-

It is a well known fact that from time immemorial people have been trying to find out the real shape and dimensions of Earth. For this purpose many attempts had been made in the ancient times by the scholars of Greece, India, Egypt and Middle East. Different results were obtained at different periods of times by different scholars. Abu Rehan Muhammad Ibn-e-Ahmad-al-Biruni had also made a very successful attempt, perhaps in the year 1018 A.D., to measure the radius and circumference of Earth while he was in detention in the ancient Fort of Nandna, which is situated in the hilly area of the famous salt ranges in District Jhelum, Tehsil Chakwal, about five miles north of Dhariala, in West Pakistan. Dhariala is situated on Jhelum-Pind Dadan Khan Road, about 40 miles from Jhelum. The approximate latitude of Nandna as given by al-Biruni himself is 32° N, but more accurately as determined from Survey of Pakistan topographical map the latitude and longitude of Nandna Towers are 32° 43' N and 73°-14' E respectively, the difference is mainly due to the different points of reference selected by al-Biruni and Survey of Pakistan for observations.

Al-Biruni obtained marvellously accurate results at that time by a very simple method which did not require an elaborate arrangement comprising large survey parties to work for several months and travel hundred of miles as was being done previously in different countries to measure the distance of one degree of latitude along the meridian on the surface of Earth with the help of the Pole Star. As explained by him in his Qanun-ul-Masudi Volume II Chapter VII page 528, his simple method consisted of only two steps; in the first step he measured the height of a peak which was overlooking towards the dead-level plains, and in the second step he measured the angle of the dip of visible horizon from the same peak. This whole process could be completed only by one person single handed within few hours. The idea of this method had been suggested previously by Abu Tayyib Sind Bin Ali, the famous astronomer of the Court of Caliph Mamun-ur-Rashid, but he himself could not employ this method successfully, because he could not get an ideal site for it. But al-Biruni was fortunate enough to get that ideal site at Nandna and he did not miss the chance to make use of it. The result obtained by al-Biruni about a thousand years ago is so astonishingly accurate that even if we compare it with the most modern result, there is only a negligible error of minus 0.19 per-cent, as will be explained later.

2. IDEAL SITE:-

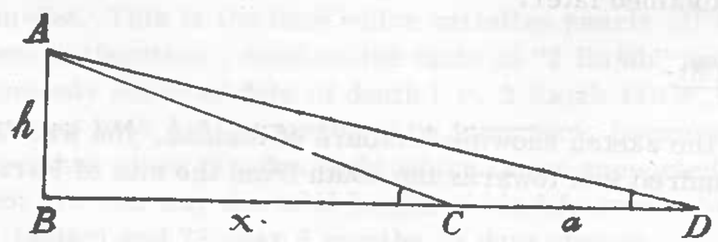
If you see the sketch showing contours of Nandna, you will find that at a distance of few hundred feet towards the south from the site of Fort Nandna, where al-Biruni was kept in detention by Sultan Mahmud of Ghazna for the reasons best known to him, there are two high peaks of the same hillock, one towards the East and the other towards the West. Both of these peaks overlook towards the southern plains which extend to several hundred miles, but the situation of the Eastern peak is ideal for the measurement of the angle of dip of southern horizon, because it is clear of any hinderance by any other peak in the way. So it was this Eastern peak which was selected by al-Biruni for his marvelous experiment.

As is evident from the sketch showing contours of Nandna, the height of this peak selected by al-Biruni is approximately 1795 feet above mean sea-level and it overlooks to the southern plains which are at an average height of approximately 750 feet above mean sea-level. The height of this peak with respect to the southern plains as found out by al-Biruni by simple trigonometrical method was 1055.18 feet which fairly tallies with the modern contours of that area.

If a straight line is drawn from the above mentioned peak in the southern direction, immediately a sudden steep fall of about 1055 feet will be noticed and then the line will pass through the dead-level plains of the three main rivers of West Punjab for several hundred miles. This line will first cross the river Jhelum at a distance of about 10 miles from the peak, then it will cross the river Chenab near Chiniot at a distance of about 60 miles from the peak, then it will cross the river Sutlej near Burewala at a distance of about 170 miles from the peak, then it will cross the border of West Pakistan near Fort Abbas at a distance of about 250 miles from the peak, and then it will enter the Indian Territory and will pass through the desert of Rajasthan. This shows that the site of observation selected by al-Biruni was an ideal one. The height of the peak of observation with respect to the adjoining plain was only 1055.18 feet, therefore the distance of the visible horizon from the peak was only 39 miles approximately, which falls a bit East of Kot Mumin situated between the rivers Jhelum and Chenab. The site was so ideal that even if the height of the peak were 10 miles (which is not possible on Earth), the distance of the visible horizon over the level plains throughout would have been 250 miles approximately i.e. in the vicinity of Fort Abbas situated near the Eastern border of West Pakistan, and then the result obtained would have been even more accurate.

3. ACTUAL EXPERIMENT:-

First of all al-Biruni came down to the southern plains and measured the height of the peak from there by the following simple and easy method.



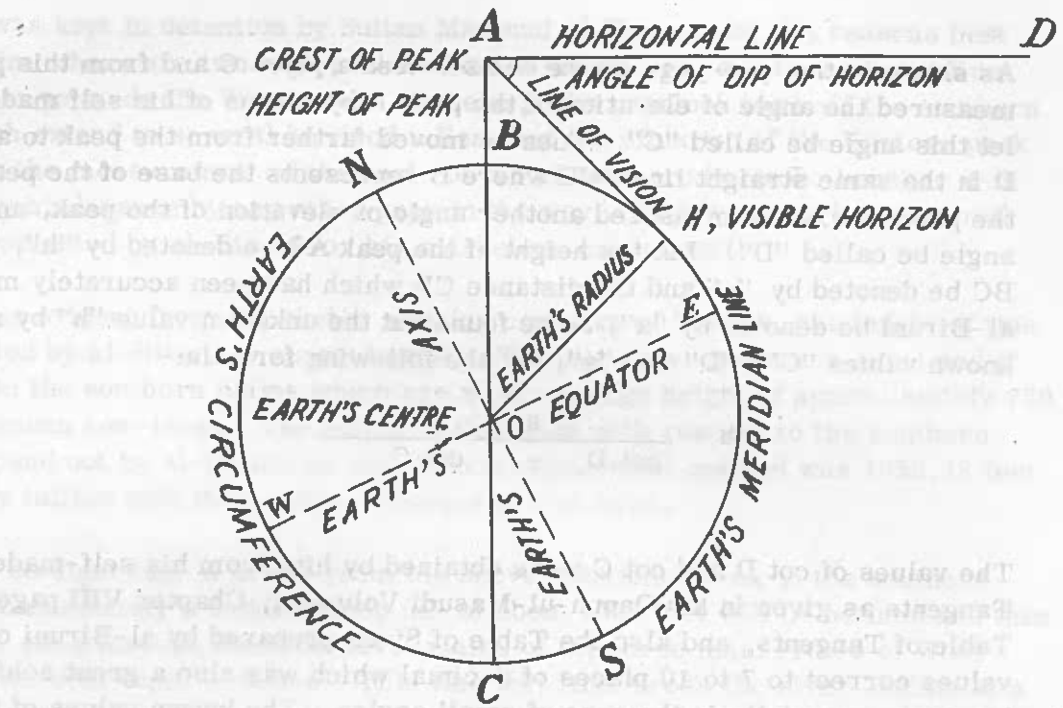
As shown in the pre-page figure, he selected a place C and from this place he measured the angle of elevation of the peak A by means of his self made theodolite; let this angle be called "C". Then he moved farther from the peak to another place D in the same straight line BCD where B represents the base of the peak. From the place C also he measured another angle of elevation of the peak, and let this angle be called "D". Let the height of the peak AB be denoted by "h", the distance BC be denoted by "x" and the distance CD which had been accurately measured by al-Biruni be denoted by "a", so he found out the unknown value "h" by means of the known values "C", "D" and "a", by the following formula:-

$$h = \frac{a}{\cot D - \cot C}$$

The values of cot D and cot C were obtained by him from his self-made Tables of Tangents as given in his Qanun-ul-Masudi Volume I, Chapter VIII page 371. This Table of Tangents, and also the Table of Sines prepared by al-Biruni could give the values correct to 7 to 10 places of decimal which was also a great achievement on his part, especially in the case of small angles. The known values of the angles "C" and "D", and the distance "a" have not been given by al-Biruni but these can easily be found out now by practical experiments. He has given the calculated value of the height "h" as $652\frac{1}{20}$ cubits, but more accurately it was 652-3'-18" cubits as can be seen from his actual calculations of the radius of Earth; this quantity when converted into lowest sub-divisions of "1/60th" system, becomes 2347398" cubits and when converted into decimal system it becomes 452.055 cubits.

After finding out the height of the peak with respect to the adjoining plains, he climbed that peak and while looking towards the south he measured angle of dip of horizon as 34' minutes of a degree. Then he found out the values of sine 34' and cos 34' from his self made Tables of Sines (vide his Qanun-ul-Masudi Volume I Chapter VI page 308) as 0'-35"-36'''-14''''-40''''' and 59'-59"-49'''-2''''-28''''' respectively, which when converted into decimal fractions become 0.0098900205 and 0.9999492644 respectively, and when converted into the lowest sub-divisions of "1/60th" system become 7690480''''' and 777560548''''' respectively. In this "1/60th" system the values are given in the form of divisions or sub-divisions of 1/60th parts upto any lower sub-divisions required. The values given by Al-Biruni are upto 5th sub-divisions of 1/60th parts which can be easily converted into decimal fractions if the sub-divisions are divided by 60'x60'x60'x60'x60' = 777600000''''' which actually represents the sinus-totus or Sine 90^{o}, or "1".

Then he calculated the radius of Earth in the following simple way:



In the above exagerated diagram suppose BCH is the circumference of Earth, AB is the perpendicular height of the peak A, OH=OB is the radius of Earth, AH is the line of vision from the peak A to the visible horizon H, AD is the horizontal line as seen from the peak, DAH is the angle of dip of horizon which is 34'in this case, HAO is the complementary angle of the dip of horizon. Now in the right angled triangle AHO, AO is the hypotenuse and if it is supposed to be the sinus totus, its value can be taken as "1" or 777600000''''. Then OH=OB=Sine of angle HAO=cosine of angle DAH=cosine of angle of dip of horizon=cosine 34'=777560548''''. Hence AB =AO-OB=sinus totus minus cosine 34'=777600000'''' - 777560548'''' = 39452''''. NS is the axis of Earth, and EW is the equator of Earth.

Now, $\frac{OB}{AB} = \frac{777560548''''}{39452''''}$, or $OB = \frac{AB \times 777560548''''}{39452''''}$, but OB is the radius of Earth and AB is the height of the peak, therefore,

$$\text{Radius of Earth} = \frac{2347398'' \times 777560548}{39452} \text{ cubits}$$

$$= \frac{1825244075254104''}{39452} \text{ cubits}$$

$$= 46264931442'' \text{ cubits}$$

$$= 12851369\text{-}50' - 42'' \text{ cubits.}$$

After finding out the above value of radius of Earth, he calculated the circumference of Earth as 80780039-1'-33" cubits and length of one degree of

latitude equal to 1/360th part of the circumference i.e. 224388-59'-50" cubits. An Arabic mile is equal to 4000 cubits and a cubit is equal to 1.61825 feet or one Arabic mile is equal to 1.225947 English miles (vide Encyclopedia of Islam by Dr. Arnold); therefore one degree of latitude is equal to $\frac{1.61825 \times 224388\text{-}59'\text{-}50''}{5280}$,

$$= \frac{363117.4947}{5280} \text{ English miles}$$

$$= 68.772 \text{ English miles}$$

4. COMPARISON OF VALUES:-

Just like other scholars of his time, al-Biruni had also considered the globe of Earth as a sphere and as such he had calculated the length of one degree of latitude on the surface of the sphere. Whatever value he had found out at about 32^{O}N latitude, he thought that the same value was true for any place between the equator and the poles. But in modern times it is revealed that Earth is not a perfect sphere and is slightly flattened at its North and South poles. The actual figure of the earth is an irregular surface, and is not suitable for expressing the position of points on it. In order to specify positions of points on the earth and to carry out various calculations, a regular geometrical figure known as reference spheroid or ellipsoid is introduced, which is a surface formed by the revolution of an ellipse about its minor axis. The science which deals with the shape & dimensions of the earth is known as Geodesy and is very much developed in these days. But in the days of al-Biruni the world knew very little about Geodesy and it was a very courageous step taken by him to undertake such a highly technical task about a thousand years ago.

Now it has been practically determined that the length of one degree on the same meridian is not the same throughout as thought by ancient scholars, but it varies from latitude to latitude; it is minimum at the equator and gradually increases towards the poles, and at poles it is maximum. In view of this fact the correct way to judge the accuracy of the values of one degree, obtained by the ancient scholars at different latitudes, is to compare these values with the corresponding values obtained by the latest method.

For the sake of comparison we take two more values prevalent at that time, in addition to the value of al-Biruni. One value was obtained by a party of astronomers of Caliph Mamun-ur-Rashid in a plain desert near Mosul between 36^{O} N and 37^{O} N latitudes and was 56^{2}/3 Arabian miles per degree, which amounts to 69.470 English miles per degree. This value was considered to be the standard value in the Middle East at the time of al-Biruni. Similarly in India the standard value at that time, as given in Ghurrat-uz-Zijat, was $13\frac{29}{30}$ yojanas per degree, and this value was verified by Vijaya Nandi, the author of Karana Tilaka, in the plains near Varanasi between 25^{O} N and 26^{O} N latitudes. As one yojana is equal to 16000 cubits or 4.9 English miles, this value amounts to 68.436 English miles per degree. Lastly the value determined by al-Biruni by the simplest method between 32° N and 33° N latitudes was 68.772 English miles as already explained.

In modern times the values of meridian lengths per degree of latitude determined on the basis of the latest reference Ellipsoid 1967 are considered to be the most accurate values. The dimensions of this Reference Ellipsoid 1967 are based on the values of the astronomical constants as adopted by the International Astronomical Union in 1964. In this Ellipsoid the equatorial semi axis is 6378.160 Kilometres, the polar semi axis is 6356.775 Kilometres, flattening is 0.0033529237, and square of eccentricity is 0.00669460532856. As per the calculations of this Ellipsoid and taking 1 Kilometre equal to 0.621372 English miles the length of one degree of meridians between 36° N and 37° N latitude is 68.954 English miles, between 25° N and 26° N latitude it is 68.837 English miles, and between 32° N and 33° N latitude it is 68.909 English miles.

So if we compare the value of the astronomers of Caliph Mamun-ur-Rashid i.e. 69.470 miles per degree with the latest value at the same latitude i.e. 68.954 miles per degree, we find a plus error of 0.516 miles which amounts to 0.75 percent positive error; if we compare the value of Indian astronomers i.e. 68.436 miles per degree with the latest value at the same latitude i.e. 68.837 miles per degree we find a minus error of 0.401 miles which amounts to 0.58 percent negative error; and if we compare the value of al-Biruni i.e. 68.772 miles per degree with the latest value at the same latitude i.e. 68.909 miles per degree, we find a minus error of 0.137 miles which amounts to 0.19 percent negative error. In this way we see that the value of al-Biruni is much more accurate than the other two values, because in the value of Indian astronomers the percentage error is nearly three times more than that of al-Biruni, and in the value of Mamun's astronomers the percentage error is nearly four times more than that of al-Biruni. The scholars can very well appreciate how easily the wonderfully correct value was obtained by al-Biruni about a thousand years ago, in spite of many practical difficulties in his way, which will be explained later.

Although it will not be fair to compare al-Biruni's values in the following manner, yet we are giving few more ordinary comparisons for general interest. As per al-Biruni the circumference of Earth comes to 24757.92 miles and as per reference ellipsoid 1967 the mean circumference, by the standard formula, comes to 24873.75 miles; the difference is only 115.83 miles. Similarly, the Earth's radius as per al-Biruni is 3938.77 miles and as per reference ellipsoid 1967 the mean radius is 3958.78 miles; the difference is only 20.01 miles. The so called radius of Earth calculated by al-Biruni as 3938.77 miles is actually the radius of curvature of the meridian at about 32° N latitude; so if we find the latest value of the radius of curvature as per reference ellipsoid 1967 at 32° N latitude by the relevant formula, we get 3947.80 miles; the difference is only 9.03 miles, which is a negligible error.

It will not be out of place to mention here that this correct measurement of Earth's dimensions by al-Biruni was about 700 years before the French scientist Picard came to the conclusion about the correct circumference of Earth. Before him, the European scientists and mathematicians including Newton had believed that the circumference was 17000 miles. Later Newton corrected his views and based his theories on Picard s Experiments. Had he known the achievement of al-Biruni, he would have not faced all that trouble in formulating his Gravitational Laws, and theories of Planetary Motions.

5. PRACTICAL DIFFICULTIES:-

There were tremendous practical difficulties and numerous sources of error in that unique type of experiment which al-Biruni had performed at Nandna to find out the dimensions of Earth, but ne had very scholarly managed to cover up all such draw backs and short comings of the experiment. It appears that Providence had also helped him in his bold step, because many minor types of errors, which could rise to a considerable extent if added together, had automatically nullified each other miraculously by having plus and minus signs. Here it is proved that "God Helps those who help themselves". In order to give an idea of the precautions which al-Biruni had to take in performing the experiment, three main difficulties are described below. The effect of atmospheric refraction etc must have also been accounted for by al-Biruni through his personal experience of such measurements while taking the reading of the distant horizon.

The greatest difficulty of the experiment was to find out the exact value of the angle of dip of horizon which was very small, but the self-made theodolite of al-Biruni could only read upto minutes of a degree by interpolation. The manufacture of such a precision instrument in those days was itself a very great achievement of al-Biruni. The importance of this reading can be well realized by the fact that an error of 1 minute of a degree in the angle of dip of horizon at that stage could make an error of approximately 250 miles (average) in the calculated value of the radius of Earth. But al-Biruni had so accurately measured the angle of dip of horizon that there was no appreciable error in his final result.

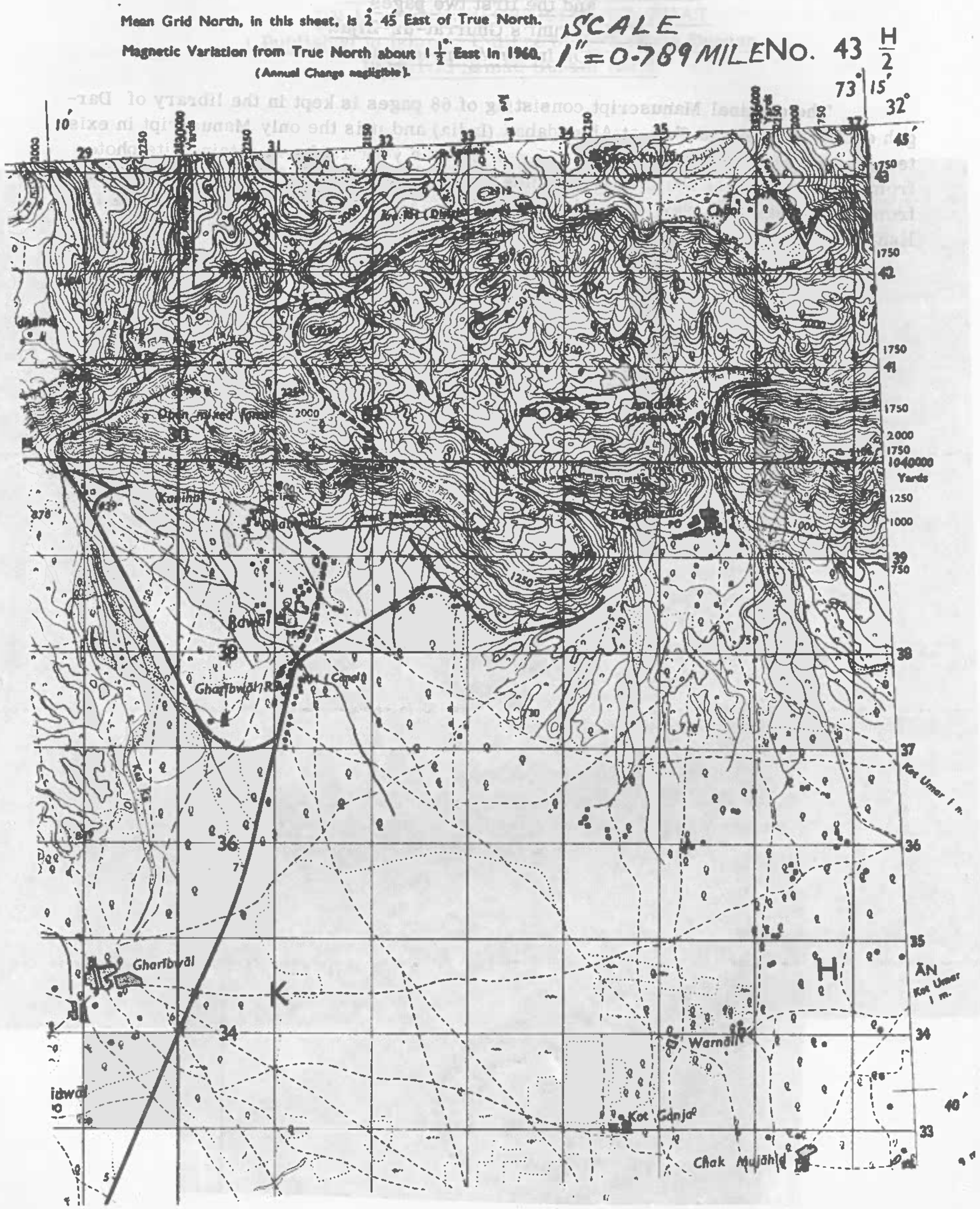
The second main difficulty of the experiment was to find out the most accurate value of the cosine of the angle of dip of horizon which was very small, but the self-made Table of sines of al-Biruni could not provide the requisite standard of accuracy. The importance of this value can be well realized by the fact that an error of 0.000001 in the value of the cosine of the angle of dip of horizon at that stage could make an error of approximately 75 miles (average) in the calculated value of the radius of Earth. But al-Biruni had so judiciously found out the value of cosine of the angle of dip of horizon that there was no appreciable error in his final result.

The third main difficulty of the experiment was to find out the correct height of the peak with respect to the effective level of the adjoining plains, but al-Biruni was not in a position to take innumerable levels at very short intervals between the peak and the visible horizon which was about 39 miles away and then to calculate the effective level of the adjoining plains by the help of the graph of these innumerable levels. The importance of the height of peak with respect to the adjoining plains can be well realized by the fact that an error of 1 foot in the height of peak could make an error of approximately $1\frac{1}{2}$ miles (average) in the calculated value of the radius of Earth. But al-Biruni had so intelligently selected an effective spot of the plains for his measurements of the height of the peak that there was no appreciable error in his final result.

SAIYID SAMAD HUSAIN RIZVI
C. & G. (London); B.Sc. (Eng.) (Alig.)
Executive Engineer, M.E.S., Pakistan.

N.B. - In the following pages the photo-copy of the original Arabic Manuscript of al-Biruni's book Ghurrat-uz-Zijat, newly discovered by me (i.e. 67 pages and the title page) is also given. This photo-copy was obtained by me from the Library of Dargah of Pir Muhammad Shah at Ahmadabad (INDIA). After giving this photo-copy, I have also given few specimens (i.e. 52 printed pages) of the research work carried out by me on this Ghurrat-uz-Zijat, for general interest of the scholars. My complete research work on this book had already been published about ten years ago in eight instalments in the Quarterly Journal, Islamic Culture of Hyderabad Deccan (INDIA), beginning from April 1963 A.D.

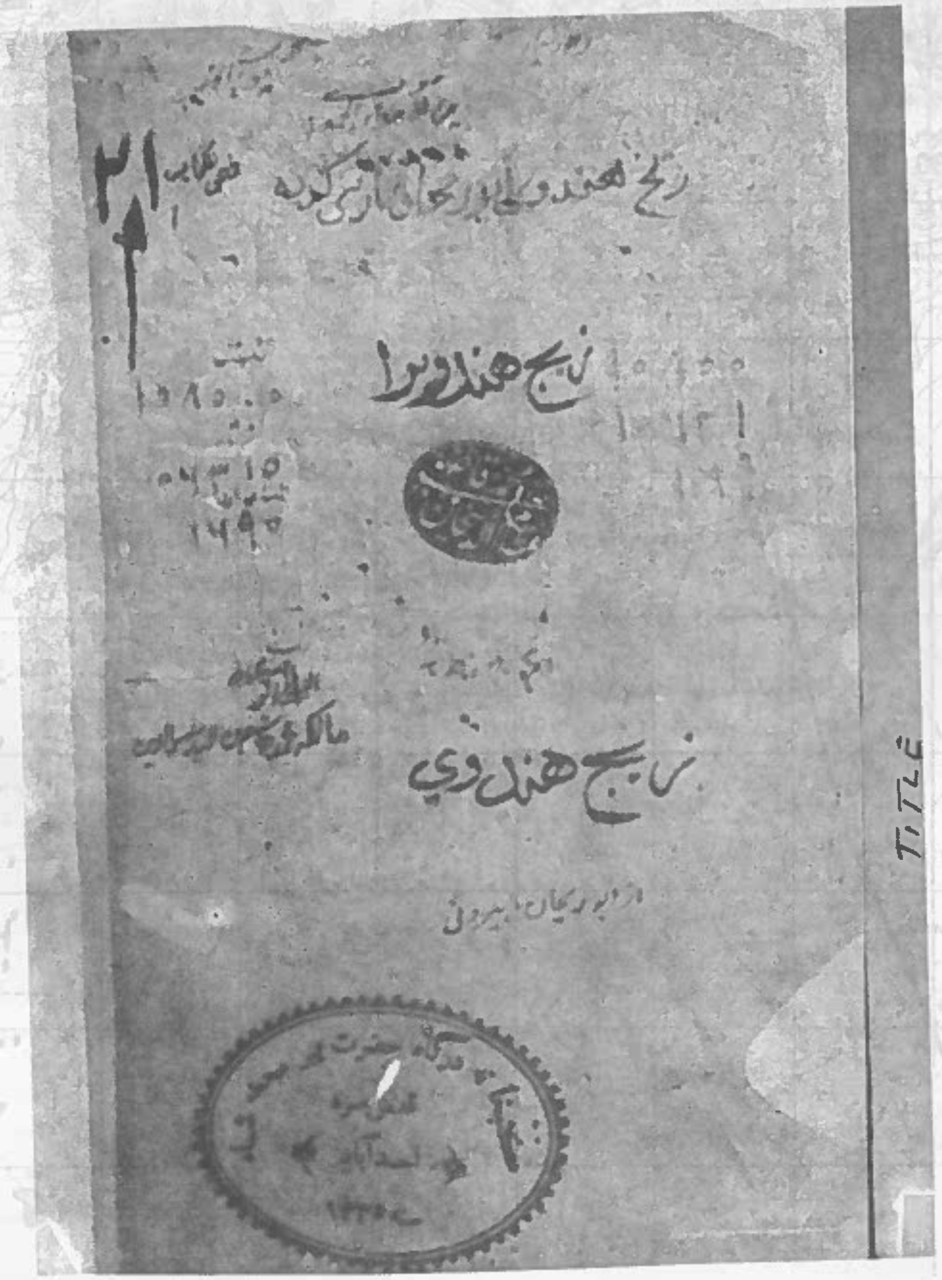
CONTOURS OF NANDNA AREA
(where al-Biruni had accurately measured the Dimensions of Earth)



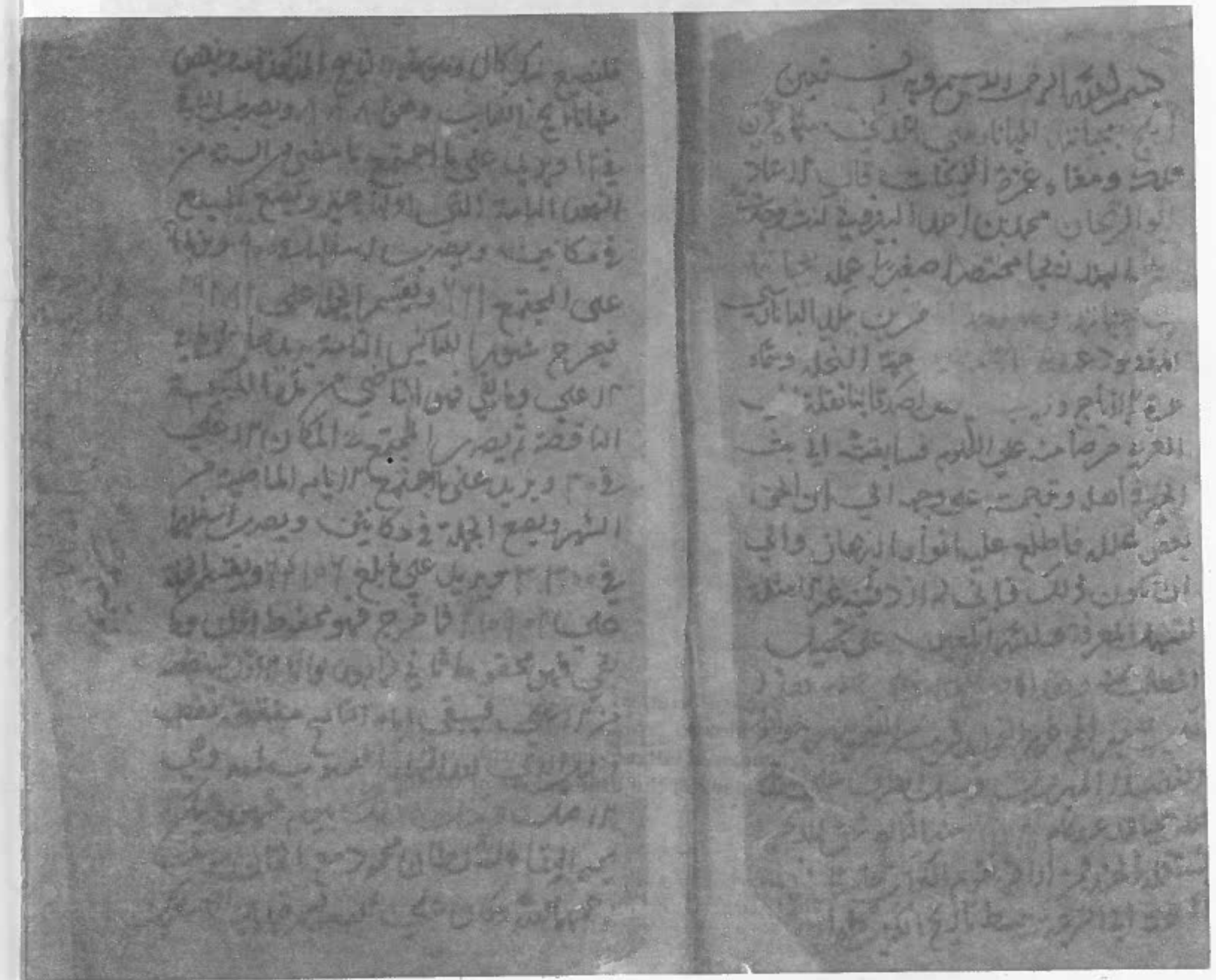
Photos of the Title Page and the first two pages of al-Biruni's Ghurrat-uz-Zijat. (On Indian Astronomy)

The original Manuscript consisting of 68 pages is kept in the library of Dargah of Pir Muhammad Shah at Ahmadabad (India) and it is the only Manuscript in existence in the whole world. I had discovered it in the year 1959 and obtained its photos from India with great difficulties; its clue had been found out by me in the year 1953 from Dr. Muhammad Nazim's famous book Life and Times of Mahmud of Ghazna Published in the year 1931 by Cambridge University Press. (vide its pages 55 and 239).

Saiyid Samad Husain Rizvi



SPECIMENS OF RESEARCH WORK ON AL-BIRUNI'S GHURRAT-UZ-ZIJAT (Published in Islamic Culture of Hyderabad Deccan) By Saiyid Samad Husain Rizvi



General view of Nanda fort
As viewed from North to South
(where al-Biruni has acurately
measured the dimensions of earth.)

IT would be a great surprise to the learned world, especially to the orientalists, if I say that I am going to reveal to them the contents of a book which was considered to have been lost long ago. From the writings of Abu Rayhan Muhammad al-Beruni it was known that a very valuable book on Indian Astronomy, named *Karana Tilaka*, existed at the beginning of the eleventh century A. D. and that it was very popular throughout India in those days. Beruni gave numerous quotations from this book in his various works, but nowhere did he specifically mention that he had completed an Arabic translation of the whole book. Shankara Balkrishna Dixit had also declared in his well-known book on the History of Indian Astronomy that *Karana Tilaka* was no longer in existence anywhere in the world. But now we find that a marvellous Arabic translation of this book made by Beruni himself exists in the library of Dargah of Pir Muhammad Shah, Ahmadabad under the name of *Ghurrat-uz-Zijat*. I consider myself most fortunate that I am placing this important book before the public for the first time.

Beruni has mentioned the name of *Karana Tilaka* or *Ghurrat-uz-Zijat* at many places and has given ample quotations from this work. For instance, the following references may be seen:-

(1) كتاب تحقيق ما للهند Hyd. Ed., p. 121 (1.7), p. 266 (1.16), p. 280 (1.10), p. 346 (1.18), p. 384 (1.14), p. 392 (1.17), p. 410 (1.12) p. 419 (1.10), p. 420 (1.2), p. 511 (1.1 & 19). p. 513 (1.20)

(2) القانون المسعودی Hyd. Ed., p. 973 (1.7), p. 1313 (1.15 & 20).

(3) تمهيد المستقر Hyd. Ed., p. 27 (1.18), p. 32 (1.4).

(4) افراد المقال Hyd. Ed., p. 107 (1.1), p. 136 (1.8), 152 (1.7)

So far as the Arabic manuscript of *Ghurrat-uz-Zijat* is concerned, I think it was first referred to by Dr. Muhammad Nazim in his well-known work "The Life and Times of Sultan Mahmud of Ghazna" published in 1931 by the Cambridge University Press. On page 55 of this book is mentioned the date on which Sultan Mahmud and Khan Yousuf Qadir Khan met near Samarkand, and a footnote says, "This date is given by al-Beruni in his unique and hitherto unknown work named *Ghurrat-uz-Zijat*" Again, in appendix O at page 239 of the same book a further clarification is given.

"*Ghurrat-uz-Zijat* being a translation into Arabic of the Sanskrit *Karana Tilaka* of Vijaya-Nanda, son of Jayananda of Benares. (Pir Muhammad Shah's Dargah Library, Ahamadabad)."

From the above-mentioned references it is clear that the manuscript of *Ghurrat-uz-Zijat* kept in Pir Muhammad Shah's Dargah Library is a very authentic and valuable document, especially because its original Sanskrit book *Karana Tilaka* has definitely been lost, and I have not been able to trace it anywhere. The real importance of *Karana Tilaka* lies in the fact that it is a missing link between *Karana Khandakhadyaka* of Brahmagupta and *Karana Kuthala* of Bhaskaracharya. Shankara Balkrishna Dixit was very anxious to trace out a Karana based wholly on modern Suryasiddhanta so that its antiquity could be proved, but unfortunately he could not find it out. Had he been alive today, he would have been extremely happy to see that his dream has taken a real shape in the form of the Arabic translation of *Karana Tilaka* which is entirely based on modern *Suryasiddhanta*. And now we can say without any doubt that the so called modern *Suryasiddhanta* is not " modern " at all, it is much more ancient than the 10th century A. D.

Another important and interesting fact about *Karana Tilaka* is that, according to it, the revolutions of Chandrochcha and Rahu in a Chaturyuga are, respectively, 488211 and 232234, while these revolutions, according to the old Suryasiddhanta, are 488219 and 232226 and, according to the modern Suryasiddhanta, are 488203 and 232238. By comparing the above-mentioned three different versions, we see clearly that *Karana Tilaka* is a connecting link between the old and the modern Suryasiddhantas. It will not be out of place to mention here that Makaranda, the author of the famous *Makaranda Sarani* had further modified these revolutions to 488199 and 232242, respectively, in order to bring the calculated values closer to the observed conditions of solar and lunar eclipses. It will be seen that the Chandrochcha revolutions are being gradually decreased while the Rahu revolutions are being gradually increased. All this is being done to enable one to calculate the accurate time of a solar and lunar eclipse, but unfortunately the error has not been eliminated upto this time. As a matter of fact the main reason for this error does not lie in the number of revolutions of Chandrochcha and Rahu, but it lies in the basic assumption that at the beginning of Kaliyuga all the mean planets were on the first point of Aries. Actually the mean planets were not exactly on the first point of Aries at the beginning of Kaliyuga; they were at various distant places somewhere near the first point of Aries. So if we want to rectify our calculations, we should modify our basic assumption and not go on changing the number of revolutions.

Apart from the above-mentioned facts, there are many other peculiarities of *Karana Tilaka* which will be described side by side with the translation of that book. The original Arabic manuscript of *Ghurrat-uz-Zijat* consists of single articles arranged serially, but I have divided the whole text into fourteen chapters and have also suggested an appropriate heading for each chapter. I have not changed the order of the original text except at one or two places where it was considered most essential. It appears that the original Sanskrit manuscript possessed by Beruni was full of mistakes of the scribe and the same mistakes have consequently crept into its Arabic translation. The

Arabic scribe of the existing manuscript has made many mistakes in his turn. Also there are many mistakes in the solution of the numerical examples which are due to careless calculations most probably made by one of his disciples, because very often he used to get such tasks done by his pupils. I have been able to correct all these and other mistakes in the text and have explained such corrections fully in the commentary on each article.

The following is the list of contents of *Ghurrat-uz-Zijat* or *Karana Tilaka* and its first two articles with translation and commentary.

بسم الله الرحمن الرحيم

وبه نستعين

زيج بجاناند البانارسى الذى سمّاه كَرَنَ تلَكَ و معناه غُرَّة الزيجات

فهرس الموضوعات

الباب الاول - فى استخراج الاصل من التواريخ المختلفة - (١) سبب تاليف الكتاب (٢) استخراج الأصل من التاريخ شك كال (٣) انتخاب التاريخ (٤) مثال لاستخراج الأصل من التاريخ شك كال (٥) استخراج تاريخ يزدجرد من هذا التاريخ (٦) استخراج تاريخ الهجرة من هذا التاريخ. (٧) استخراج تاريخ الاسكندر من هذا التاريخ (٨) استخراج الأصل من احد هذه التواريخ الثلاثة (ا) استخراج الأصل من تاريخ الهجرة (ب) استخراج الأصل من تاريخ يزدجرد (ج) استخراج الأصل من تاريخ الاسكندر (د) استخراج شك كال من الأصل - (٩) معرفة اصحاب نوب الازمنة (ا) معرفة والى السنة (ب) معرفة والى الشهر (ج) معرفة والى اليوم (د) معرفة والى الساعة.

الباب الثانى - فى استخراج اوساط النيرين وتقويمها - (١) استخراج وسط الشمس. (٢) استخراج وسط القمر (٣) استخراج اوج القمر (٤) استخراج وسط الراس وتقويمه (٥) معرفة تصحيح اوساط البلدان (٦) معرفة بعد الاوساط من نصف الليل الى وقت آخر (٧) استخراج جيب القوس. (٨) استخراج قوس الجيب (٩) معرفة تقويم النيرين.

الباب الثالث ـ في معرفة الاركان الخمسه ـ (١) معرفة طول النهار والليل (٢) معرفة منازل القمر (٣) معرفة يوم القمر من شهره (٤) معرفة الجوكات متي الشهر (٥) معرفة كرنات في الشهر

الباب الرابع ـ في استخراج اوساط الكواكب الخمسة ـ (١) معرفة استخراج وسط المريخ (٢) معرفة استخراج سرعة عطارد (٣) معرفة استخراج وسط المشترى (٤) معرفة استخراج سرعة الزهره (٥) معرفة استخراج وسط زحل.

لباب الخامس ـ في تقويم الكواكب الخمسة ـ (١) ذكر اوجنت الكواكب (٢) معرفة استخراج تعديل الحصة (٣) معرفة استخراج تعديل الخاصة (٤) معرفة استخراج تقويم الكواكب الخمسة (٥) معرفة بهت الكواكب (٦) معرفة رجوع الكواكب واستقامتها (٧) جوزهرات الكواكب (٨) تعديل جوزهرات الكواكب (٩) استخراج عرض الكواكب.

الباب السادس ـ في الامثلة الثلثه ـ (١) استخراج عرض القمر (٢) معرفة ميل القمر المعدل (٣) استخراج مقدار النهار (٤) معرفة خط نصف النهار (٥) معرفة عرض البلد من قبل ظل نصف النهار (٦) معرفة عرض البلد من ظل الاستوا (٧) معرفة ظل الاستواء من قبل عرض البلد (٨) معرفة تمام ارتفاع نصف النهار (٩) معرفة سهم النهار (١٠) معرفة الماضي من النهار من قبل الظل في الوقت (١١) معرفة الظل في الوقت من قبل الماضي من النهار (١٢) معرفة مطالع البلاد (١٣) معرفة الطالع من الماضي من النهار (١٤) معرفة الماضي من النهار من قبل الطالع.

الباب السابع ـ في خسوف القمر (١) معرفة مقدار النيرين (٢) معرفة مقدار الراس (٣) معرفة خسوف القمر.

الباب الثامن ـ في معرفة كسوف الشمس.

الباب التسع ـ في جدول انحراف التصوير.

الباب العاشر ـ في ظهور الكواكب واختفائها.

الباب الحادي‌عشر ـ في معرفة اجتماع الكواكب (١) معرفة مقادير الكواكب (٢) معرفة اجتماع الكواكب.

الباب الثاني عشر ـ في معرفة اجتماع الكواكب بالثوابت.

الباب الثالث عشر ـ في معرفة رؤية الهلال (١) معرفة رؤية الهلال (٢) معرفة ما مضى من القمر.

الباب الرابع عشر ـ في معرفة بتيبات و بيدهرت.

الباب الاول

في استخراج الأصل من التواريخ المختلفة.

(١) سبب تأليف الكتاب: ـ

زيج بجيانند البانارسي الذي سماه كرن تلك و معناه غرة الزيجات ـ قال الاستاذ ابو الريحان محمد بن احمد البيروني كنت وجدت في الهند زيجا مختصرا صغيرا عمله بجيانذ بن جيانسند و هو احد المفسرين ببلد البانارسي المقصود عندهم بالتعظيم من جهة الخلة و سماه غرة الازياج و احب بعض اصدقائنا نقله الى العربي حرصا؟ منه على العلوم فسابقته الى بث الخير في اهله و ترجمته على وجهه الى ان ألحق بعض علله فاطلع عليه انوار البرهان و الى ان تكون ذلك فاني لم ازد فيه غير الامثلة لتسهل المعرفة و الله المعين على تحصيل المطالب بمنه ـ .

(٢) استخراج الأصل من التاريخ شك كال: ـ .

و هذا فص الكتاب على نظام ـ هذا كتاب صغيرا لحجم غزير الفوائد قريب من الفهم بنير خواطر الفضلاء المبرزين و يسهل الطرق على المبتدئين، عمله بجيانذ عند تمام ٨٨٨ سنة التاريخ الشق الذي يستعمله الهند ـ فمن اراد تقويم الكواكب جامعا فيه الصحة الى السرعة يبسط تاريخ الكتاب كله أياما فليضع شك كال و هو سنو التاريخ المذكور تامة و ينقص منها تاريخ الكتاب وهو ٨٨٨ و يضرب الباقي في ١٢ و يزيد على ما اجتمع ما مضى من السنة من الشهور التامة التي اولها جيتر و يضع المبلغ في مكانين و يضرب اسفلها في ٩٠٠ و يزيد على المجتمع ٦٦١ و يقسم الجملة على ٢٩٢٨٢ فيخرج شهور الكبائس التامة يزيدها على ما في الاعلى و ما بقى فهو الماضي من مدة الكبيسة الناقصه ثم يضرب المجتمع في المكان الاعلى في ٣٠ و يزيد على ما اجتمع الايام الماضية من الشهر و يضع الجملة في مكانين و يضرب اسفلها في ٣٣٠٠ و يزيد على ما بلغ ٦٤١٠٦ و يقسم الجملة على ٢١٠٩٠٢ فما خرج فهو محفوظ اول و ما بقى فهو محفوظ ثاني لما بعد ـ فأما الاول فينقصه من الاعلى فيبقى ايام التاريخ مفتتحة بنصف الليل الذي بعد النهار المحسوب و هي الأصل ـ

CHAPTER I—*Relating to Ahargana.*

1. Karana Book of Vijaya Nandi of Varanasi the name of which is Karana Tilaka and its (Arabic) translation is Ghurrat-uz-Zijat.
2. The substance of the book and its arrangement.
3. Selected date for the example.
4. Example for the selected date.
5. To find the date of Yazdjird era from this date.
6. To find the date of the Hijra era from this date.
7. To find the date of the Roman era from this date.
8. To find the ahargana from the date of the Hijra era.
9. To find the ahargana from the date of Yazdjird era.
10. To find the ahargana from the date of the Roman era.
11. To find the date of the Ṣhaka-kala from the ahargana.
12. To find the Lord of the year.
13. To find the Lord of the month.
14. To find the Lord of the day.
15. To find the Lord of the hour.

CHAPTER II—*Relating to the sun and the Moon.*

1. To find the mean place of the sun.
2. To find the mean place of the moon.
3. To find the place of Chanrochcha.
4. To find the mean and true place of Rahu.
5. Correction for the mean longitude of the place.
6. Correction for any time other than the midnight.
7. To find sine of an arc.
8. To find the arc of a sine
9. To find true places of the sun and the moon.

CHAPTER III—*Relating to the Punchanga.*

1. To find the duration of a day and a night.
2. To find the Nakshatra.
3. To find the tithi in a month.
4. To find the Yoga in a month.
5. To find the Karana in a month.

CHAPTER IV-*Relating to the mean places of the planets.*

1. To find the mean place of Mars.
2. To find the Budhochcha.
3. To find the mean place of Jupiter.
4. To find the Shukrochcha.
5. To find the mean place of Saturn.

CHAPTER V—*Relating to the true places of the planets.*

1. Uchchas of the planets.
2. To find the Mandphala.
3. To find the Sighraphala.

4. To find true places of the five planets.
5. To find true daily motions of the planets.
6. To find retrogression and direction of the planets.
7. Nodes of the planets.
8. Equation of the Nodes of the planets.
9. To find the latitude of the planets.

CHAPTER VI-*Relating to the three Problems of Diurnal Motion.*

1. To find the latitude of the Moon.
2. To find the true declination of the Moon.
3. To find the duration of a day.
4. To find the meridian line.
5. To find the latitude from the shadow at noon.
6. To find the latitude from equinoctial shadow.
7. To find the equinoctial shadow from the latitude.
8. To find the co-altitude of the Sun at noon.
9. To find the versed-sine of the day.
10. To find the past period of the day from the shadow at any time.
11. To find the length of shadow at any time from the past period of the day.
12. To find the duration of ascendants at any place.
13. To find the ascendant from the past period of the day.
14. To find the past period of the day from the ascendant.

CHAPTER VII—*Relating to the Lunar eclipse.*

1. To find the measures of the Moon and the Sun.
2. To find the measure of the shadow.
3. Detailed description of the lunar eclipse.

CHAPTER VIII *Relating to the solar eclipse.*

CHAPTER IX— *Relating to the shapes of the lunar and solar eclipse.*

CHAPTER X— *Relating to the heliacal rising and setting of the planets.*

CHAPTER XI— *Relating to the conjunctions of the planets.*

1. To find the measures of the planets.
2. To find the conjunctions of the planets.

CHAPTER XII— *Relating to the conjunctions of the planets and stars.*

CHAPTER XIII- *Relating to the visibility of the Moon*

1. To find the visibility of the Moon.
2. To find the lighted portion of the Moon.

CHAPTER XIV- *Relating to the Vyatipala and Vaidharita.*

CHAPTER I

Relating to Ahargana

1. Karana Book of Vijaya Nandi of Varanasi the name of which is Karana Tilaka and its (Arabic) translation is Ghurrat-uz-Zijat:-

Master Abu Rayḥan Muhammad ibn Ahmad-al-Beruni said :-

" In India I came across a Karana book which was very concise and was written (in Sanskrit) by Vijaya Nandi, son of Jaya Nandi, one of the commentators of the city of Varanasi. The Hindus have a high regard for it from the religious point of view and the author has named it as *Karana Tilaka.* Some of my friends, who are fond of knowledge, have expressed their desire to get it converted into Arabic. So I hasten to do this work in order to spread knowledge among those who deserve it. I am translating it in such a way that its reasonings are expressed clearly, and with this aim in view I have not added anything from my own side except the examples which are meant to make the understanding of the subject easy. And it is God Who, through His kindness, helps in achieving the objects."

COMMENTARY

(a) *Correction of the Manuscript :-*

No correction is required.

(b) *Translation of the technical terms :-*

(i) *Ahargana :-* It is a Sanskrit term which means "number of civil days". Beruni has translated this term into Arabic as "Asl" meaning root or base, but I have preferred to use the original Sanskrit term in the English translation.

(ii) *Karana Book:-* It is the translation of the Arabic term "Zij". In English I could not find a suitable equivalent. A Karana book is generally a book which is based on some astronomical canon known as Siddhanta, but it gives much easier methods of calculations as compared with those of Siddhanta. As will be seen later, this Karana book is based on modern Surya Siddhanta.

(iii) *Vijaya Nandi:-* In the Arabic manuscript this name is written as "Bijaya Nand" but its correct pronunciation is the same as I have given in the translation. This author flourished in Vārānasi in the middle of the 10th century A. D. Another Indian astronomer of this name flourished in the 5th century A. D. or even earlier than that, Dr. Edward C. Sachau seems to have got confused, because he thinks both these astronomers identical, although there is a time gap of more than five centuries between these two personalities (Vide *Alberuni's India*, 1910, Vol. II, page 306 Annotations)

(iv) *Varanasi:-* In the Arabic manuscript this name is written as Banarasi, but I have preferred to write it as Varanasi, because it is more correct and at present also the city is known by this name. This is a famous city of India situated on the banks of the river Ganges and its religious status is still maintained.

(v) *Ghurrat-uz-Zijat:-* This is the Arabic translation of the Sanskrit name "Karana Tilaka," and Beruni has himself mentioned this name in his Indica (vide *Alberuni's India*, 1910, vol. II, page 90). Dr. Edward C. Sachau has made a serious mistake in

assuming this book to be identical with the *Kitab-algurrah* (vide *Alberuni's India* 1910, Vol. II, page 388 Annotations), although at another place he clearly states that the title of Vijaya Nandi's book *Karana Tilaka* would be in Arabic Gurraht-uz-Żijat (vide *Alberuni's India.* 1910, Vol II page 345 Ānnotations). This name is given by Beruni to his Arabic translation of *Karana Tilaka.*

(vi) *Beruni :-* He was the most learned Muslim author of the early 11th century A.D. He wrote more than 150 books on various subjects and translated many Sanskrit books into Arabic. The present book is perhaps the only Arabic translation of a Sanskrit book and probably the only manuscript available, but has hitherto been unknown to the world. Beruni had made this translation before writing his Indica, but he could not revise and rectify the manuscript afterwards; that is why he did not mention anywhere that he had made a translation of *Karana Tilaka.* He quoted only those passages of this translation in his *Indica*, which he had verified from *Pulisa Siddhanta* available with him. He did not quote those passages of this translation which were based on *Surya Siddhanta* because the *Surya Siddhanta* was not available with him and hence he could not verify the correctness of those passages. Due to these unverified passages, he did not think it proper to mention the name of this translation as a complete and authentic work. He, however, had given an indication of it in his letter to one of his friends nearly 13 years before his death, where he had mentioned that there were various incomplete translations of Sanskrit books with him at that time. Beruni was born in 973 A. D. and he died in 1048 A. D. As stated in the *Encylopaedia Britannica* and judged from his writings, Beruni was a Shia by faith and believed in the twelve Imams. His complete life has been given in the Urdu book named "Alberuni" written by Syed Hasan Barni, who died on 16 October, 1959.

(vii) *Commentator :-* This is a literal translation of the Sanskrit term "Tikakara", for which Beruni has used the Arabic word "Mufassir." In Sanskrit an Acharya is one who writes a Siddhanta on astronomical principles, but a Tikakara is one who merely follows the Siddhantas and, on the basis of these Siddhantas, writes a Karana book introducing easy calculations.

(c) *Elaboration of the principle :-*

No elaboration of any principle is required here.

(d) *Explanation of the text :-*

In this article Beruni has expressed his views about the Sanskrit book *Karana Tilaka* written by Vijay Nandi, son of Jaya Nandi of Varanasi. From his statement it is clear that this book was very popular among the Hindus throughout the whole country. It appears that in those days, that is at the beginning of the 11th century A. D., it was the only Karana book which was considered a standard book by the Indian astronomers and astrologers. Most probably the main reason for its popularity was that its calculations were based on the modern *Surya Siddhanta* which is considered by the Hindus as a revelation from Sun God to Maya Danava. It will be observed later on that even the ahargana of this book is based on the modern *Surya Siddhanta* and not on *Pulisa Siddhanta* as believed by Dr. Sachau and Dr. Schram (vide *Alberuni's India*, 1910, vol. II, page 375, Annotations). The methods of calculations given in this book are so accurate that they can be adopted even today without any appreciable error, although the usual range of a Karana book is supposed to be only two or three centuries. The most important fact about this book is that its text has proved the antiquity of the modern *Surya Siddhanta*, because it is based on it and now we can say without any doubt that the time of the so called modern *Surya Siddhanta* is much earlier than the 10th century A. D.

Beruni has not given a literal translation of the text of *Karana Tilaka*; he has given only a summary of this book in his own words. By adopting this method of reproduction he has made a few mistakes because he had some wrong notions in his mind about certain technical terms of the Hindus such as *tithi*, *Karana*, *Yoga*, etc. Moreover, the Sanskrit manuscript possessed by Beruni seems to be carelessly written and Beruni has copied the same figures in his translation without any verification. Such mistakes have been rectified by me in this English translation and full justification of this rectification has also been given.

The most unfortunate part of it is that Beruni could not get hold of a copy of the modern *Surya Siddhanta* as stated by him in his *India* (vide *Alberuni's India*, 1910, vol. I, page 154), and therefore he could not verify many formulae mentioned in *Karana Tilaka* He could not even know that *Karana Tilaka* was actually based on the modern *Surya Siddhanta*, otherwise he would have stated this fact somewhere. The solved examples given by Beruni from his own side are very useful but are not free from petty mistakes of calculation; these mistakes have been rectified in the English translation. At certain places such solved examples are not given by Beruni, and so I have tried to give some additional examples in the commentary.

(e) *Additional examples* :–

No examples are required.

2. *The substance of the book and its arrangement* :–

This book is small in bulk, important for use and easy to understand. It enlightens the brains of big scholars and smoothes the path for beginners. It is written by Vijaya Nandi at the end of the year 888 Shaka-kala which is in use amongst the Hindus.

Any one who wants to compute the true places of the planets completely, correctly and quickly, should find out the total number of civil days since the date of this book (upto the selected date). Take the complete years of Shaka-kala of the selected date and deduct from these the years of this book, i.e. 888. Multiply the remainder by 12 and add to the product the complete (Synodic lunar) months of the current year of Shaka-kala beginning from (Shukla Pratipada of) Chaitra. Put the sum at two places and multiply the lower one by 900; add to the product 661 and divide the sum by 29282; the quotient represents the number of complete adhimasas which are to be added to the number already kept at the upper place; the remainder represents the past period of the incomplete adhimasas (which is to be ignored, because only complete adhimasas are taken into account). Now multiply the sum at the upper place by 30 and add to the product the past tithis of the current month (at Lanka upto the midnight which follows the day you have selected). Put this sum at two places and multiply the lower one by 3300; add to the product 64106 and divide the sum by 210902. The quotient represents the first fixed number and the remainder represents the second fixed number. Both these fixed numbers should be kept in memory, because these will be used later on (for computation of the Moon). Now deduct the first fixed number from the number already kept at the upper place; the remainder will represent the total number of civil days (passed since the date of this book at Lanka, upto the midnight which follows the selected day.) This is the ahargana.

COMMENTARY

(a) *Correction of the Manuscript* :–

No correction of the manuscript is required; only a few phrases have been added within brackets to make the text more intelligible.

(b) *Translation of the technical terms* :–

(i) *Shaka-kala* :– This is an era which depends on solar years and is generally 78 years less than the Christian era, so that the year 1957 A.D. corresponds to 1879 Shaka-kala. When we write 1957 A.D., it means that 1956 years of the Christian era have passed and the 1957th year is current; but when we write 1879 Shaka-kala it means that 1879 years of this era have passed and the 1880th year is current. In other words, we always write the number of past years to indicate the Shaka-kala, but to indicate the Christian era we write the number of the current year. This is just a convention. Shaka-kala may be used either with solar months or synodic lunar months. When used with solar months, the first month of the year begins when the sun passes the first point of Aries (without any regard to degrees of precession). and nowadays it generaly happens on 14 April; the second month begins when the sun passes the first point of Taurus and it generally happens on 15 May, and so on. But when this era is used with synodic lunar months, the first month of the year corresponds to the first month of the year of Vikrami Samvat. Vikrami Samvat also is indicated by the number of past years and it is 135 years longer than the Shaka-kala, so that 1879 Shaka-kala corresponds to 2014 Vikrami Samvat. In *Karana Tilaka* the Shaka-kala is used with synodic lunar months and luni-solar years. In this luni-solar system normally each year consists of 12 synodic lunar months, but generally after every two years the third year consists of 13 synodic lunar months in order to keep the luni-solar years in step with the solar years If the synodic lunar month is reckoned from full moon to full moon, the arrangement is known as purnimanta; if the month is reckoned from new moon to new moon, this arrangement is known as amanta. The purnimanta arrangement is used all over North India, and the amanta arrangement is generally used in South India. Each month of purnimanta is divided into two parts, dark half and light half, but in the amanta arrangement each month is divided into light half and dark half. From new moon to full moon it is light half and from full moon to new moon it is dark half. In both purnimanta and amanta arrangements the light half of each month is identical, but in the purnimanta arrangement this light half is considered the second half of the month, the preceding black half being the first half, and in the amanta arrangement this light half is considered the first half of the month, the succeeding black half being the second half. Thus, if we take the purnimanta arrangement, then the month of Chaitra begins just after the full moon, generally occurring between the middle of March and the middle of April; but if we take the amanta arrangement, then the month of Chaitra begins just after the new moon, generally occurring in the month of March. In this way the months of purnimanta arrangement begin nearly 15 days earlier than those of amanta arrangement, but the new year always begins in both the arrangements on the same day, i. e., with the beginning of the light half of Chaitra. Hence it will be seen that, according to the purnimanta arrangement, the new year begins from the middle of Chaitra and, according to the amanta arrangement, the new year begins from the beginning of Chaitra. There are very elaborate methods of calculations for this luni-solar system and these will be explained at some other proper place. The names of the synodic lunar months are given below in the correct order:-

1.	Chaitra	7.	Ashwina
2.	Vaishakha	8.	Kartika
3.	Jyeshtha	9.	Margashirsha
4.	Ashadha	10.	Pausha
5.	Shravana	11.	Magha
6.	Bhadrapada	12.	Phalgana

ii *Tithi*:- Each of the two halves of a synodic lunar month is divided into 15 parts and each 15th part is known as synodic lunar day or simply a *tithi*. The light half is known as Shukla Paksha and the dark half is known as Krishna Prakasha. The last tithi of the light half is known as purnima and the last tithi of the dark half is known as amavasaya. The names of the remaining 14 tithis are identical for both the halves and are given below:-

1.	Pratipada.	8.	Ashtami
2.	Dutiya.	9.	Naumi
3.	Tritiya.	10.	Dashami
4.	Chaturthi	11.	Ekadashi
5.	Panchami.	12.	Dwadashi
6.	Shashti.	13.	Trodashi
7.	Saptami.	14.	Chaturdashi

When the sun and the moon meet together at one point of the Zodiac, it is the end of amavasya and beginning of Shukla Pratipada; and when the sun and the moon come exactly opposite to one another at a distance of 180 degree of the Zodiac, it is the end of purnima and beginning of Krishna pratipada. In fact the time during which the moon moves 12 degrees away from the sun is one tithi, and so in one synodic lunar month there are 30 tithis equivalent to one complete synodic revolution of the moon comprising 360 degrees. A tithi may begin or end at any moment during day and night, but a civil day is generally named according to the tithi which is present at the time of sunrise of that day. In the Hindu system a civil day is reckoned from sunrise to sunrise.

(iii) *Adhimasa*:- As mentioned above, after every two years the third year consists of 13 synodic lunar months, instead of 12. The additional synodic lunar month is known as adhimasa and it may fall at any part of the year. It is supposed that during each synodic lunar month the sun should transit one sign of Zodiac. But every third year it so happens that the transit of the sun does not take place during any one synodic lunar month; such a synodic lunar month is known as adhimasa and so the next month is given the same name. In this way two months of the same name are reckoned in one year and the actual number of months becomes 13, instead of 12. It is important to note that for the sake of finding out the transit of the sun during a month, the amanta arrangement is always used, and a synodic lunar month for this purpose is reckoned from new moon to new moon and not from full moon to full moon. For example, if there is no transit of the sun in the Shukla Paksha of Chaitra, and its succeeding Krishna Paksha, then we will say that this Chaitra is an adhimasa and so its next month will also be named as Chaitra. The first Chaitra will be considered inauspicious for all religious festivals, etc., but the second Chaitra will be considered just a normal month. Very rarely after a very long time it may also happen that during one synodic lunar month there occur two consecutive transits of the sun. In such cases the particular synodic lunar month is dropped from the serial order and is known as kshaya-masa. Whenever a kshaya-masa occurs, there must occur two adhimasas in that year. Such kshaya-masas can only occur in the months of Kartika, Margashirsha, and Pausha, because the daily motion of the sun becomes maximum during these months.

(c) *Elaboration of the Principle*:– The first and the main problem in computing the planetary positions is to find out the total number of civil days passed after a particular day upto the selected date. In the Christian calendar this problem is very simple, but in the Hindu calendar of the luni-solar system this problem becomes a little complicated. In this article a method is given by which the synodic lunar days or the tithis are converted into civil days and after making an allowance of tithis of adhimasas, the correct number of civil days passed is found out. First the number of past solar years is found out and it is converted into solar months by multiplying it by 12; then the past synodic lunar months are also added to this solar month, and with the help of this sum the number of past complete adhimasas is calculated ignoring the remainder which represents the incomplete adhimasa; then these adhimasas are also added to the above-mentioned sum and then it is converted into a sort of mixed number of solar days and synodic lunar days by multiplying it by 30, and to this product the past tithis of the current month are added; then with the help of this sum the number of tithis in excess of the corresponding number of civil days is calculated ignoring the remainder which represents excess tithis; then these excess tithis are deducted from the above-mentiond sum and the remainder represents the number of civil days passed after a particular day fixed by the author of the book. This number is known as ahargana.

It will be observed that in the above-mentioned method at one place we add synodic lunar months to solar months, and at another place we add synodic lunar days to a sort of mixed number of lunar and solar days. This is apparently a wrong process; but as neither of the two remainders in the two divisions is taken into account in calculating the ahargana, this irregularity does not affect the final result.

(d) *Explanation of the Text*:– The starting point for calculating the ahargana as fixed by Vijaya Nandi, the author of *Karana Tilaka*, is the midnight at Lanka at the beginning of Chaitra Shukla Pratipada 888 Shaka-kala. From this day the ahargana, or the number of civil days that have passed upto any selected day, is calculated to compute the planetary positions on the Zodiac. This fixed moment corresponds to the midnight at Lanka between 23rd and 24th of March, 966 A. D. In future references we shall always mention various dates preceding the midnight under consideration and accordingly we may say that the above-mentioned fixed day corresponds to Saturday, the 23rd of March, 966 A. D., 30 Isfindar-muz-Mah 334 Yazdjird, 28 Rabi-ul-Awwal, 335 A. H. 23 Azar 1277, Roman era,

and the Amavasya preceding Chaitra Shukla Pratipada 888 Shaka-kala. On this particular day the ahargana of *Karana Tilaka* was zero; it was "1" on the next day, i. e., on Sunday, the 24th of March, 966 A. D., 1 Farwardin-Mah 335 Yazdjird, 29 Rabi-ul-Awwal 355 A. H., 24 Azar 1277 Roman era, and Chaitra Shukla Pratipada 888 Shaka-kala (or 1023 Vikrami Samvat). In this way the ahargana increases one by one from midnight to midnight at Lanka; if the ahargana is 1, it is Sunday, if it is 2 it is Monday if it is 3, it is Tuesday and so on.

The location of Lanka is supposed to be at the equator of the earth and at 75° 45′ E longitude with reference to Greenwich; so if we suppose the beginning of each day of the Julian period at the Greenwich mean noon, the above-mentioned fixed midnight corresponds to 2073972.28958 day of the Julian period. Similarly, the mean noon of the next day at Lanka when the ahargana of *Karana Tilaka* was "1" corresponds to 2073972.78958 day of the Julian period. If we assume the number of days of the Julian period as a whole number considering the decimal fraction as one, we may say that the first day of the ahargana of Karana Tilaka corresponds to the day 2073973 of the Julian period.

According to the modern Surya Siddhanta (*vide* English translation of Surya Siddhanta, 1935, edited by E. Burgess, formerly Missionary of the A.B.C. F.M. in India), the number of civil days in a Chaturyuga is 1577917828, and within this period the number of complete cycles for the sun is 4320000 and for the moon it is 57753336. A Chaturyaga is subdivided into four smaller yugas, e. g., Satya-yuga Treta-yuga, Dwapara-yuga and Kali-yuga. The duration of these smaller Yugas in terms of solar years is 1728000, 1296000, 864000 and 432000, respectively. The first three yugas of the present Chaturyuga have already passed and the fourth yuga, i. e., the Kali-yuga, has started since B. C. 3102 or 3179 before Shaka-kala. At the beginning of Kali-yuga the mean places of all the planets were at the first point of Aries. If we take more accurate figures, then the number of civil days that passed since the first midnight after which the Kali-yuga started up to the above-mentioned fixed midnight is 1485507; in other words, the Kali-yuga ahargana on 23 March, 966 A. D. is 1485507. At the beginning of Kali-yuga the mean places of all the planets were at the first point of Aries, and there was no incomplete adhimasa.

From the above-mentioned figures it is clear that in a Chaturyuga there are $4320000 \times 12 = 51840000$ solar months and $57753336 - 4320000 = 53433336$ synodic lunar months. Therefore, the number of excess synodic lunar months or the adhimasas is $53433336 - 51840000 = 1593336$. So the number of adhimasas in each solar month is $\frac{1593336}{51840000}$.

In order to simplify this fraction we can select by inspection an easy whole number 900 as numerator, and its corresponding denominator (say D) can be found out in the following manner:-

$$\frac{900}{D} = \frac{1593336}{51840000}$$

$$\text{or } D = \frac{900 \times 51840000}{1593336} = 29281 \frac{1528584}{1593336}$$

The fraction being equal to more than half can be taken as one, so that D = 29282 and therefore the fraction is $\frac{900}{29282}$.

Now, the number of solar years that elapsed up to the beginning of 888 Shaka-kala since the beginning of Kali-yuga is 888 + 3179 = 4067; it is equal to 4067 × 12 = 48804 solar months. Hence the number of adhimasas that passed since the beginning of Kaliyuga up to the beginning of 888 Shaka-kala is

$$\frac{1593336}{51840000} \times 48804 = 1500 \frac{1170144}{51840000}$$

It shows that 1500 complete adhimasas had already passed and been accounted for, and only a fraction of 1501 st. adhimasa had been accumulated. This fraction is yet to be accounted for in all the calculations to be made since the beginning of 888 Shaka-kala.

We have already seen that the approximate value of adhimasa in one solar month is $\frac{900}{29282}$, and if we adopt the same denominator for the fraction $\frac{1170144}{51840000}$, the numerator (say N) can be found out in the following way:—

$$\frac{N}{29282} = \frac{117044}{51840000}$$

$$\text{or } N = \frac{29282 \times 117044}{51840000} = 660 \frac{49756608}{51840000}$$

The fraction being equal to more than half can be taken as one, so that D = 661, and the fraction = $\frac{661}{29282}$

Hence if the number of solar months that elapsed since the beginning of 88 Shaka-kala is "S", then the number of adhimasas that accumulated during "S" solar months is given by the expression $\frac{900 \times S + 661}{29282}$.

We have seen that in a Chaturyuga the total number of synodic lunar months is 53433336, so the total number of tithis is 53433336 × 30 = 1603000080.

But the number of civil days in a Chaturyuga is 1577917828, so the excess synodic lunar days or the unaratra days are 1603000080 − 1577917828 = 25082252. Therefore, the number of unaratra days in one tithis is $\frac{25082252}{1603000080}$. In order to simplify the fraction select the numerator by inspection as 3300, and as such the corresponding denominator (say d) can be found out in the following way:-

$$\frac{3300}{d} = \frac{25082252}{1603000080}$$

$$\text{or } d = \frac{3300 \times 1603000080}{25082252} = 210902 \frac{2936696}{25082252}$$

The fraction being equal to less than half may be ignored, so that d = 210902, and the fraction is $\frac{3300}{210902}$

Now, at the beginning of 888 Shaka-kala the total number of solar days that elapsed since the beginning of kaliyuga is 48804 × 30 = 1464120: the number of tithis in 1500 synodic lunar months is 45000; the sum of these two is 1509120. The number of unaratra days during this period is

$$\frac{1509120 \times 2508225}{1603000080} = 23613 \frac{48724920}{1603000080}$$

The complete number of these unaratra days is deducted from 1509120 and the remainder 1485507 is the number of civil days or the ahargana, but the fraction $\frac{48724920}{1603000080}$ remains unaccounted for, which represents the portion of incomplete unaratra day. In order to account for this fraction we adopt the same denominator and the numerator (say n) can be found out as follows:-

$$\frac{n}{210902} = \frac{48724920}{1603000080}$$

$$\text{or } n = \frac{210902 \times 48724920}{1603000080} = 64105 \frac{1510650000}{1603000080}$$

The fraction being equal to more than half can be taken one, so that n = 64106, and the fraction is $\frac{64106}{210902}$

Therefore for "L" synodic lunar days since the beginning of 888 Shaka-kala, the unaratra days can be found out by the expression

$$\frac{3300 \times L + 64106}{210902}$$

From this expression whatever we get as the quotient is the first fixed number and whatever we get as the remainder, is the second/number; the first fixed number represents the complete unaratra days and the second fixed number represents the incomplete unaratra day. Both of these fixed numbers will be utilised later on for the computation of the moon's position in the Zodiac.

Dr. Schram could not find out the actual basis of the above mentioned expression and so he had erroneously supposed that the method employed was based on the Pulisa's theory in which the number of civil days in a Chaturyuga is 1577917800; but actually it is based on the theory of the modern Surya Siddhanta, in which the number of civil days in a Chaturyuga is 1577917828. Due to this misunderstanding, Dr. Schram, after giving the detailed calculations in his support, had wrongly concluded that "The fraction of unaratra days $\frac{232}{703}$ is equal to $\frac{69600\frac{464}{703}}{210902}$ or nearly to $\frac{69601}{210902}$. Therefore we must add 69601 before dividing by 21092. Alberuni has, instead of this number 69601, the number 64106, 4 instead of 9, and the last three numbers reversed" (vide *Alberuni's India*, 1910, vol. II page 376, Annotations).

From the above reasoning it is quite clear that the supposition of Dr. Schram is totally baseless and the figures given by Beruni are absolutely correct.

(d) *Additional examples:*– It will be seen that from this method we can find out the ahargana only for the period after 888 Shaka-kala. But from similar reasoning it is quite possible to evolve an additional method to find out the negative ahargana for any date before 888 Shaka-kala. Such a negative ahargana will be counted backward from the end of 887 Shaka-kala and separate methods can also be evolved for every planet to compute its position by means of this negative ahargana. This method of finding the negative ahargana along with completely solved examples is given in Appendix 'A' of this English translation.

It will not be out of place to give a simple method here for finding out the ahargana of Karana Tilaka from the dates of the Christian calendar. This method can be employed for any date after 1601 A. D.

"Deduct 1601 from the current Christian Era and multiply the remaining number by 365. Keep this product at one place. Now divide the same remaining number by 4 and add the quotient to the above-mentioned product kept at one place. Then divide the above-mentioned quotient by 25 and deduct this second quotient from the above-mentioned product. Then divide this second quotient by 4 and add this third quotient to the same above-mentioned product. To this sum add the number of days of the current year with effect from 1st January, including the current day. Finally, add 231841 to this total number; then you will get the ahargana of Karana Tilaka which begins with Sunday." Actually 231841 is the ahargana of Karana Tilaka on 31st December, 1600 A. D. which was a Sunday.

For example, let us calculate the ahargana on 10 May, 1957 A. D., when the 100th anniversary of the Freedom Struggle of India was celebrated all over Indo-Pak sub-continent. We deducted 1601 from 1957 and got 356 as remainder; we multiplied this remainder by 365 and got the product 129940. We kept this product at one place. Then we divided the same remaining years 356 by 4 and got the quotient 89; we added it to the above-mentioned product and got 130029; then we divided 89 by 25 and got 3 as the quotient, which was deducted from the above-mentioned product and we got 130026 as remainder. When we divided 3 by 4 we got zero as quotient, so we ignored it. To this we added 130 days of the current year and got 130156; to this we added 231841 and got the total sum 361997. This is the ahargana of Karana Tilaka for 10 May, 1957, which is a Friday, because after dividing this ahargana by 7 we get 6 as remainder, which gives us Friday by counting it from Sunday.

3. SELECTED DATE FOR THE EXAMPLE :–

To give an example for it we take a famous day, i.e., the day of the meeting of Sultan Mahmud with Khan Yousuf, (may God bestow His mercy on both of them). This meeting took place between the two armies at a valley near Samarqand on Thursday 27 Safar 416 A.H. (in the manuscript 25 Safar 416 A. H.), 29 Naisan 1336 Roman, Ram 21 Urdibihishtmah 294 Yazdjird, and according to the (Amanta System of) Hindus it was 29th tithi of Vaishakha, the second lunar month of 947 Shakakala (in the manuscript 27th tithi of Jeshtha, the third lunar month of 948 Shakakala).

COMMENTARY

(a) *Correction of the Manuscript* :–

There are two very serious mistakes in the manuscript which appear to be due to some misunderstanding on the part of Beruni. He selects an important date as an example and makes all his calculations on the basis of that date. That date, as he has himself mentioned, is 29 Naisan 1336 Roman, 21 Urdibihisht 394 Yazdjird, Thursday. This date actually corresponds to 27 Safar, 416 A. H. (if we reckon the months from the actual visibility of the Crescent), and not to 25 Safar, 416 A. H., as Beruni has mentioned. A method to find out the visibility of the new moon in his *Ghurrat-uz-Zijat* is given which will be fully explained and exemplified when we reach that chapter. According to this method, we can clearly see that the new moon of Safar at Multan (for which place Beruni has made all his calculations in the examples) was sighted on 2 Naisan, 1336 Roman. If we convert this date into the Christian Calendar, we get 2 April, 1025 A. D. which was a Friday. Therefore, on 29 Naisan, 1336 Roman (which corresponds to 29 April, 1025 A. D.) it was 27 Safar, 416 A. H. If we adopt the "Conventional" method of the Muslim Calendar (which was used by the Muslim astronomers and was independent of the actual visibility of the new moon), we get 28 Safar instead of 27 Safar. But I have preferred to accept 27 Safar, because it depends on the actual visibility of the new moon. In any case, the date 25 Safar, as given by Beruni, is incorrect.

Similarly, the date in question corresponds to 29 Vaishakha 947 Shakakala, (according to the Amanta System of reckoning, which will be explained later), and not to 27 Jeshtha, 948 Shakakala as given by Beruni. Even if we use "Purnimanta" system of reckoning, (which will be explained later) we find that the date in question corresponds to 14 Jeshtha and not to 27 Jeshtha. Perhaps Beruni was confused in understanding the complicated calculations of "Amanta" and "Purnimanta" systems. I have seen that even the Hindu astronomers are often confused in these two systems, especially while making calculations for "adhimasa" or the intercalary month. The mistake of writing 948 Shakakala instead of 947 Shakakala, is obviously due to the fact that under this system, unlike all other Calendars, the number of the total years passed is used, instead of the number of the current year. For example, when we write 29 Naisan 1336 Roman, we mean that 1335 complete years of the Roman era have passed and the 1336th year is now passing. But when we write 29 Vaishakha 947 Shakakala, we mean that 947 complete years of Shakakala have passed and the 948th year is now passing. Here Beruni has used the figure 948 in the usual sense that 948th year of Shakakala is passing which should have not been used. Therefore, I have corrected this figure in the text accordingly and have written 947 Shakakala, instead of 948 Shakakala. I wonder how could Beruni make this mistake here, although in his *Indica* he has clearly stated that the year of the Hijra 416 corresponds to 947 Shakakala (*vide* al-Beruni's *India*-1910-Vol. II, p. 9, l. 19).

(b) *Translation of Technical Terms*

(ii) AMANTA AND PURNIMANTA SYSTEMS: A brief description of these two systems has already been given earlier in this article. Here a more detailed description is given in order to show how complicated calculations are involved in finding out a date. It has already been explained that from the time of conjunction of the Sun and the Moon upto the time of their opposition, the period is known as the "bright half" or "Shukla Paksha" of the lunar month, and from conjunction to opposition it is the "dark half" or "Krishna Paksha." The following concrete example will show how these "halves" or "Pakshas" are combined in "Amanta" and "Purnimanta" Systems and how an "intercalary" month or "Adhimasa" is calculated. It is also shown in which "Paksha" the transit of the Sun or "Sankranti" has taken place. The names of the twelve signs of Zodiac or "Rasis" through which the Sun passes are:– (1) Aries, (2) Taurus, (3) Gemeni, (4) Cancer, (5) Leo, (6) Virgo, (7) Libra, (8) Scorpio, (9) Sagitarius, (10) Capricorn, (11) Aquarius and (12) Pisces. Their Sanskrit names are: (1) Mekha, (2) Brikha, (3) Mithuna, (4) Karka, (5) Singha, (6) Kanya, (7) Tula, (8) Vrishchika, (9) Dhanu, (10) Makara, (11) Kumbha, and (12) Meena.

Purnimanta			*Amanta*
	Shukla Paksha---	(No Sankranti)	
Vaiskakha Adhimasa	Krishna Paksha---	(Mekha Sankranti)	Chaitra
	Shukla Paksha-----	(No Sankranti)	
	Krishna Paksha----	(No Sankranti)	Vaishakha Adhimasa
Vaishakha	Shukla Paksha-----	(Brikha Sankranti)	
Jeshtha	Krishna Paksha----	(No Sankranti)	Vaishakha
	Shukla Paksha-----	(Mithuna Sankranti)	

Ashadha	Krishna Paksha----	(No Sankranti)	Jeshtha
	Shukla Paksha-----	(Karka Sankranti)	

From the above example it will be seen that both in the Shukla Paksha and the next Krishna Paksha there was no Sankaranti and, therefore, this month has been dropped out and is termed as Vaishakha Adhimasa. These two particular Pakshas make one complete month of Amanta System, but they do not make a complete month of the Purnimanta System, because under the Purnimanta System these two Pakshas belong to two different months. So for an Adhimasa it is essential that the two adjacent Pakshas in which no Sankranti occurs, must belong to one complete month of the Amanta system, otherwise, it will not be termed as an Adhimasa. For example, the following case may be examined:

Purnimanta			*Amanta*
	Shukla Paksha-----	(Mekha Sankranti)	Chaitra
Vaishakha	Krishna Paksha----	(No Sankranti)	
	Shukla Paksha-----	(Brikha Sankranti)	Vaishakha
Jeshtha	Krishna Paksha----	(No Sankranti)	
	Shukla Paksha-----	(No Sankranti)	Jeshtha
Ashadha	Krishna Paksha----	(Mithuna Sankranti)	
	Shukla Paksha-----	(No Sankranti)	Ashadha
	Krishna Paksha----	(Karka Sankranti)	

It should be carefully seen that, although there are two adjacent Pakshas in which no Sankranti occurs, yet there is no Adhimasa, because these two Pakshas do not make a complete month of the Amanta System; One Paksha belongs to Vaishakha and the other Paksha belongs to Jeshtha. No doubt these two Pakshas make a complete month of Jeshtha according to the Purnimanta System, but, for the calculation of Adhimasa, we always consider the Amanta System.

Now turning to the actual figures of the text, we find that the date in question was 29 Vaishakha, i.e., 14 of Krishna Paksha (15 tithis being reckoned in Shukla Paksha); and we may call this date as Vaishakha Krishna Paksha Chaturdashi (Amanta). The same date according to the Purnimanta system is 14 Jeshtha, i.e., 14 of Krishna Paksha; and we may call the same date as Jeshtha Krishna Paksha Chuturdashi (Purnimanta). In northern India the Purnimanta System is used in performance of all religious ceremonies, except in determination of Adhimasa, in which case the Amanta System is used. In astronomical calculations the Amanta system is considered easier and perhaps for this reason Beruni has used this System in his text. But it appears that he had not thoroughly studied these systems, as is evident from his *Indica* also.

(ii) HIJRA CALENDAR: This Calendar is used by all the Muslims of the world for religious performances. The commencement of the month is taken after the actual visibility of the new moon. The day is reckoned from evening to evening. Each month has either 29 days or 30 days. Generally, these two types of months occur alternately, but it is also possible that in extreme cases there are three consecutive months of 29 days each, or four consecutive months of 30 days each. There are various astronomical methods to find out whether the new moon will be visible at a particular place on a particular day or not, but these methods are so complicated that it is very difficult to prepare a calendar on the basis of the actual sightibility of the new moon. Therefore, the Muslim astronomers had devised a "conventional" calendar independent of the actual sightibility of the new moon. This calendar gives a date which is usually one or two days ahead of the actual date determined by the visibility of the new moon. For this conventional method the astronomers have decided to reckon the days of the month in the following way :–

1.	Moharram	30	2.	Safar	29
3.	Rabi' I	30	4.	Rabi' II	29
5.	Jamadi I	30	6.	Jamadi II	29
7.	Rajab	30	8.	Sh'aban	29
9.	Ramaḍan	30	10.	Shawwal	29
11.	Dhiq a'd	30	12.	Dhilhijj	29

In this way there are only 354 days in a year; but during a period of 30 years there are eleven leap years, in which the twelvth month has 30 days, instead of 29 days. These leap years are 2nd, 5th, 7th, 10th, 13th, 15th, 18th, 21st, 24th, 26th, and 29th years from the beginning of each period of 30 years. The complete years of the Hijra era may be divided by 30, and for the remainder the above-mentioned order may be considered to find out the leap year. In this way the total number of days which elapsed since the beginning of the Hijra era up to 28 Safar 416 A.H. comes to 147120. The first day of the Hijra era was a Thursday, therefore 147120th day of this era was also a Thursday (this can be verified by dividing the total number of days by 7 and counting the remainder from Thursday). Apart from this conventional method, we have seen that the actual date of the Hijra era was 27 Safar, which shows that there was a difference of one day between these two systems.

(iii) Roman Calendar: In this calendar one year is supposed to be of 365¼ days and for this reason three consecutive years are supposed to have 365 days each and the fourth year is supposed to be a leap year of 366 days. There are twelve months in a year and each month has a fixed number of days except the fifth month which has twenty-eight days in ordinary years and twenty-nine days in leap years. The names of the months and the days in each month are given below :–

1.	Tashrin I	31	2.	Tashrin II	30
3.	Kanun I	31	4.	Kanun II	31
5.	Shubat	28	6.	Azar	31
7.	Naisan	30	8.	Ayar	31
9.	Haziran	30	10.	Tamoz	31
11.	Ab	31	12.	Elol	30

It will be seen that the number of days in each month is the same as in the Christian calendar in which February corresponds to Shubat. The difference between the Roman calendar and the Christian calendar is that in the Roman calendar the year begins with the month Tashrin I (which corresponds to October) but in the Christian calendar the year begins with January (which corresponds to Kanun II). Moreover, the Roman era starts 311 years earlier than the Christian era, so that if we deduct 311 years from the Roman era, we get the Christian era. In this way 29 Naisan 1336 Roman corresponds to 29 April, 1025 A.D. The total number of days of the Roman era which elapsed since its beginning up to 29 Naisan 1336 Roman is 487820. The first day of this era was Monday, therefore 487820th day of this era was Thursday (this can be verified by dividing the number of days by 7 and counting the remainder from Monday). To find out whether a particular year is a leap year or not, we should add 1 to the current year of the era and then if it is divisible by 4, it is a leap year. So the year 1336th was not a leap year but its preceding year was a leap year.

(iv) Yazdjird Calendar :– In this calendar each year is supposed to be of 365 days without any fraction, so that there is no leap year in this system. There are twelve months in a year and each month has 30 days, each of which has a separate name. The remaining 5 days are added either at the end of the eighth month or at the end of the twelvth month; these five days also have separate names. The names of the months are given below :–

Farwardinmah, Urdibihishtmah, Khurdadmah, Tirmah, Murdadmah, Shahrewarmah, Mehrmah; Abanmah, Azurmah, Dimah, Behmanmah, Isfindarmuzmah.

Similarly, the names of days of each month are given below :–

(1)	Hurmuz	(2)	Behman	(3)	Urdibihisht
(4)	Shahrewar	(5)	Isfindarmuz	(6)	Khurdad
(7)	Murdad	(8)	Debazur	(9)	Azur
(10)	Aban	(11)	Khur	(12)	Mah
(13)	Tir	(14)	Josh	(15)	Debimehr
(16)	Mehr	(17)	Sarosh	(18)	Rishn
(19)	Farwardin	(20)	Behram	(21)	Ram
(22)	Bad	(23)	Debidin	(24)	Din
(25)	Erad	(26)	Ashtad	(27)	Asman
(28)	Zamiad	(29)	Marisfind	(30)	Aneran

The names of the remaining five days are different in different books, but the common names are :–

(1)	Ahnud	(2)	Ashtud	(3)	Isfindmad
(4)	Hasht	(5)	Hashtosh		

The total number of days of this era which elapsed since the beginning up to Ram (i.e., 21st) of Urdibihishtmah 394 Yazdjird, is 143496. The first day of this era was Tuesday; therefore 143496th day of this era was Thursday (this can be verified by dividing the total number by 7 and counting the remainder from Tuesday).

(v) SHAKAKALA :- In this calendar very complicated calculations are involved and a luni-solar system is used as already explained in the previous article. Each year consists of either twelve or thirteen synodic lunar months. In 4320000 years there are 1577917828 civil days and 1603000080 synodic lunar days. The total number of days which elapsed since the beginning of this era upto 29 Vaishakha 947 Shakakala (Amanta) is 345950. The first day of the era was a Tuesday; therefore the 345950th day of this era was a Thursday (this can be verified by dividing the total number by 7 and counting the remainder from Tuesday).

(c) *Elaboration of the principle* :-

No elaboration of any principle is required.

(d) *Explanation of the text* :-

No explanation of the text is required.

(e) *Additional example* :-

No additional example is required.

4. EXAMPLE FOR THE SELECTED DATE :-

(Actually this example is given for 27 Jeshtha 947 Shakakala, (Amanta) which corresponds to 26 Rabi-ul-Awwal 416 A. H., 27 Ayar 1336 Roman, Farwardin 19 Khurdadmah 394 Yazdjird, Thursday).

We took complete years of Shakakala 947 and subtracted from it 888, getting the remainder 59. We multiplied it by 12 and to the product added the past two months and then placed the sum at two places above and below. We multiplied the lower figure by 900 getting product 639000; to it added 661, getting 639661. We then divided it by 29282 and got the quotient 21 which is the number of Adhimasas, and the remainder was 24739. Then we added these Adhimasas to the figure already written at the upper place getting the sum 731. We multiplied it by 30, getting the product 21930; to it we added the past Tithis of the third month, i.e., 27, getting the sum 21957. We wrote that figure at two places, above and below. Then we multiplied the figure at the lower place by 3300, getting the product 7245800, to it we added 64106, getting the sum 2522206. We divided it by 210902, getting the quotient 343 which may be called the first "fixed number." The remainder is 182820 which may be called the second "fixed number." We deducted the first "fixed number" from the figure already written at the upper place, getting the remainder 21614, which is the "*Ahargana.*"

Although we had said that we would not add anything to this book from our side except the examples, yet we feel the necessity of adding the description as how to find out the dates used in our country from the *Ahargana* described by the author of this book, because it is evident that non-Hindus do not know the dates used by the Hindus.

COMMENTARY

(a) *Correction of the Manuscript* :-

There is no correction required in this article. As explained in the previous article, Beruni has made a mistake in calculating the luni-solar date of the Hindus. He has selected a date to quote as an example and that date is 29 Naisan, 1336 Roman, which actually corresponds to 29 Vaishakha 947 Shakakala (Amanta) or 14 Jeshtha 947 Shakakala (Purnimanta). In more technical words these dates may be called as Vaishakh Krishnapaksha Chaturdashi (Amanta) or Jeshtha Krishna Paksha Chaturdashi (Purnimanta). Whatever system, whether Amanta or Purnimanta, is used, we have to reckon the past Tithis from Chaitra Shukla Pratipada which is common to both systems and the same result is obtained. So we see that according to these dates full one month and twenty-nine Tithis had passed since Chaitra Shukla Pratipada. But Beruni has erroneously supposed that it was 27 Jeshtha (Amanta) and according to this he supposed that full two months and 27 Tithis had passed since Chaitra Shukla Pratipada. Therefore, he wrongly calculates the Ahargana for 29 Naisan 1336 Roman as 21614. If we correctly calculate the Ahargana for 29 Naisan 1336 Roman, which corresponds to 29 Vaishakha 947 Shakakala (Amanta), we get 21586, i.e., 28 days less than the figure calculated by Beruni. Unfortunately, this difference of 28 was divisible by 7 and so Beruni found the same Thursday according to his ahargana and consequently thought his ahargana to be correct, because if the day of the week tallies with the Ahargana, the Ahargana is taken to be correct. If we divide the Ahargana by 7 and count the remainder from Sunday, we get the day of the week. It will be seen that both from 21614 and 21586 we get 5 as remainder after dividing by 7, which gives Thursday.

At first I thought of making correction in the Ahargana and changing the figure 21614 to make it 21586. But afterwards I found that if the ahargana was changed, the whole calculations of the book would have to be changed, because the entire calculations are based on this ahargana. Therefore, I have not changed the ahargana, but have clearly shown that this ahargana is not in accordance with 29 Naisan 1336 Roman, but it actually corresponds to 27 Ayar 1336 Roman, 19 Khurdadmah 394 Yazdjird, 26 Rabi-ul-Awwal, 416 A. H., Jeshtha Krishna Paksha Dawadashi (Amanta) or Ashadha Krishna Paksha Dawadashi (Purnimanta) 947 Shakakala, Thursday, 27 May 1025 A. D. This fact should also be borne in mind that 26 Rabi-ul-Awwal has been calculated according to the actual sightibility of the new moon. According to the "Conventional" system, as explained in the previous article, it is 27 Rabi-ul-Awwal, i.e., one day more than the actual date. In future we will always use the actual Islamic date according to the visibility of the moon and not according to the "Conventional" system.

(b) *Translation of the Technical Terms* :-

No clarification of any kind is required.

(c) *Elaboration of the principle* :-

No elaboration of any principle is required.

(d) *Explanation of the Text.*

No explanation is required.

(e) *Additional example.*

As an additional example we can calculate the correct Ahargana on ~~92~~ 29 Naisan 1336 Roman which corresponds to 29 Vaishakha (Amanta) 947 Shakakala, as shown below:

We subtracted 888 from the Shakakala 947 and got 59; multiplied it by 12 and to the product added one month already passed, getting 709; placed this figure at two places and multiplied the lower figure by 900, getting 638100; to it added 661 geting 638761; divided it by 29282 and got the quotient 21 which is the number of Adhimasas; and the remainder was 23839; then added the Adhimasas to the figure writing 730; multiplied it by 30 getting 21900; to it added the past tithis 29, of the second month Vaishakha getting 21929; wrote it at two places and multiplied the lower figure by 3300 getting 72365700; to it added 64106 getting 72429806; then divided it by 210902 getting the quotient 343 as the first fixed number and the remainder 90420 as the second fixed number; now deducted the first fixed number from the figure at the upper place and got 21586 which is the required Ahargana for the given date.

5. To find the date of Yazdjird era from this date :–

When we want to know the date of Yazdjird from this date, we divide the corresponding Ahargana by 365 and add 334 to the quotient to get the complete years of Yazdjird era (and the next year is the current year). From the remainder we go on deducting 30 for each month beginning from Farwardinmah. For Abanmah we deduct 35, instead of 30, and in this way we come to the month whose day cannot be deducted, then that month is the current month and the remaining days are the day, passed (up to the midnight which immediately follows that day).

Example :– We divided the Ahargana already calculated by 365, getting the quotient 59; we added 334 to it, getting the sum 393 which represents the complete years of Yazdjird era (and the next year, i.e., 394th is the current year). After division the remainder was 79; from it we deducted 30 days of Farwardinmah and 30 days of Urdibihishtmah, getting the remainder 19. Therefore. these are the past days of Khurdadmah (up to the midnight which immediately follows that day).

Commentary

(a) *Correction of the Manuscript* :–

In this article also Beruni had made a mistake by stating that "deduct 29 from the Ahargana and then divide the remainder by 365." I have deleted this phrase from the text, because Beruni had said so due to the wrong Ahargana calculated by him. He had tried to get the correct result from wrong Ahargana and therefore he had made this mistake, otherwise there is no necessity of deducting 29 from the Ahargana. Accidentally the first day of the Ahargana of Karanatilaka exactly corresponds to the first day of the

335th year of Yazdjird era. Therefore we can use the Ahargana of Karana Tilaka without subtracting any thing from it. As each year of Yazdjird era consists of 365 days, it is very easy to find out the number of the years passed by dividing the ahargana by 365; and after further adding 334 years already passed at the beginning of the ahargana, we get the complete years of the Yazdjird era.

(b) *Translation of the Technical Terms.*

No clarification is required.

(c) *Elaboration of the principle.*

On the first day of Yazdjird era, i.e., on 1 Farwardinmah, 1 Yazdjird, it was Tuesday, 16 June 632 A. D. and the Julian day was 1952063. If we divide the Julian day by 7 and count the remainder from Tuesday we get the day of the week, because the first Julian day was a Tuesday. The Julian day actually is counted from midday to midday at Greenwich but the days of the week are counted from midnight to midnight. Therefore there is always a confusion in the relation of the Julian day and the day of the week. Ordinarily, if we take the Julian day before the noon at Greenwich, the first day of this period corresponds to Tuesday; but if we take the Julian day after the noon at Greenwich, the first day of this period corresponds to Monday. I have adopted such Julian day for all of my references which begins from Tuesday. This Julian date was derived by Joseph Scaliger, and named by him after his father whose name was Julius Scaliger. He derived a cycle of 7980 years as the common multiple of the three cycles used by the Roman, this multiple being computed to date from noon of the first of January 4713 B. C. and to finish by the end of 3267 A. D. If we need more accuracy of time, we can use decimal fractions also to show the exact moment of a happening. For example, the mean noon at Lanka (its longitude being 75' 45'E) when the Ahargana of KaranaTilaka was "1," can be represented by 2073972.78958 Julian day, but for ordinary use we only say that it was 2073973 Julian day; and when the Ahargana of KaranaTilaka was 0, the Julian day was 2073972. It means that if we convert any date into the Julian day and then subtract from it 2073972, we get the ahargana of Karanatilaka; or if we add 2073972 to the ahargana of Karanatilaka, we get the Julian day.

Actually the Yazdjird calendar is neither a solar calendar nor a lunar calendar. This calendar is mostly used now by the Zoroastrians of Persia for their religious ceremonies. It was started from the date of the coronation of the Persian King Yazdjird, son of Shahryar. In order to make it a pure solar calendar, the Persian King Jalal-ud-Din Malikshah, son of Alparsalan Saljuqi ordered the famous poet and astronomer Umar Khyyam to reform it, and since then the majority of the Persians are using the reformed calendar which is known as the "Maliki" or "Jalali" calendar. The Parsis of India and Pakistan still use the same old Yazdjird calendar for their religious performances, but their calendar is known as "Parsi calendar" and is exactly 30 days behind the actual Yazdjird calendar used in Persia. The reason for this

"Lag" is said to be the unaccounted period of one month lost in the migration of the Parsis from Persia to India.

(d) *Explanation of the Text*:-

The correction made by me in the actual method of the text has already been explained. Due to this correction the figures of the calculation given in the "Example" have also been changed. I am giving here these changed as well as the original figures of the manuscript.

(i) The following phrase has been deleted from the text after the word من هذا التاريخ :

نقصنا من الأصل على ايام التاريخ ٢٩ ابدا

(ii) The following phrase has been deleted from the example in the beginning:-

اذا نقصنا من الأصل الذى عملناه ٢٩ فبقى ٢١٥٨٥ قسمناه على ٣٦٥

and instead of the above phrase, the following phrase has been added:-

إذا قسمنا الأصل فى ٣٦٥

(iii) The figure 50 has been deleted from the example and in its place 79 has been written.

(iv) The following phrase has been deleted from the example:-

و بقى معنا ٢٠ فهى الايام الماضية من ارديبهشت ماه

and in its place the following phrase has been inserted:-

و بقى معنا ٤٩ و القينا منه لارديبهشت ماه ل و بقى ١٩ فهى الايام الماضية من خرداد ماه (بنصف الليل الذى بعد النهار المحسوب له)

(e) *Additional Example*:-

No additional example is required.

6. To find the date of Hijra era from this date:-

When we need to find the Arabic date from the date of the Hindus given in this book, we add 87 in the ahargana and multiply the sum by 60 (and divide this product by 911704, and deduct the quotient from the same product). Whatever we get, we divide it by 21262 and add 354 to the quotient, getting the complete years of the Hijra era (and the next year is the current year). Then we divide the remainder which represents ghatis by 60 and the quotient represents complete days; if the remainder is 30 or more, then we further add "1" to the quotient otherwise we ignore it. Now we start deducting alternately 30 and 29 from the complete days to get complete months beginning from Moharram, till we get such a remainder from which the days of the next month cannot be deducted. This remainder represents the past days of that month (for the coming midnight of the day in question).

Example:- We added 87 to the ahargana getting the sum 21701; we multiplied it by 60 getting the product 1302060 (we divided this product by 911704 and deducted the quotient from that product getting 1302059); we divided it by 21262 getting the quotient 61 which we added to 35 getting the sum 415 which represents the complete years of the Hijra era (and the next i.e., the 416th year is the current year). From the remaining ghatis we got 85 days, from these days we took 30 days from Moharram, 29 days for and the remaining 26 days belonged to the next month (i.e., to Rabi'-ul-Awwal) which had passed (upto the coming midnight of the day in question).

Commentary

(a) *Correction of the Manuscript*:

Here also Beruni makes a mistake when he says "we add 57 to the ahargana." I have corrected this figure and have written 87 instead of 57. This mistake also is due to the wrong ahargana calculated by Beruni; he had made a mistake in calculating the ahargana and then he had tried to get the correct date of the Hijra era from that wrong ahargana. Moreover, he had supposed that 29 Naisan 1336 Roman corresponded to 25 Safar 416 A.H., but actually it was 27 Safar 416 A.H., so a mistake of 2 days was made in supposing a wrong date. Similarly, he had calculated the ahargana for that date as 21614, but the correct ahargana was 21586; so a mistake of 28 days was made in calculating wrong ahargana. In this way a total mistake of 30 days was made by Beruni in calculating the constant number 57; the correct number should have been $57 + 30 = 87$.

Moreover, the method given by Beruni here can hold good only within a span of a few centuries; if we use this method today, i.e., after a thousand years, there may occur a mistake of one or two days in the Arabic date. So in order to make this method useful for a span of millions of years, I have added a phrase in the original text within a bracket, stating "and divide this product by 911704 and deduct the quotient from the same product." This is just an additional phrase and has no effect on the original text given by Beruni. Actually Beruni had no intention to give a method which might be useful for millions and millions of years, because such a long period of time is outside the scope of a "*Zijj*" or "*Karana*," which normally covers a period of a few centuries and "simplicity of calculation" is its essence.

(b) *Translation of the technical terms*:

Ghati:- In Hindu calculations a day (from sunrise to sunrise) is divided into sixty equal parts and each of this part is called a "ghati". This ghati is further subdivided into sixty parts, each part being called a "pala"; each pala is subdivided into sixty parts, each part being called a "vipala." There are further subdivisions of time but usually we calculate upto "vipala" only.

(c) *Elaboration of the Principle*:

The basic principle of this method is that the number of civil days is converted into the number of synodic lunar days. The civil days are first converted into ghatis by multiplying them by 60. Then these ghatis are divided by the figure 21262 which represents the total from the same elementary constants which were used in formulating the method for ahargana. It has already been shown that in a Chaturyuga the number of synodic lunar months is 53433336 and the number of civil days is 1577917828. When these civil days are converted into ghatis by multiplying them by 60, we get 94675069680 ghatis. Therefore, the number of ghatis in a synodic lunar year is obtained by dividing these ghatis by the number of synodic lunar years. The synodic lunar years are obtained by dividing the number of months by 12 and getting the quotient 4452778. So the number of ghatis in a lunar year $= \frac{94675069680}{4452778}$

$$= 21262\ \frac{103844}{4452778}$$

The fraction being less than half is to be ignored. So the approximate number of ghatis in a lunar year is found to be 21262, and this is the figure by which we divide the total ghatis in order to get the complete years of Hijra era.

Now if we do not want to ignore the above-mentioned fraction, we account for it in the following way:-

In 21262 ghatis the difference $= \frac{103844}{4452778}$

So in "1" ghati the difference

$$= \frac{103844}{4452778 \times 21262}$$

$$= \frac{103844}{94674965836}$$

$$= \frac{1}{911703\frac{79504}{103844}}$$

The fraction being more than half can be takens as "1" so that we get

$$= \frac{1}{911704}$$

Therefore it is clear that for more accurate results we should divide the "total ghatis" by 911704 and the quotient should be deducted from the "total ghatis"; then the remainder will represent a much more accurate figure of the total ghatis. This is the correction which I have mentioned in my additional phrase given in the text within a bracket.

It can be shown with the help of "back calculation" that, when the ahargana of Karana Tilaka was zero, it was 28th of Rabi-'ul-Awwal, 355 A. H. It reveals that at the beginning of the ahargana of Karana Tilaka 354 complete years and 87 complete days of the Hijra era had already passed. That is the reason why Beruni tells us to add 87 days in the ahargana of Karana Tilaka,

number of ghatis in a synodic lunar month. This figure is obtained

so that we may get the complete days that have elapsed since the beginning of the 35th year of the Hijra era. After converting these civil days into synodic lunar days, months and years, he further tells us to add 354 to the number of the lunar years obtained from the ahargana; this is due to the fact that 354 of the Hijra era had already passed at the beginning of the ahargana of Karana Tilaka.

(d) *Explanation of the text*:-

As already mentioned, Beruni had made a mistake in the calculation of the ahargana for 29 Naisan 1336 Roman and from that wrong ahargana he had tried to derive correct date of the Hijra calendar. For this reason the method given by him was wrong which has now been corrected. Consequently, the calculations of the example have also gone wrong. The changes made by me in the original text are shown below:-

(i) I have deleted the following phrase from the text

زدنا على الأصل نز

and in its place the following phrase has been inserted:-

زدنا على الأصل فز

(ii) I have added the following phrase in the text within brackets for greater accuracy:-

(و قسمنا المجتمع على ٩١١٧٠٤ و القينا ما خرج من المجتمع)

(iii) I have added the following phrase in the text within brackets to give better understanding:-

(بنصف الليل الذى بعد النهار المحسوب له)

(iv) I have added the following phrase in the example within brackets for greater accuracy:-

(قسمناه على ٩١١٧٠٤ و القينا منه ما خرج فبقى ١٣٠٢٠٥٩)

(v) I have deleted the following phrase from the example:-

للحرم ل و يبقى كه وهى الأيام الماضية

and in its place the following phrase has been inserted:-

للحرم ل و للصفر كط و يبقى كو وهى الأيام الماضية من
ربيع الأول (بنصف الليل الذى بعد النهار المحسوب له)

(vi) The following further changes have been made in the figures of the example:-

I have written فز instead of نز, 21701 instead of 21671, 1302060 instead of 1300260, ف instead of نه and 85 instead of 55.

(e) *Additional Example*:-

No additional example is required.

7. To find the date of Roman era from this date:-

When we intend to find the da… of the Roman era, we add 175 to t… ahargana and multi… by … the product we add 30 and …

divide it by 21915; to the quotient we add 1276 getting the complete years of the Roman era (and the next year is the current year). The remainder represents the ghatis which are converted into days; then we start subtracting the date of the month beginning from Tashrin I, as we have done in all other dates. If it is a leap year, we take 29 days for Shubat, and if it is not a leap year we take 28 days. The indication for a leap year is that while converting the ghatis into days, we get complete days and no remainder is obtained.

EXAMPLE:– We added 175 to the ahargana getting 21789; we multiplied it by 60 and got 1307340; to it we added 30 and got 1307370; we divided it by 21915 and got the quotient 59; to it we added 1276 and got the sum 1335 which represents the complete years of the Roman era (and the next, i. e. the 1336th year is the current year). The remainder was 14385 ghatis which, while converted into days, gave 239. As we get some remainder also while converting ghatis into days, it is not a leap year. From it we deducted the number of days of seven months beginning from Tashrin I, i. e. 212, and got the remainder 27. This is the number of days of Ayar already passed (up to the coming midnight of the day in question).

COMMENTARY

(a) *Correction of the manuscript* :–

There are two main corrections made by me in this article. As already explained, Beruni had calculated a wrong ahargana and from this wrong ahargana he had tried to get the correct date of the Roman era. For this purpose he had said, "we add 147 to the ahargana"; but due to the mistake of 28 days made by Beruni in ahargana, this figure should have been 147 + 28 = 175. Therefore I have deleted the figure 147 in the text and have written 175 in its place. Due to this correction, most of the figures of the example have also been changed which will be explained later.

The second mistake made by Beruni is that he has not taken into account the 30 ghatis already accumulated for the previous two years; without adding these 30 ghatis into the total ghatis of the ahargana we cannot find out correct leap years. So I have added a phrase to the text stating, "to the product we add 30" in order to eliminate the mistake made by Beruni. All such petty mistakes clearly show that Beruni wrote this book in a hurry and had never a chance to see it again. These mistakes are never a proof of Beruni's lack of knowledge.

(b) *Translation of the technical term* :–

We have already described the names of the months of the Roman era and no further clarification is required.

(c) *Elaboration of the Principle* :–

Beruni has based this method on the fact that every fourth year of the Roman era is a leap year and consists of 366 days, instead of 365 days. Therefore the mean duration of a year in this system is 365¼ days or 21915 ghatis. Therefore, if we convert the ahargana into ghatis by multiplying it

by 60 and then divide the total number of ghatis by 21915, we get the complete years of the Roman era. The difference of 15 ghatis per year becomes 60 ghatis in the fourth year and that year is considered as a leap year, but it is very important to remember that in the Roman era you cannot find out the leap year simply by dividing the current year by 4 as we do in the case of the Christian era. In the Roman era you should add "1" to the current and then divide by 4 to find out the leap year. In the leap year of the Roman calendar the fifth month Shubat has 29 days instead of 28 days.

It can be shown by "back calculation" that when the ahargana of Karanatilaka was zero, it was 24th of Azar 1277 Roman. It shows that before the start of the ahargana of Karana Tilaka, 175 days of the 1277th year had already passed from Tashrin I to 24 Azar (i. e., 31 + 30 + 31 + 31 + 28 + 24 = 175). So if we add 175 to the ahargana of Karanatilaka, we get the complete days from the start of the 1277th year, i. e., 1276 complete years had already passed. That is why we add 1276 to the complete years obtained by the ahargana in order to get the complete years of the Roman era. Moreover, the 1275th year was a leap year which means that before the start of that year a difference of 15 ghatis per year had already accumulated to 60 ghatis, or full one day. Therefore, a difference of 15 ghatis was accumulated in the 1275th year and so a total difference of 30 ghatis had already accumulated before the start of the 1277th year. This is why we add 30 ghatis to the total ghatis of the ahargana to get the correct number of ghatis since the beginning of the 1277th year of the Roman era.

The Roman calendar was started 12 years after the death of Alexander. This calendar is seldom used in the modern world, because it is not a pure Solar calendar. Actually the length of a Solar year (i. e., a tropical year, if we use a more technical term) is equal to 365. 2422 civil days, and not 365. 25 civil days, as is assumed in the Roman calendar. Therefore a difference of 0.0078 day per year accumulates to 3. 12 days in four centuries. Therefore if we decide that the first, second and third centuries will not have a leap year but the fourth century will have a leap year, then the above mentioned difference of three days can be eliminated, but a difference of 0. 12 days per four hundred years will still remain unaccounted for; it may be ignored for smaller spans of time.

In very early days the Roman calendar was converted into the Christian calendar by changing the "beginning of the year" and counting the year from the death of Christ. So there was a difference of 311 years between the two calendars and when it was the first year of the Christian calendar, it was 312th year of the Roman calendar. The Christian year was started from Kanun II, instead of Tashrin I, and this month was named as January, keeping the number of days of each month the same as the Roman calendar. This calendar was known as Julian calendar after Julius Caesar who had taken much interest in it. This Julian calendar must not be confused with the "Julian date" or "Julian day" which is absolutely a different thing. The names of the Christain months along with their Roman equivalents and number of days are given below, showing also the comparative years :–

(and a difference of 15 ghatis was accumulated in the 1276th year)

	Month		Month	Days	
1582 A.D.	January	...	Kanun II	...31	1893 Roman
	February	...	Shubat	...28/29	
	March	...	Azar	...31	
	April	...	Naisan	...30	
	May	...	Ayar	...31	
	June	...	Haziran	...30	
	July	...	Tamoz	...31	
	August	...	Ab	...31	
	September	...	Elol	...30	
	October	...	Tashrin I	...31	1894 Roman
	November	...	Tashrin II	...03	
	December	...	Kanun I	...31	

The Julian calendar was not a pure Solar calendar, as already explained. People started feeling that the dates were going out of step with respect to the season. In the year 730 A.D. Venerable Bede, the medieval English scholar, showed that the deviation was more than three days; Roger Bacon in 1200 A.D. found an error of 7 or 8 days; soon after 1300 A.D. Dante emphasised the need for calendar reform. In 1474 A.D., Pope Sixtus IV invited the scientist Regeomontanus to revise the calendar and bring it into line with the seasons, but the Pope's death interrupted the plan. It was not until 1582 A.D. that Pope Gregory XIII revised the calendar, dropped the 10 days of accumulated error, and decided on the rule that 5 October 1582 A.D. should be considered 15 October 1582 A.D., and that only such centesimal years were to be leap years as were divisible by 400. This shows that the same day was called 5 October and also 15 October, and the dates such as 6, 7, 8, 9, 10, 11, 12, 13, and 14 October 1582 A.D. do not exist at all. The ahargana of Karana Tilaka was 225189 on that peculiar day; it was a Friday, and 5 Tashrin I, 1894 Roman. The year 1582 was a leap year according to the old style, and the year 1584 also was a leap year according to the new style; so there was a difference of only two years between two leap years. This is the calendar which is so widely used in the world and is known as the Gregorian calendar. It is more nearly staisfactory than the Julian calendar but still not perfectly so, and after a few thousand years another slight revision (such as making only such Millenniums into leap years as are divisible by 4000) will be necessary. The problem is one that cannot be solved except approximately, since the periods of rotation and revolution of the earth are in no simple ratio.

The Julian calendar was promptly adopted throughout the Roman Empire, but the Gregorian calendar was not so fortunate, because it was formulated after the Reformation, and the Pope's decision was not immediately accepted everywhere. The Catholic nations adopted it at once, but the Protestant nations fell into line only gradually. Germany adopted the Gregorian calendar in 1700 A.D.; England adopted it in 1751, and by that time the two calendars were out of step by 11 days. The year began in England on January 1, instead of March 25, for the first time in 1752 A.D., although the change had been made in Scotland in 1600 A.D. Russia adopted the Gregorian calendar only after the Revolution of 1918, so that many Russians

now living were born under the old style, i. e., the Julian calendar. The Russian Orthodox Church still uses the Julian calendar; that is why the Russian Easter may differ by one or two weeks from the date of Easter in the rest of the Christian world.

(d) *Explanation of the Text*:

As already explained Beruni has made two basic mistakes in this articel due to which the following changes have been made :–

(i) I have deleted the following phrase from the text:

زدنا على الأصل ١٤٧

and in its place the following phrase has been inserted.

زدنا على الأصل ١٧٥

(ii) I have added the following phrase to the text:

و زدنا على المجتمع ل

(iii) I have added the following phrase to the example:

و زدنا على المجتمع ل فصار ١٣٠٧٣٧٠ .

(iv) In the example I have written 175 instead of 147, 21789 instead of 21761, 1307340 instead of 1305660, 14385 instead of 12675, 239 instead of 211 لسبعة instead of لستة, 212 instead of 182; كز instead of كط, and ايار instead of نيسان

(e) *Additional Example*:

In this book methods are given to calculate ahargana from any calendar, but in the modern world the widely used calendar is the Gregorian calendar, which unfortunately was not existing at the time of Beruni. But from the same reasoning we have found a method of calculating the ahargana of Karana Tilaka from the Gregorian calendar. With the help of this method we found out the ahargana for 10 May 1957, as 361997. We have also shown that the difference between the Julian day and the ahargana of Karana Tilaka is 2073972, so the ahargana can also be found by deducting 2073972 from the Julian day if we know it. Therefore the best method of finding out the ahargana in modern days is to find out the Julian day first which can be found out with the help of any modern ephemris, and then deducting 2073972 from it. For example, the Julian day on 10 May 1957 A.D. was 2435969 and when we deducted 2073972 from it, we got 361997 which is the required ahargana and exactly tallies with the previous ahargana obtained by another method.

To calculate the Roman date from this ahargana, we added 175 to the ahargana getting 362172, multiplied it by 60 getting 21730320, added 30 to it getting 21730350, divided it by 21915 getting the quotient 991, added 1276 to it getting 2267 which represents the complete years of the Roman era and the next, i. e. 2268th year is the current year. We converted the remainder 12558 ghatis into complete days by dividing it by 60, getting 209 days and 45 ghatis, deducted from it the days of the month beginning from Tashrin I up to Ayar, i.e., 31 + 30 + 31 + 31 + 28 + 31 = 182 days getting the remainder 27 days which belong to Naisan, which is the next month. We have taken 28 days for Shubat, because the 2268th year of the Roman era is not a leap year which is evident from the fact that we got a remainder of 45 ghatis while converting the total ghatis into complete days; had it been a leap year there would have been no remainder. So we see that the date was 27 Naisan, 2268 Roman. We have also shown that there is a difference of 311 years between the Roman and the Christian calendars, so that if we deduct 311 from 2268 we get 1957 which is the Christian year. Similarly, at present there is a difference of 13 days between the Roman and Christian months; the difference of 10 days was created in the year 1582 A. D. by Pope Gregory XIII and a further difference of 3 days has occurred in the years 1700, 1800, and 1900 A. D. which were not considered leap years in Gregorian system. Therefore if we add 13 days to the Roman date, we get 10 Ayar which corresponds to 10 May of the Christian calendar. Therefore it has been verified that our calculations are correct, because we get the correct date 10 May 1957 A. D.

To calculate the Arabic date from this ahargana, we added 87 to the ahargana getting 362084, multiplied it by 60 getting the product 21725040, divided it by 911704 getting the quotient 23, deducted it from the same product getting 21725017, divided it by 21262 getting the quotient 1021 and the remainder 16515, added 354 to the quotient getting 1375 which represent the complete years of Hijra era and the next 1376th year is the current year. Then we converted the remaining 16515 ghatis into days by dividing it by 60 getting the quotient 275 which represents the complete days of the 1376th year passed. It is very important to remember that in these calculations, when we say "so many complete days passed up to such and such particular date" that particular date is also included in the total of the complete days passed, because we always count upto the coming midnight of the particular date. Now from 275 complete days we deducted alternately 30 and 29 days for the months beginning from Moharram. So we deducted 266 days upto Ramaḍan and the remaining 9 days belong to Shawwal. Therefore the Arabic date on 10 May, 1957 was 9 Shawwal 1376 A./H. This date has been verified by me from my personal diary in which it is written that Eid was celebrated all over India and Pakistan on 2nd May, 1957 A. D. which was 1 Shawwal 1376 A. H.

To calculate the Persian date from this ahargana, we divided the ahargana by 365 getting the quotient 991 and the remainder 282, added 334 to the quotient getting 1325 which represents the complete years of the Yazdjird calendar and the next, i. e. 1326th year is the current year. From the remaining 282 days we deducted the days of the months beginning from Farwardinmah, i. e., 30 + 30 + 30 + 30 + 30 + 30 + 30 + 35 + 30 = 275 days getting the remainder 7 days which belong to Dimah which is the next month. So the Persian date on 10 May, 1957 A. D. was Murdad, 7 Dimah, 1326 Yazdjird. This date has been verified by me from a calendar printed in Persia. The Parsis of India and Pakistan do not include the five additional days in Abanmah as we have done in the above-mentioned calculations; they add these five special days at the end of the year after Isfindarmuzmah Moreover, they are exactly 30 days backward from the old Persian calendar

but still they call their calendar "Yazdjird calendar" because except the two above-mentioned variations, the Parsi calendar is exactly similar to the Yazdjird calendar. Therefore, if we want to find out the date of the Parsi calendar from the above-mentioned date, we get Mah, 12 Azurmah 1326 Yazdjird. This date also has been verified by me from a calendar printed in India. The reformed form of this calendar, known as Maliki or Jalali calendar, is absolutely different from it and neither years nor months nor days tally with each other. The Jalali year starts from the day when the sun crosses the first point of Aries; so sometimes there are 365 days and sometimes 366 days in a year which can only be determined by actual calculations. Therefore we can say that the Jalali calendar is a pure Solar calendar and does not require any modifications or alterations from time to time. This calendar was derived, as has already been stated, by the famous poet and astronomer, Umar Khyyam. Since space does not permit me to give a detailed description of this calendar, I leave it at this stage.

(٣) انتخاب التاريخ :-

و تمثل ذلك بيوم مشهور و ليكن يوم التقاء السلطان محمود مع الخان يوسف رحمهما الله وكان على ثنية بسمرقند بين العسكرين يوم الخميس كز من صفر (و فى المخطوطة كه من صفر) سنة تيو للهجرة و كط من نيسان سنة غشلو من تاريخ اليونانيين و روز رام كا من ارديبهشت ماه سنة شصد ليزدجرد و هو عند الهند يوم التاسع و العشرين القمرى من بيشاك امانت ثانى شهور سنة ٩٤٧ لشق (و فى المخطوطة يوم السابع و العشرين القمرى من جيرت ثالث شهور سنة ٩٤٨ لشق)

(٤) مثال لاستخراج الاصل من التاريخ شك كال :-

(وهو يوم السابع و العشرين القمرى من جيرت امانت ثالث شهور سنة ٩٤٧ لشق يوم الخميس كو من ربيع الاول سنة تيو للهجرة و كز من ايار سنة غشلو من تاريخ اليونانيين وروز فروردين يط من خرداد ماه سنة شصد ليزدجرد) فوضعنا شك كال التامة ٩٤٧ ونقصنا منها ٨٨٨ فبقى ٥٩ ضربناها فى ١٢ وزدنا على ما خرج ما مضى من الشهور و هو٢ ووضعناها فى موضعين وضربنا اسفلهما فى ٩٠٠ فصار ٦٣٩٠٠٠ زدنا عليه ٦٦١ فكان ٦٣٩٦٦١ قسمناه على ٢٩٢٨٢ فخرج شهور الكبيسة ٢١ و بقى ٢٤٧٣٩ زدنا شهور الكبيسة على ما فى الموضع الاعلى فصار ٧٣١ ضربناه فى ٣٠ فبلغ ٢١٩٣٠ و زدنا عليه الايام القمرية الماضية من الشهر الثالث و هى ٢٧ فاجتمع ٢١٩٥٧ وضعناه فى مكانين و ضربنا اسفلهما فى ٣٣٠٠ فاجتمع ٧٢٤٥٨١٠٠ زدنا عليه ٦٤١٠٦ فصار ٧٢٥٢٢٢٠٦ قسمناه على ٢١٠٩٠٢ فخرج ٣٤٣ المحفوظ الاول و بقى ١٨٢٨٢٠ - المحفوظ الثانى نقصان المحفوظ الاول مما فى المكان الاعلى فبقى ايام التاريخ ٢١٦١٤ و هى الاصل

و نحن و ان ضمنا ان لا نزيد فى الكتاب غير الامثلة فلابـد ههنا من زيادة استخراج التواريخ المستعملة فى بلادنا من هذا التاريخ الذى جعله صاحب الزيج اصلا لاعماله فان جل اصحابنا لا يهتدون الى تاريخ الهند ـ

(٥) استخراج تاريخ يزدجرد من هذا التاريخ :ـ

اذا اردنا تاريخ يزدجرد من هذا التاريخ قسمنا الاصل على ٣٦٥ فما خرج زدنا عليه ٣٣٤ فيجتمع سنو يزديرد التامة و مابقى القينا لكل شهر من عند فروردين ماه ٣٠ و لآبان من بين الشهور له الى ان لا يبقى ما عندنا بايام شهر فيكون الماضى من الشهر الذى انتهينا عليه (بنصف الليل الذى بعد النهار المحسوب له)

مثال :ـ اذا قسمنا الاصل على ٣٦٥ فخرج نط زدنا عليه ٣٣٤ فبلغ ٣٩٣ و تلك سنو يزدجرد التامة و بقى من القسمة ٧٩ فالقينا منه لفروزدين ماه ل وبقى معنا ٤٩ و القينا منه لارديبهشت ماه ل و بقى ١٩ فهى الايام الماضية من خرداد ماه (بنصف الليل الذى بعد النهار المحسوب له)

(٦) استخراج تاريخ الهجرة من هذا التاريخ :ـ

و اذا اردنا تاريخ العرب من تاريخ الهند فى هذا الزيج زدنا على الاصل فز وضربنا المبلغ فى ستين (و قسمنا المجتمع على ٩١١٧٠٤ و القينا ماخرج من المجتمع وقسمنا المجتمع) على ٢١٢٦٢ فما خرج زدنا عليه ٣٥٤ فيجتمع سنو تاريخ الهجرة التامة و مابقى يرفعه من الدقائق الى صحاح الايام بالقسمة على ٦٠ و متى فضلت الدقائق الباقية عن الرفع على ٣٠ جبرناها الى ايام يوما و ان لم يفضل نلغيها ثم نلقى من هذا الايام لشهر ل و لشهر كط و نبتدئ بالمحرّم الى ان تمنع القاء شهر مما معنا فيكون الباقى مامضى من الشهر الذى انتهينا اليه (بنصف الليل الذى بعد النهار المحسوب له)

مثال :ـ انا زدنا على الاصل فز فاجتمع ٢١٧٠١ ضربناه فى ستين بلغ ١٣٢٠٦٠ (قسمناه على ٩١١٧٠٤ و القينا منه ماخرج فبقى ١٣٠٢٠٥٩) قسمناه على ٢١٢٦٢ فخرج ٦١ و زدنا عليه ٣٥٤ فصار ٤١٥ و ذلك سنو تاريخ الهجرة تامة و بقى من الدقائق فه يوما فصارت الايام ٨٥ و منها للمحرم ل و للصفر كط و يبقى كو و هى الايام الماضية من ربيع الاول (بنصف الليل الذى بعد النهار المحسوب له)

(٧) استخراج تاريخ الاسكندر من هذا التاريخ :ـ

اذا اردنا تاريخ الاسكندر زدنا على الاصل ١٧٥ و ضربنا مابلغ فى ستين وزدنا على المجتمع ل و قسمنا المجتمع على ٢١٩١٥ فماخرج زدنا عليه ١٢٧٦ فيجتمع سنو تاريخ الاسكندر تامة وترفع مايبقى من الدقائق الى الصحاح فيكون اياما و يبقى منها لكل شهر من تشرين الاول عدد ايامه كما فعلنا فى سائر التواريخ فان كانت السنة كبيسة جعلنا حصة شباط من الايام كط و ان لم يكن كبيسة جعلناها كح و علامة السنة الكبيسة ان تنجبر الدقائق بالرفع فلا يبقى منها شيء ـ

مثال :ـ انا زدنا الى الاصل ١٧٥ فصار ٢١٧٨٩ فضربناه فى ٦٠ فبلغ ١٣٠٧٣٤٠ و زدنا على المجتمع ل فصار ١٣٠٧٣٧٠ قسمناه على ٢١٩١٥ فخرج نط و زدنا عليه ١٢٧٦ فاجتمع ١٣٣٥ و هو سنو تاريخ الاسكندر تامة و بقى من الدقائق ١٤٣٨٥ و يكون مرفوعه ٢٣٩ و السنة لاجل الدقائق القاصرة عن الارتفاع غير كبيسة فالقينا لسبعة اشهر منها من عند تشرين الاول ٢١٢ و بقى كز و هى الايام الماضية من ايار (بنصف الليل الذى بعد النهار المحسوب له)

و مالم يتقرر عكس ماذكرناه لم يشترك اصحابنا و الهند فى استعمال هذا الزيج ـ

IV

Relating to the Mean Places of the Planets

1. To Find the Mean Place of Mars:–

Add 218½ to the Ahargana and divide the sum by 687, then the quotient will represent the number of revolutions which we do not require; from the remainder find out signs, etc., according to the method already described and keep the result in memory. Now consider the ahargana again and divide it by 8719, then the quotient will be minutes and will be added to the result kept in memory; deduct 2½ minutes from this sum, then the remainder will be the mean place of Mars (for the midnight which follows the day in question).

Example:– We added 218½ to the Ahargana getting 21832½; divided it by 687 getting the quotient 31 revolutions which were ignored; from the remainder we obtained 9S 10' 36' 41" which we kept in memory. Now we divided the Ahargana by 8719 getting the quotient 2 min. 29 sec. which we added to the result already kept in memory getting 9S 10' 39' 10"; from it we deducted 2½ min. getting the remainder 9S 10' 36' 40" which is the mean place of Mars (for the midnight which follows the day in question).

Commentary

(a) *Correction of the Manuscript*:–

In this article only one minor correction is made and that is that I have written 8719 instead of 8718; this error of the manuscript seems to be due to the mistake of the original Sanskrit scribe.

(b) *Translation of Technical Terms*:–

It has already been explained in the past.

(c) *Elaboration of the Principle*:–

The formula for calculating the mean place of Mars as given in this article is shown below where "A" stands for Ahargana:–

$$\frac{A+218\frac{1}{2}}{687}\text{ rev.} + \frac{A}{8719}\text{ min.} - 2\tfrac{1}{2}\text{ min.}$$

The general form of this formula is given below, where X, Y, Z and D are constants:–

$$\frac{A+Z}{X}\text{ rev.} + \frac{A}{Y}\text{ min.} - D\text{ min.}$$

If we ignore the kshepakas Z and D, and put A equal to 1577917828, we get the revolutions in a Chaturyuga or a Mahayuga; now we will see how this formula has been derived by the author of Karana Tilaka:

$$\frac{1577917828}{687}\text{ rev.} + \frac{1577917828}{8719\times 21600}\text{ rev.}$$

$$= 2296823.621\text{ rev.} + 8\text{-}379\text{ rev.} = 2296832\text{ rev.}$$

It shows that the number of revolutions of Mars in a Chaturyuga as adopted in Karana Tilaka are exactly in accordance with Surya Siddhanta (*vide* chapter I, sloka 30). Now, the motion of the mean place of Mars in one day is $\frac{2296832}{1577917828}$ rev. If we select the numerator of a simplified fraction as "1", we can find out its denominator "X" in the following way:–

$$\frac{1}{X} = \frac{2296832}{1577917828}, \text{ or } X = \frac{1577917828}{2296832} = 686\,\frac{2291076}{2296832},$$

the fraction being equal to more than half can be taken as "1" so that X = 687. In order to compensate for this approximation error we can have another fraction giving the result in minutes and having its numerator "1"; then its denominator Y can be found out in the following way:–

$$\frac{1}{21600Y}\text{ rev.} = \frac{2296832}{1577917828}\text{ rev.} - \frac{1}{687}\text{ rev.} \doteq \frac{5756}{1084029547836}\text{ rev.}$$

$$\text{or } Y = \frac{1084029547836}{5756\times 21600} = 8718\,\frac{124095036}{124329600},$$

the fraction being more than half can be taken as "1" so that Y=8719. Beruni has given 8718 instead of 8719 but I have corrected this figure in the manuscript accordingly.

Now to find out the kshepakas Z and D, we calculate the mean place of Mars in the beginning of 888 Shakakala as follows:–

$$\frac{1485507\times 2296832}{1577917828}\text{ rev.} = 2162\,\frac{501669688}{1577917828}\text{ rev.}$$

Ignoring the complete revolutions we have an incomplete revolution in the beginning of 888 Shakakala, and in order to incorporate this partial revolution we can find out the kshepaka Z so that,

$$\frac{Z}{687} = \frac{501669688}{1577917828}, \text{ or } Z = \frac{687\times 501669688}{1577917828} = 218\tfrac{1}{2} - \frac{127969762}{1577917828}.$$

So if we take Z = 218½, we should deduct the following figure from the result in order to compensate for the negative fraction, and call this value D, so that,

$$D = \frac{127969762}{1577917828\times 687}\text{ rev.} = \frac{127969762\times 21600}{1577917828\times 687}\text{ min.}$$

$$= 2\ \frac{59608776352 8}{1084029547836}\text{ min.} = 2 + \cfrac{1}{\cfrac{1}{1 + \cfrac{1}{1+0000}}}\text{ min.} = 2\tfrac{1}{4}\text{ min. app.}$$

A more accurate value of D could have been taken as,

$$D = 2 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{4}}}\text{ min.} = 2\frac{5}{9}\text{ min.}$$

but the author has preferred the simpler value, i.e., 2¼ min; the difference between these two values is app. 3 sec. which is negligible for all practical purposes. In the solved example we have calculated the mean place of Mars as 9 S 10° 36′ 40″; if we calculate the same directly by the method of Surya Siddhanta, we get a bit more accurate value 9 S 10° 36′ 37″.

2. To Find the Sheeghrochcha of Mercury :-

Multiply the Ahargana by 9000 and from the product deduct 336262; divide the remainder by 791727, then the quotient will be the revolutions which we do not require; from the remainder find out the signs, etc., and keep the result in memory. Now divide the Ahargana by 10060, then the quotient will be minutes, etc., deduct it from the result kept in memory, then the remainder will be the sheeghrochcha of Mercury (for the midnight which follows the day in question).

Example :- We multiplied the ahargana by 9000 getting the product 194526000; from it we deducted 336262 getting the remainder 194189738, which we divided by 791727 getting the quotient 245 revolutions which we ignored; from the remainder we obtained 3S 8° 29′ 56″, which we kept in memory. Now we divided the Ahargana by 10060 getting the quotient 2′ 9″, which we deducted from the result kept in memory getting the remainder 3S 8° 27′ 47″, which is the sheeghrochcha of Mercury (for the midnight which follows the day in question).

Commentary

(a) *Correction of the Manuscript* :-

No correction of the manuscript is required except a slip of the pen of the Arabic scribe which has been corrected by me and I have changed the original figure ١٩٤٥٦٦... to read as ١٩٤٥٢٦... which is the correct figure. In the solved example a missing phrase has been inserted by me within brackets.

(b) *Translation of the Technical Terms* :-

Sheeghrochcha of Mercury :- This term can also be written as Budha-sheeghrochcha or simply Budhochcha just like Chandrochcha, but I

have preferred to write it as sheeghrochcha of Mercury; Beruni has used the term سرعة عطارد for the original Sanskrit term Budha-sheeghrochcha.

(c) *Elaboration of the Principle* :-

The formula for calculating the sheeghrochcha of Mercury as given in this article is shown below where A stands for Ahargana :-

$$\frac{9000\,A - 336262}{791727}\text{ rev.} - \frac{A}{10060}\text{ min.}$$

The general form of this formula can be taken as follows where X, Y and Z are constants :-

$$\frac{9000\,A - Z}{X}\text{ rev.} - \frac{A}{Y}\text{ min.}$$

Now we will see how this formula has been derived by the author of Karana Tilaka. If we ignore the kshepaka Z and put A equal to 1577917828, we get the revolutions in a Chaturyuga or Mahayuga,

$$\frac{9000 \times 1577917828}{791727}\text{ rev.} - \frac{1577917828}{10060 \times 21600}\text{ rev.} = 17937060\text{ rev.}$$

This value exactly tallies with the value given in Surya Siddhanta (*vide.*, chapter I, sloka 31).

Now the mean daily motion of the sheeghrochcha of Mercury is $\frac{17937060}{1577917828}$ rev., and if we select a numerator 9000 for a simplified fraction, we can find out its denominator X in the following way,

$$\frac{9000}{X} = \frac{17937060}{1577917828},\ \text{or } X = \frac{1577917828 \times 9000}{17937060} = 791727\,\frac{5749380}{17937060}.$$

The fraction being less than half is ignored so that we have X = 791727. But to compensate for the above error of approximation we can find out another fraction giving the result in minutes and having its numerator as 1; then its denominator Y can be found out in the following way :-

$$\frac{1}{21600Y} = \frac{9000}{791727} - \frac{17937060}{1577917828} = \frac{5749380}{1249280148208956}$$

$$\text{or } Y = \frac{1249280148208956}{5749380 \times 21600} = 10059\,\frac{87058336956}{124186608000}$$

The fraction being more than half is taken as 1, so that Y = 10060. Now to find out the kshepaka Z we calculate the sheeghrochcha of Mercury in the beginning of 888 Shakakala as follows :-

$$\frac{1485507 \times 17937060}{1577917828}\text{ rev.} = 16886\,\frac{907745812}{1577917828}\text{ rev.} = 16887\text{ rev.} - \frac{670172016}{1577917828}\text{ rev.}$$

The negative fraction shows the value of the sheeghrochcha of Mercury in the beginning of 888 Shakakala. Therefore the value of Z can be found as follows :–

$$\frac{Z}{791727} = \frac{670172016}{1577917828}, \text{ or } Z = \frac{791727 \times 670172016}{1577917828} = 336261\frac{1052950524}{1577917828},$$

The fraction being more than half is taken as 1, so that Z = 336262 (negative). If we want to have a positive value of Z, we can use the figure (791727–336262) = 455465; but the author of Karana Tilaka has used the negative kshepaka, because it is smaller than the positive one and is easier for calculations. In the solved example we have calculated the sheeghrochcha of Mercury as 3 S 8° 27′ 47″; if we calculate the same directly by the method of Surya Siddhanta, we get a bit more accurate value 3S 8° 27′ 48″.

3. To Find the Mean Place of Jupiter :–

Multiply the Ahargana by 100 and from the product deduct 47898; divide the remainder by 433232, then the quotient will be the complete revolutions which we do not require; from the remainder find out signs, etc., and keep the result in memory. Now, divide the Ahargana by 22200, then the quotient will be seconds which is to be deducted from the result kept in memory; the remainder will be the mean place of Jupiter (for the midnight which follows the day in question).

Example :– We multiplied the Ahargana by 100 getting the product 2161400; from it we deducted 47898 getting the remainder 2113502; divided it by 433232 getting the quotient 4 revolutions which were ignored; from the remainder got 10S 16° 14′ 35″ which we kept in memory. Now we divided the Ahargana by 22200 getting the quotient 1″; deducted it from the result kept in memory getting the remainder 10 S 16° 14′ 34″; this is the mean place of Jupiter (for the midnight which follows the day in question).

Commentary

(a) *Correction of the Manuscript* :–

I have made one major correction in the text of this article and have changed the figure ۲۲۰۷ to read as ۲۲۲۰۰ which is the correct value; I have also changed the figure ٤٧٨٩٧ to read as ٤٧٨٩٨ which is the correct value. Due to these two changes the figures of the solved example have also been changed as shown in the following table. The justification of these corrections will be given afterwards.

Original in Manuscript	Changed after correction	Original in Manuscript	Changed after correction
نقصنا منه ٤٧٨٩٧	نقصنا منه ٤٧٨٩٨	اقسم الاصل على ۲۲۰۷	اقسم الاصل على ۲۲۲۰۰
فبقى ۲۱۱۳۵۰۳	فبقى ۲۱۱۳۵۰۲	فخرج ها ها ها ى	فخرج ها ها ها ا
فمابقى ى يو يد لح	فمابقى ى يو يد له	فبقى ى يو يد كيح	فبقى ى يو يد لد

(b) *Translation of the Technical Terms* :–

No new terms have been translated in this article.

(c) *Elaboration of the Principle* :–

The formula for calculating the mean place of Jupiter as given in this article is shown below where A stands for the Ahargana :–

$$\frac{100\,A - 47898}{433232} \text{ rev.} - \frac{A}{22200} \text{ sec.}$$

The general form of this formula can be taken as follows where X, Y and Z are constants,

$$\frac{100\,A - Z}{X} \text{ rev.} - \frac{A}{Y} \text{ sec.}$$

Now we will see how this formula has been derived by the author of "Karana Tilaka." If we ignore the kshepaka Z and put A equal to 1577917828, we get the revolutions in a Chaturyuga or Mahayuga,

$$\frac{100 \times 1577917828}{433232} \text{ rev.} - \frac{1577917828}{22200 \times 1296000} \text{ rev.} = 364220 \text{ rev.}$$

This value exactly tallies with the value given in Surya Siddhanta (*vide.* chapter I, sloka 31).

Now the daily motion of the mean place of Jupiter is $\frac{364220}{1577917828}$ rev. and if we select a numerator 100 for a simplified fraction, we can find out its denominator X in the following way :–

$$\frac{100}{X} = \frac{364220}{1577917828}, \text{ or } X = \frac{1577917828 \times 100}{364220} = 433232\frac{1188}{18211}.$$

The fraction being less than half is ignored so that X = 433232. But to compensate for this error of approximation we can find out another fraction giving the result in seconds and having numerator as 1; then its denominator Y can be found out in the following way :–

$$\frac{1}{1296000\,Y} = \frac{100}{433232} - \frac{364220}{1577917828} = \frac{23760}{683604496460096},$$

$$\text{or } Y = \frac{683604496460096}{23760 \times 1296000} = 22200\frac{784460096}{30792960000}.$$

The fraction being less than half is ignored so that we get Y = 22200. Beruni has given this value as 2207 instead of 22200 which shows that either the Sanskrit manuscript possessed by Beruni was incorrect or Beruni could not decipher it correctly. In the Sanskrit script there is a close resemblance between O and 7, therefore it is just possible that Beruni might have read the last zero of 22200 as 7 and missed its first digit 2, thus making the figure 2207. If we suppose the figure 2207 to be correct, we get the total number of revolutions of mean place of Jupiter in a Chaturyuga equal to 364219½ app.

which is absurd, because we should always have "complete" revolutions in a Chaturyuga; moreover, these complete revolutions should also be divisible by 4.

Now to find out the kshepaka Z we calculate the mean place of Jupiter in the beginning of 888 Shakakala as follows :-

$$\frac{1485507 \times 364220}{1577917828} \text{ rev.} = 342 \frac{1403462364}{1577917828} \text{ rev.} = 343 \text{ rev.} - \frac{174455464}{1577917828} \text{ rev.}$$

The negative fraction shows the value of the mean place of Jupiter in the beginning of 888 Shakakala and the corresponding value of Z can be found out as follows :-

$$\frac{Z}{433232} = \frac{174455464}{1577917828}, \text{ or } Z = \frac{433232 \times 174455464}{1577917828} = 47898 \frac{581454104}{1577917828}$$

The fraction being less than half is ignored so that we have Z = 47898; but Beruni has given this value as 47897 which is incorrect and I have corrected it now; the reason for this mistake is not understood clearly. We have just seen that the figure 2207 given by Beruni is wrong; another proof of it is this that if we assume 2207 to be correct, the revolutions in Chaturyaga come to be 364219.5031067, and the corresponding value of Z comes to be 48101, instead of 47898. In the solved example we have calculated the mean place of Jupiter as 10S 16° 14′ 34″; if we calculate the same directly by the method of Surya Siddhanta, we get a bit more accurate value 10S 16° 14′ 33″.

4. To Find the Sheeghrochcha of Venus :-

Multiply the Aharagana by 600 and to the product add 14862; divide the sum by 134819, then the quotient will be revolutions which we do not require; from the remainder find out signs, etc., and keep the result in memory. Now divide the Ahargana by 9980, then the quotient will be minutes, etc., which is to be deducted from the result kept in memory; the remainder is the mean place of Jupiter (for the midnight which follows the day in question).

Example :- We multiplied the Ahargana by 600 getting the product 12968400; to it we added 14862 getting 12983262 which we divided by 134819 getting the quotient 96 revolutions which we do not require; from the remainder we got 3S 18° 30′ 49″ which we kept in memory; then we divided the Ahargana by 9980 getting the quotient 2′ 10″ which we deducted from the result kept in memory getting the remainder 3S 18° 28′ 39″ which is the sheeghrochcha of Venus (for midnight which follows the day in question).

Commentary

(a) *Correction of the Manuscript* :-

No correction is required in this Article.

(b) *Translation of the Technical terms* :-

Sheeghrochcha of Venus - This term can also be written as Shukra-

sheeghrochcha or simply Shukrochcha just like Chandrochcha, but I have preferred to write it as sheeghrochcha of Venus; Beruni has used the term سرعة الزهرة for the original Sanskrit term Shukra-sheeghrochcha.

(c) *Elaboration of the Principle* :-

The formula for calculating the sheeghrochcha of Venus as given in this Article is shown below where A stands for Ahargana :-

$$\frac{600\,A + 14862}{134819} \text{ rev.} - \frac{A}{9980} \text{ min.}$$

The general form of this formula can be taken as follows where X, Y and Z are constants :-

$$\frac{600\,A + Z}{X} \text{ rev.} - \frac{A}{Y} \text{ min.}$$

Now we will see how this formula has been derived by the author of "Karana Tilaka." If we ignore the kshepaka Z and put A equal to 1577917828, we get the revolutions in a Chaturyuga or a Mahayuga,

$$\frac{600 \times 1577917828}{134819} \text{ rev.} - \frac{1577917828}{9980 \times 21600} \text{ rev.} = 7022376 \text{ rev.}$$

This value exactly tallies with the value given in Surya Siddhanta (*vide.* Chapter, I, sloka 32).

Now the daily mean motion of sheeghrochcha of Venus is $\frac{7022376}{1577917828}$ rev., and if we select a numerator 600 for a simplified fraction, we can find out its denominator X in the following way :-

$$\frac{600}{X} = \frac{7022376}{1577917828}, \text{ or } X = \frac{1577917828 \times 600}{7022376} = 134819 \frac{986856}{7022376},$$

The fraction being less than half is ignored so that we have X = 134819. But to compensate for the above error of approximation we can find out another fraction giving the result in minutes and having its numerator as 1; then its denominator Y can be found out as follows :-

$$\frac{1}{21600\;Y} = \frac{600}{134819} - \frac{7022376}{1577917828} = \frac{986856}{212733303663132},$$

$$\text{or } Y = \frac{212733303663132}{986856 \times 21600} = 9979 \frac{20045544732}{21316089600}.$$

The fraction being more than half may be taken as 1, so that Y = 9980. Now to find out the kshepaka Z we calculate the sheeghrochcha of Venus in the beginning of 888 Shakakala as follows,

$$\frac{1485507 \times 7022376}{1577917828} \text{ rev.} = 6611 \frac{173943724}{1577917828} \text{ rev.}$$

The fraction shows the value of the sheeghrochcha of Venus in the beginning of 888 Shakakala. Therefore the value of Z can be found out as follows :-

$$\frac{Z}{134819} = \frac{173943724}{1577917828}, \text{ or } Z = \frac{134819 \times 173943724}{1577917828} = 14861 \frac{1482084048}{1577917828}$$

The fraction being more than half is taken as 1, so that we have $Z = 14862$.

In the solved example we have calculated the sheeghrochcha of Venus as 3S 18° 28′ 39″; if we calculate the same directly by the method of Surya Siddhanta, we get a bit more accurate value 3S 18° 28′ 38″.

5. To Find the Mean Place of Saturn :-

Multiply the Ahargana by 4 and from the product deduct 678⅔; divide the remainder by 43063, then the quotient will be the revolutions which we do not require; from the remainder find out signs, etc., and keep the result in memory. Now divide the Ahargana by 3876, then the quotient will be seconds; deduct it from the result kept in memory, then the remainder will be the mean place of Saturn (for the midnight which follows the day in question).

Example – We multiplied the Ahargana by 4 getting 86456; from it we deducted 678⅔ getting 85777⅓; divided it by 43063 getting the quotient 1 revolution which we ignored; from the remainder we got 11S 27° 5′ 7″ which were kept in memory. Now we divided the Ahargana by 3876 getting the quotient 6″ which we deducted from the result kept in memory getting the remainder 11S 27° 5′ 1″, which is the mean place of the Saturn (for the midnight which follows the day in question).

Commentary

(a) *Correction of the Manuscript* :-

In the text of this article only two main corrections are made by me; I have corrected the figure 768⅔ to read as 678⅔, and the figure 3879 to read as 3876. This error appears either due to the mistake of the original Sanskrit scribe or due to some misunderstanding on the part of Beruni. The figures of the solved example have also been corrected by me as follows :-

Original in manuscript	Changed after correction	Original in manuscript	Changed after correction
نقصنا منه ٧٦٨م	نقصنا منه ٠٧٨م	قسمنا الاصل على ٣٨٧٩	قسمنا الاصل على ٣٨٧٦
فبقى ٨٥٦٨٧ لك	فبقى ٨٥٧٧٧ لك	فخرج ها ها ها او	فخرج ها ها و
٥ عا قى يا كو يط نج	و ٤ بقى يا كزه ز	فبقى يا كو يط نب	فبقى يا كز ه ا

(b) *Translation of the Technical Terms* :-

No new terms have been translated in this article.

(c) *Elaboration of the Principle* :-

The formula for calculating the mean place of Saturn as given in this article is shown below where A stands for Ahargana,

$$\frac{4A - 678\frac{2}{3}}{43063} \text{ rev.} - \frac{A}{3879} \text{ sec.}$$

The general form of this formula can be taken as follws where X, Y and Z are constants,

$$\frac{4A - Z}{X} \text{ rev.} - \frac{A}{Y} \text{ sec.}$$

Now we will see how this formula has been derived by the author of "Karana Tilaka." If we ignore the kshepaka Z and put A equal to 1577917828, we get the revolutions in a Chaturyuga or Mahayuga,

$$\frac{4 \times 1577917828}{43063} \text{ rev.} - \frac{1577917828}{3879 \times 1296000} \text{ rev.} = 146568 \text{ rev.}$$

This value exactly tallies with the value given in Surya Siddhanta (*vide* chapter I, sloka 32).

Now the mean daily motion of the Saturn is $\frac{146568}{1577917828}$ rev. and if we select a numerator 4 for a simplified fraction, we can find out its denominator X in the following way :-

$$\frac{4}{X} = \frac{146568}{1577917828}, \text{ or } X = \frac{4 \times 1577917828}{146568} = 43063 \frac{13528}{146568},$$

The fraction being less than half is ignored, so that we have $X = 43063$. But to compensate for the above error of approximation we can find out another fraction giving the result in seconds and having its numerator as I; then its denominator Y can be found out in the following way:

$$\frac{1}{1296000Y} = \frac{4}{43063} - \frac{146568}{1577917828} = \frac{13528}{67949875427164},$$

$$\text{or } Y = \frac{67949875427164}{1296000 \times 13528} = 3875 \frac{12259427164}{17532288000},$$

The fraction being more than half can be taken as I, so that we have $Y = 3876$. Beruni has given this value as 3879 due to some misunderstanding, but now I have corrected this figure to read as 3876.

Now to find out the kshepaka Z we calculate the mean place of the Saturn in the beginning of 888 Shakakala as follows :-

$$\frac{1485507 \times 146568}{1577917828} = 137 \frac{1553047540}{1577917828} = 138 - \frac{24870288}{1577917828},$$

The negative fraction shows the value of the mean place of Saturn in the beginning of 888 Shakakala. Therefore the value of Z can be found out as follows :-

$$\frac{Z}{43063} = \frac{24870288}{1577917828}, \text{ or } Z = \frac{43068 \times 24870288}{1577917828} = 678 \frac{1160924760}{1577917828}$$

The fraction being more than half could have been taken as I, but due to the small denominator 43063, the author of "Karana Tilaka" has taken a more accurate value of this fraction as,

$$\cfrac{1}{1 + \cfrac{1}{2 + \dots}} = \frac{2}{3},$$

so that we have Z = 678¾; the error due to this approximation is plus 2″ app. Beruni has given this value as 768¾, which is clearly an error of interchange of two digits 6 and 7.

In the solved example we have calculated the mean place of the Saturn as 11S 27° 5′ 1″; if we calculate the same directly by the method of Surya Siddhanta, we get a slightly more accurate value 11S 27° 4′ 59″.

(N.B.) The readers will note that the mean places of all the planets (except Chandrochcha and Rahu) as calculated by the method of Karana Tilaka are approximately the same as calculated directly by the method of Surya Siddhanta; therefore we can definitely say that "Karana Tilaka" is based on Surya Siddhanta. There is a slight doubt in the case of the Moon, because the kshepaka (i.e. 180) given in the original manuscript differs from the correct kshepaka (i.e. $\frac{1}{18}$) calculated in accordance with Surya Siddhanta; the difference between these two kshepakas is equal to 37 sec. app., as already explained in the commentary of article 2, chapter II. Although I have corrected the kshepaka given in the original manuscript, yet there can be a very remote possibility that the author of "Karana Tilaka" might have introduced a very small "beeja" (i.e. correction) equal to minus 37 sec. based on his personal observations. Ganesha Devagya, the author of "Graha Laghava," has also mentioned a similar "beeja" for the Moon equal to minus 9 min. in 1442 Shakakala (*vide* "Graha Laghava," chapter I, sloka 16). Such corrections, however, have no bearing on the basic principles of a karana book.

In the beginning of the 16th century A.D. the planetary revolutions in a Chaturyuga were further modified by the author of Makaranda Sarani; for example, the revolutions of Chandrochcha and Rahu, according to Old Surya-Siddhanta as well as Khandakhadyaka, are 488219 and 232226, respectively; according to modern Surya Siddhanta, these are 488203 and 232238, respectively; according to Karana Tilaka, these are 488211 and 232234, respectively; and according to Makaranda Sarani, these are 488199 and 232242, respectively. The following table will give an idea of similar modifications according to five different authorities :-

"Revolution in a Chaturyuga"

Planets	Old Suryasiddhanta and Khandakhadyaka	Modern Surya-Siddhanta and Karana Tilaka	Makaranda Sarani
Sun	4320000	4320000	4320000
Moon	57753336	57753336	57753336
Mars	2296824	2296832	2296832
Sheeghrochcha of Mercury	17937000	17937060	17937044
Jupiter	364220	364220	364212

Sheeghrochcha of Venus	7022388	7022376	7022364
Saturn	146564	146568	146580
Civil days	1577917800	1577917828	1577917828

الباب الرابع

فی استخراج اوساط الکواکب الخمسة

معرفة استخراج وسط المریخ :-

زد علی الاصل ٢١٨ و نصف ثم اقسم المبلغ علی ٦٨٧ فیخرج ادواره لایحتاج الیها و استخرج البروج و ما تلاها مما یبقی کما تقدم ذکره و احفظها ثم عد الی الاصل فاقسمه علی ٨٧١٩ فیخرج دقائق یزید ها علی ما حفظت ثم انقص من الجملة دقیقتین و نصف فیبقی وسط المریخ (لنصف اللیل الذی بعد النهار المحسوب)٠

مثال :- انا زدنا علی الاصل ٢١٨ و نصف فصار ٢١٨٣٢ و نصف و قسمناه علی ٦٨٧ فخرجت الادوار الملغاة ٣١ و مما بقی ط ی لو ما نیه حفظنا ها ثم قسمنا الاصل علی ٨٧١٩ فخرج ها ها ب کط نیه زدنا ها علی المحفوظ فصار ط ی اط ی نیه نقصنا منه دقیقتین و نصف فیبقی ط ی لو م نیه وذلك وسط المریخ (لنصف اللیل الذی بعد النهار المحسوب)٠

(٢) معرفة استخراج سرعة عطارد :-

اضرب الاصل فی ٩٠٠٠ و انقص من المبلغ ٣٣٦٢٦٢ و اقسم الباقی علی ٧٩١٧٢٧ یخرج ادوار ملغاة و استخرج البروج و ما بعدها مما یبقی و احفظها ثم اقسم الاصل علی ١٠٠٦٠ فیخرج دقائق و توابعها و انقصها مما حفظت فیبقی سرعة عطارد (لنصف اللیل الذی بعد النهار المحسوب)

مثال :- انا ضربنا الاصل فی ٩٠٠٠ فصار ١٩٤٥٢٦٠٠٠ (نقصنا منه ٣٣٦٢٦٢ فبقی ١٩٤١٨٩٧٣٨) قسمنا علی ٧٩١٧٢٧ فخرجت الادوار الملغاة ٢٤٥ و المحفوظ ج ح کط نو نیه ثم قسمنا الاصل علی ١٠٠٦٠ فخرج ها ها ب ط نیه نقصنا ها من المحفوظ فبقی ج ح کز مز نیه و هو سرعة عطارد (لنصف اللیل الذی بعد النهار المحسوب)٠

(٣) معرفة استخراج وسط المشتری :

اضرب الاصل فی ١٠٠ و انقص مما بلغ ٤٧٨٩٨ و اقسم ما بقی علی ٤٣٣٢٣٢ فیخرج ادوار

تامة ملغاة و استخرج البروج و ما يتلوها مما يبقى و احفظها ثم اقسم الاصل على ٢٢٢٠٠ فيخرج ثواني و انقصها مما معك فيبقى وسط المشترى (لنصف الليل الذى بعد النهار المحسوب) .

مثال :ــ انا ضربنا الاصل فى ١٠٠ فاجتمع ٢١٦١٤٠٠ و نقصنا منه ٤٧٨٩٨ فبقى ٢١١٣٥٠٢ قسمناه على ٤٣٣٢٣٢ فخرجت الادوار الملغاة اربعة و المحفوظ مما بقى ى يو يد له نيه ثم اقسم الاصل على ٢٢٢٠٠ فخرج ها ها ا نيه نقصناه من المحفوظ فبقى ى يو يد لد نيه و هو وسط المشترى (لنصف الليل الذى بعد النهار المحسوب) .

52

(٤) معرفة استخراج سرعة الزهره :ــ

اضرب الاصل فى ٦٠٠ وزد على ما اجتمع ١٤٨٦٢ و اقسمه على ١٣٤٨١٩ فيخرج ادوار ملغى و استخرج البروج و ما يتلوها مما يبقى و احفظها ثم اقسم الاصل على ٩٩٨٠ فيخرج دقائق و ما بعدها فانقصها مما عندك فيبقى سرعة الزهرة (لنصف الليل الذى بعد النهار المحسوب) .

مثال :ــ انا ضربنا الاصل فى ٦٠٠ فصار ١٢٩٦٨٤٠٠ و زدنا عليه ١٤٨٦٢ فبلغ ١٢٩٨٣٢٦٢ قسمناه على ١٢٤٨١٩ فخرجت الادوار المطروحة ٩٦ (و المحفوظ ج يح ل مط نيه ثم قسمنا الاصل على ٩٩٨٠) فخرج ها ها ب ى نيه نقصناه من المحفوظ فيبقى ج يح كح لط نيه و ذلك سرعة الزهرة (لنصف الليل الذى بعد النهار المحسوب) .

(٥) معرفة استخراج وسط زحل :ــ

اضرب الاصل فى ٤ و انقص مما اجتمع ٦٧٨ و ثلث و اقسم ما بقى على ٤٣٠٦٣ فيخرج ادوار ملغى و استخرج البروج و ما يتلوها من الباق و احفظها ثم اقسم الاصل على ٣٨٧٦ فيخرج ثواني ينقصها مما معك فيبقى وسط زحل (لنصف الليل الذى بعد النهار المحسوب) .

مثال :ــ انا ضربنا الاصل فى ٤ فصار ٨٦٤٥٦ و نقصنا منه ٣٧٨ م فبقى ٨٥٧٧٧ لث قسمناه على ٤٣٠٦٣ فخرج دور واحد و مما بقى يا كز ه ز نيه حفظناه ثم قسمنا الاصل على ٣٨٧٦ فخرج ه ها ه و نيه نقصناه مما حفظنا فبقى يا كز ه ا نيه و ذلك وسط زحل (لنصف الليل الذى بعد النهار المحسوب) .