MANṢŪR IBN ᶜALĪ IBN ᶜIRĀQ, ABŪ NAṢR (*fl.* Khwarizm [now Kara-Kalpakskaya, A.S.S.R.]; *d.* Ghazna [?] [now Ghazni, Afghanistan], ca. 1036), *mathematics, astronomy.*

Abū Naṣr was probably a native of Gīlān (Persia); it is likely that he belonged to the family of Banū ᶜIrāq who ruled Khwarizm until it fell to the Ma'mūnī dynasty in A.D. 995. He was a disciple of Abu'l-Wafā' al-Būzjānī and the teacher of al-Bīrūnī. Abū Naṣr passed most of his life in the court of the monarchs ᶜAlī ibn Ma'mūn and Abu'l-ᶜAbbās Ma'mūn, who extended their patronage to a number of scientists, including al-Bīrūnī and Ibn Sīnā. About 1016, the year in which Abu'l-ᶜAbbās Ma'mūn died, both Abū Naṣr and al-Bīrūnī left Khwarizm and went to the court of Sultan Maḥmūd al-Ghaznawī in Ghazna, where Abū Naṣr spent the rest of his life.

Abū Naṣr's fame is due in large part to his collaboration with al-Bīrūnī. Although this collaboration is generally considered to have begun in about 1008, the year in which al-Bīrūnī returned to Khwarizm from the court of Jurjān (now Kunya-Urgench, Turkmen S.S.R.), there is ample evidence for an earlier date. For example, in his *Al-Āthār al-bāqiya* (Chronology), finished in the year 1000, al-Bīrūnī refers to Abū Naṣr as *Ustādhī* - "my master," while Abū Naṣr dedicated his book on the azimuth, written sometime before 998, to his pupil.

This collaboration also presents grave difficulties in assigning the authorship of specific works. A case in point is some twelve works that al-Bīrūnī lists as being written in my name (*bismī*), a phrase that has led scholars to consider them to be of his own composition. Nallino has, however, pointed out that *bismī* might also mean "addressed to me" or "dedicated to me" - by Abū Naṣr - and there is considerable evidence in support of this interpretation. For instance, the phrase is used in this sense in both medieval texts (the *Mafātīḥ al-ᶜulūm* of Muḥammad ibn Aḥmad al-Khwārizmī of 977) and modern ones of which there is no doubt of the authorship. The incipits and explicits of the works in question make it clear, moreover, that they were written by Abū Naṣr in reponse to al-Bīrūnī's request for solutions to specific problems that had arisen in the course of his more general researches. Indeed, in some of al-Bīrūnī's own books he mentioned Abū Naṣr by name and stated that his book incorporates the results of some investigations that the older man carried out at his request. Al-Bīrūnī gave Abū Naṣr full credit for his discoveries - as, indeed, he gave full credit to each of his several collaborators, including Abū Sahl al-Masīḥī, a certain Abū ᶜAli al-Ḥasan ibn al-Jīlī (otherwise unidentified) and Ibn Sīnā, who wrote answers to philosophical questions submitted to him by al-Birūnī.

The extent of the collaboration between Abū Naṣr and al-Bīrūnī may be demonstrated by the latter's work on the determination of the obliquity of the ecliptic. Al-Bīrūnī carried out observations in Khwarizm in 997, and in Ghazna in 1016, 1019, and 1020. Employing the classical method of measuring the meridian height of the sun at the time of the solstices, he computed the angle of inclination as $23^o35$. On the other hand, however, al-Bīrūnī became acquainted with a work by Muḥammad ibn al-Ṣabbāḥ, in which the latter described a method for determining the position, ortive amplitude, and maximum declination of the sun. Since al-Bīrūnī's copy was full of apparent errors, he gave it to Abū Naṣr and asked him to correct it and to prepare a critical report of Ibn al-Ṣabbāḥ's techniques.

Abū Naṣr thus came to write his *Risāla fi 'l-barāhin ${}^c$alā ${}^c$amal Muḥammad ibn al-Ṣabbāḥ* ("A Treatise on the Demonstration of the Construction Devised by Muḥammad ibn l-Ṣabbāḥ"), in which he took up Ibn al-Ṣabbāḥ's method in detail and demonstrated that it must be in error to the extent that it depended on the hypothesis of the uniform movement of the sun on the ecliptic. According to Ibn al-Ṣabbāḥ, the ortive amplitude of the sun at solstice ($a_t$) may be obtained by making three observations of the solar ortive amplitude ($a_1, a_2, a_3$) at thirty-day intervals within a single season of the year. He thus reached the formula:

$$2\sin a_t = \frac{2\sin a_2\sqrt{(2\sin a_2)^2-(2\sin a_1)(2\sin a_3)}}{\sqrt{(2\sin a_2)^2-(\sin a_1+\sin a_3)^2}}.$$

The same result may also be obtained from only two observations ($a_1, a_2$) if the distance ($d$) covered by the sun on the ecliptic over the period between the two observations is known:

$$2\sin a_t = \frac{R\sqrt{\frac{R^2(\sin a_1+\sin a_2)^2}{\cos^2\frac{d}{2}}-4\sin a_1\sin a_2}}{\sin\frac{d}{2}}.$$

The value of $a_t$ is thus extractable in two ways, and the value of the maximum declination can then be discovered by applying the formula of al-Battānī and Ḥabash:

$$\sin\quad\text{ort. ampl.}\ = \frac{(\sin\delta)\times R}{\cos\phi}.$$

Al-Bīrūnī then took up Abū Naṣr's clarification of Ibn al-Ṣabbāḥ's work, citing it in his own *Al-Qānūn al-Mas$^c$ūdī* and *Taḥdīd.* He remained, however, primarily interested in obtaining the angle of inclination, and simplified Ibn al-Ṣabbāḥ's methods to that end. He thus, within the two formulas, substituted three and two, respectively, observations of the declination of the sun for the three and two observations of solar ortive amplitude. By this method he obtained values for the angle of inclination of $23^o25'19''$ and $23^o24'16''$, respectively. These values are clearly at odds with that then commonly held ($23^o35'$) and confirmed by al-Bīrūnī's own observations. Al-Bīrūnī then returned to Abū Naṣr's work, and explained the discrepancy as being due to Ibn al-Ṣabbāḥ's supposition of the uniform motion of the sun on the ecliptic, as well as to the continuous use of sines and square roots.

Abū Naṣrs contributions to trigonometry are more direct. He is one of the three authors (the others being Abu'l Wafā' and Abū Maḥmūd al-Khujandī) to whom al-Ṭūsī attributed the discovery of the sine law whereby in a spherical triangle the sines of the sides are in relationship to the sines of the opposite angles as

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C},$$

or, in a plane triangle, the sides are in relationship to the sines of the opposite angles as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

The question of which of these three mathematicians was actually the first to discover this law remains unresolved, however. Luckey has convincingly argued against al-Khujandī, pointing out that he was essentially a practical astronomer, unconcerned with theoretical problems. Both Abū Naṣr and Abu'l Wafā', on the other hand, claimed discovery of the law, and while it is impossible to determine who has the better right, two considerations would seem to corroborate Abū Naṣr's contention. First, he employed the law a number of times throughout his astronomical and geometrical writings; whether or not it was his own finding, he nevertheless dealt with it as a significant novelty. Second, Abū Naṣr treated the demonstration of this law in two of his most important works, the *Al-Majisṭī al-Shāhī* ("Almagest of the Shah") and the *Kitāb fi'l-sumūt* "Book of the Azimuth"), as well as in two lesser ones, *Risāla fi ma$^c$rifat al-qisiyy al-falakiyya* (Treatise on the Determination of Spherical Arcs) and *Risāla fi'l-jawāb $^c$an masā'il handasiyya su'ila anhā* "Treatise in Which Some Geometrical Questions Addressed to Him are Answered").

The *Al-Majisṭī al-Shāhī* and the *Kitāb fi'l-sumūt* have both been lost. It is known that the latter was written at the request of al-Bīrūnī, as well as dedicated to him, and that it was concerned with various procedures for calculating the direction of the *qibla*. Abū Naṣr's other significant work, the most complete Arabic version of the *Spherics* of Menelaus, is, however, still extant (although the original Greek text is lost). Of the twenty-two works that are known to have been written by Abū Naṣr, a total of seventeen remain, of which sixteen have been published.

In addition to the books cited above, the remainder of Abū Naṣr's work consisted of short monographs on specific problems of geometry or astronomy. These lesser writings include *Risāla fi ḥall shubha $^c$araḍat fi'l-thālitha $^c$ashar min Kitāb al-Uṣūl* ("Treatise in Which a Difficulty in the Thirteenth Book of the *Elements* is Solved"); *Maqāla fi iṣlāḥ shakl min kitāb Mānālāwus fi'l-kuriyyāt $^c$adala fīhi muṣalliḥū hādha 'l-kitāb* ("On the Correction of a Proposition in the *Spherics* of Menelaus, in Which the Emendators of This Book Have Erred"); *Risāla fi ṣan$^c$at al-asṭurlāb bi'l-ṭariq al-ṣinā$^c$ī* "Treatise on the Construction of the Astrolabe in the Artisans Manner"); *Risāla fi'l-asṭurlāb al-sarṭānī al-mujannaḥ fī ḥaqīqatihi bi'l-ṭarīq al-ṣinā$^c$i* ("Treatise on the True Winged Crab Astrolabe, According to the Artisan's Method); and *Faṣl min kitāb fi kuriyyat al-samā'* ("A Chapter From a Book on the Sphericity of the Heavens").

### Bibliography

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Julio Samsó