TWO MAPPINGS PROPOSED BY BĪRŪNĪ

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As a young man still in his thirties Abū'l-Rayḥān al-Bīrūnī (974–1048) wrote a treatise describing several methods of representing the surface of a sphere upon a plane. A recent paper ([1] in the references which follow) gives an English translation of this with a commentary, together with a facsimile of the unique manuscript of the Arabic original. Somehow or other the latter appears without any identification whatever. It is in fact pp. 300–314 of Ms Or. 14(15) in the collection of the Bibliotheek der Rijksuniversiteit te Leiden, the authorities of which kindly supplied photographs of the manuscript, together with permission to publish.

This note concerns two of the schemes Bīrūnī describes. The first is known nowadays as the *azimuthal equidistant* projection (cf. [2], p. 155). It is characterized as follows. Let the sphere be placed on a horizontal plane. Then for any great circle which passes through the point of tangency, its map is the line in the plane, tangent to the circle, distances along it being preserved. Bīrūnī puts it felicitously by saying that if it is desired to find on the plane the map of a point on the sphere, mark the latter point with a sticky ink, and then roll the sphere in the direction of the point until the point touches the plane. The mark left will be the required map ([1]), pp. 60–82).

If it is the lower half of the unit sphere which is being mapped, the map will fall inside a circle of diameter π. If, further, the point of tangency of the sphere is the point on the equator having longitude $\lambda = 90°$, then the maps of the (semi-)equator and the meridian through the point of tangency will be a pair of orthogonal diameters.

The second of Bīrūnī's mappings, also of a hemisphere, is likewise

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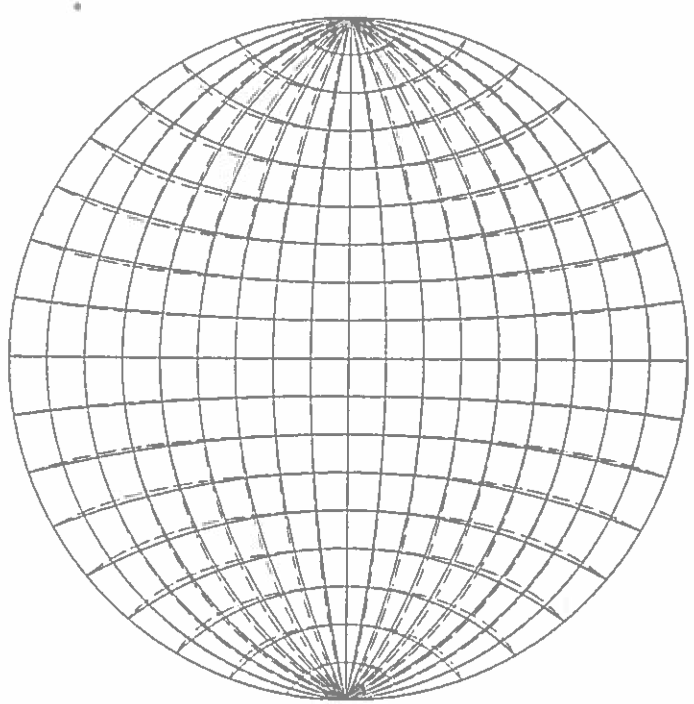

Superposed coordinate nets of a hemisphere
mapped by the azimuthal equidistant (dashed) and global projections.

the interior of a circle, and it exhibits the orthogonal pair of diameters mentioned above. The coordinate net representing the parallels of latitude and the meridians are all circular arcs. They are drawn in the following manner. Graduate each of the four quadrants and the four radii in ninety equal divisions, or such other equal parts as may be convenient. In our figure the interval is ten degrees. Then, for instance, the map of the parallel of 30° north latitude is the circle determined by the three points in the upper half of the map, one on each of the quadrants, and one on the radius to the north pole, corresponding to 30°. To draw the arc representing the meridian for, say, $\lambda = 60°$, pass a circle through the maps of the two poles and through the point on the map of the equator corresponding to 60°. Bīrūnī gives explicit algebraic expressions for locating the centers and for calculating the radii of the two families of circles. This mapping is now known as the *globular* projection ([2], p. 158).

The coordinate nets of both mappings are displayed on the figure, precisely plotted by means of a program fed to a digital computer. The unbroken curves are the circular arcs of the globular projection; those shown dashed are the corresponding curves of the azimuth equidistant projection. Both sets issue from the same points along the periphery of the circle, each curve tangent to its mate, and they pass together, in like manner, through the graduations of the orthogonal diameters. In the central region of the map, where distortion is minimal, the two sets are indistinguishable. Only in the four regions to either side, and above and below the diameters, do the two sets perceptibly diverge, and there not much.

Berggren (in [1]) has conjectured that in inventing his globular map Bīrūnī sought to simplify Ptolemy's second map projection. This may have been the case. But our figure demonstrates indubitably that the easily drawn circles of the globular map constitute a good approximation to the azimuthal projection. We suggest that Bīrūnī was cognizant of this fact when he conceived his second mapping.

References:

1. J. L. Berggren, "Al-Bīrūnī on Plane Maps of the Sphere", *Journal of the History of Arabic Science*, 6 (1982), pp. 47–96.
2. C. H. Deetz and O. S. Adams, *Elements of Map Projection*, 5th ed., Washington: U. S. Coast and Geodetic Survey, 1945 (repr. New York: Greenwood Press, 1969).