The al-Hasam, however, has failed to do so, feeling that the quantity AN is definable more generally than defined above. In all probability, he checked the theorem for (what we might call) both the "base perpendiculars" and thought that the theorem is true for any "apex perpendicular" (the perpendicular drawn on any side from the apex of the opposite angle).

(31). It has already been shown that the proposition is not valid for scalene triangles (see note 28 above); hence Ibn al-Haitham's claim that the proof given by him for this theorem is applicable to all sorts of scalene and equilateral triangles is patently wrong. Ironically, enough the latter part of his claim (regarding equilateral triangles) is however quite correct.

It would however be wrong to conjecture that Ibn al-Haitham might have propounded his theorem for equilateral triangles. The theorem comes immediately after his eighth theorem and the latter is also involved in the present theorem—whereas it follows that this theorem must also be about scalene triangles.

MENSURATION

In the twelfth volume of Ughlar it has been stated that the volume of a cone is one third the volume of a cylinder having the same base and height.

1. Area of the Base of a Cylinder

How can we find the area of the base of a cylinder? Method—Measure the circumference of the base of the cylinder and divide it by 3.14. The quotient will be the diameter of the base. Having found the diameter of the base, we can find the area of the circle by the method given in my book (The Area of a Circle).

2. Volume of a Sphere

In order to find the volume of a sphere we find the area of the greatest circle on it. The area of the greatest circle on a sphere is two times the area of the greater circle of the sphere and whose height is equal to the diameter of a sphere. This fact has been proved by geometers in their works which are readily available and I have proved it in my writings.

3. The Greatest Circle of a Sphere

The method for finding the greatest circle of a sphere is given below.

Open your compasses at will. Fix one of its legs at any point on the sphere and draw a circle on the surface of the sphere. Take off the compasses and keep it away in the open condition. Take any two points on the circle drawn on the sphere. The circumference of the circle will be divided into two areas. By using another compasses bisect the two areas of the circle on the sphere. The two points of bisection of these areas will bisect the circumference of the circle and the imaginary straight line joining these two points will be the diameter of the circle.

Fix one leg of the compasses at one of the points bisecting the circumference of the circle and open the compasses so that other leg of the compasses just reaches the other point of bisection of the circumference. The distance between the two legs will be equal to the diameter of the circle on the sphere.

Place the compasses on a plane surface and join the two extremities with the help of a straight line. Draw the right bisector of this straight line. Now take up the first compasses (used in drawing the circle on the sphere).
and kept open in the same condition) and fixing one leg at one of the extremities of the straight line (Diagon. or radius) of the right bisector at a point. The above mentioned arc cuts the right bisector, because its radius is greater than the radius of the circle drawn on the sphere. Théodorus has proved in his book entitled *The Surface and the Volume of a Sphere*, that any line joining the pole of the circle (the point at which one leg of the compass was fixed at the time of drawing the circle) to any point on the circle, along the surface of the sphere will be greater than the radius of the circle.

Join the point of intersection of the arc and the right bisector to the extremity of the line representing the diameter. Draw a line perpendicular to this line. Produce the right bisector to the opposite direction so as to meet the perpendicular at a point. The intercept of the right bisector made by the perpendicular and the line joining the intersection of the right bisector and the arc to the extremity of the diameter will be equal to the diameter of the greatest circle.

4. Diameter of Sphere

The diameter of a sphere can be calculated as described below.

1. Take half of the diameter (radius) of the circle drawn on the sphere.
2. Find the square of the radius.
3. Calculate the height of the upper portion of the right bisector by Pythagoras’ theorem.
4. Divide the square of the radius by the length of the upper portion of the right bisector.
5. Add the quotient to the length of the upper portion of the right bisector.
6. The sum will be equal to the diameter of the greatest circle and the sphere.

In order to find the area of the greatest circle on the sphere, find the square of the diameter of the sphere and subtract from it 1/7th and 1/14th part of the square of the diameter. The volume of the sphere can be found by multiplying the area of the greatest circle by 2/3 of diameter.

**Explanation:** Take a straight line AB equal in length to the diameter of the circle on the sphere. Bisect AB at C. At C erect a perpendicular CD on AB. With A as the centre and radius equal to the distance between the two legs of the compasses, used for drawing the circle on the sphere, draw an arc cutting CD at D. At A, draw a perpendicular and produce it to cut DC, produced at E. DE is equal to the radius of the sphere.

**Proof:** Let L be the pole at which one leg of the compasses is fixed to draw the circle on the sphere. Let MN be one of the diameters of the circle drawn on the sphere. Bisect MN at O. Join LO. The straight line LO passes through the centres of all the circles which can be drawn on the sphere with L as centre. Triangles IOM and ION will be congruent. Triangle IOM will be right∠ at O. (It may be noted that M is any point on the circumference of the circle and LM is equal to the distance between the two legs of the compasses. This means that all the triangles formed by joining the pole, L, the centre, O, and any point on the circumference will be right∠ at O and will have IOM as a common side. Moreover, all these triangles will be congruent.) So LO will be perpendicular to the line joining the centre O with any point on the circumference of the circle. Therefore, LO, which will be perpendicular to the plane containing the circle, will necessarily pass through the centre of the sphere. This fact has been proved by Théodorus in *The Surface and the Volume of a Sphere*.

Produce LO to meet the sphere at any point P. Join MP. Since LP passes through the centre of the sphere, it will be the diameter of the sphere.

**Theodorus in the work mentioned above also proved that triangle LMP will lie in the semicircle which will have LP, the diameter of the sphere, as its diameter. This means that the centre of this circle will be the same as that of the sphere. So LP will also be the diameter of the greatest circle.**

**Proof:** Let A be the top and C be the foot of the object (hill, pillar, cone or building, etc.); B and D the right eye and the right foot respectively of the observer in the first position; K and L the eye and right foot in the second position; D and E the top and foot of bamboo in the first position and M and N the marks on the bamboo, so that $DZ = MN = 1$ zarah and $ZT = KL$ the height of observers eye from the ground. The points S, H, K, N, M and N are in the same straight line and to L, B, F and C, AC, MF, KL, DE and HT are all horizontal.

In the rectangle MFED, $MD = LE$.

Similarly, in the rectangle KHST, $KH = LT$.

As SASK and MNK are similar,
\[
\frac{AS}{MK} = \frac{MN}{SK} = \frac{NH}{NK}.
\]

Similarly, as $A$ ASH and DHZ are similar,
\[
\frac{AD}{DH} = \frac{DZ}{ZH} = \frac{MN}{ZT}.
\]

The difference between the two positions of the observer from the two positions of bamboo and HT the height of the observers eye from the ground.

7. Method of Finding the area of a triangle:

1. Measure all the side of a triangle.
2. Find the perimeter of the triangle (the sum of the lengths of the sides).
3. Take half of this sum (semi-perimeter).
4. Subtract the length of each side from the semi-perimeter.
5. Find the product of the semi-perimeter and the three remainders of step 4
6. Take the square root of this product.
7. The square root will be the required area.

8. Practical Method of Finding the length of a side of a Triangle

1. Measure the length of the two sides forming the angle.
2. Divide the length of one side by the length of the other side.
3. Produce the two sides opposite to the direction of the arms of the angle containing them. At the ends of these lines, cut off the produced side of the second angle. From the points of cut, draw parallel lines to the original sides of the triangle. The difference between these lines is the length of the side.
4. The three remainders of step 4
5. The length of the first side
6. The length of the second side

6. Measurement for the Surveyor:

Surveyors do not in general make use of the reasoning and proof of whatever has so far been explained. They are concerned only with the practical side of measurement. In the following section some important practical methods of measurements which are of immense value to surveyors are explained. Generally, surveyors deal with plane figures which are either rectangular or connected with circular cylinders, cones and spheres. So, actually they have to deal either with triangles (which can be formed by joining the diagonals of any rectilinear figure) or with circles and its parts. Let us now discuss them in succession.

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should stand in such a way that the foot of the object (whose height is to be found), the foot of the bamboo and his own feet are in a straight line. He should move backward or forward on this very straight line so that his eyes, the top of the bamboo and the top of the object are in a straight line. For this purpose he should close one of his eyes (say left eye) to make sure that his right eye, the top of the bamboo and the top of the object are concurrent. The observer should sit down with his feet in the same position and make a mark on the ground exactly beneath the centre of the right foot. He should measure the distance of this mark from the foot of the bamboo.

Dig out the bamboo from the ground and repeat exactly the same procedure after fixing the bamboo at some other place on the same straight line joining the foot of the object to the first position of the bamboo, nearer to the object than before. The distance between the second position of the bamboo and the right foot of the observer should again be measured. He should also measure the distance between the two positions of the bamboo and the two marks of the right foot. The distance between the right foot and the bamboo in the second position will be less than the distance between them in the first position. It has been explained in this proof given below.

The Formula for Finding the Height:

Formula: Subtract the distance between the foot and the bamboo in the second position from the distance between them in the first position. Divide the distance between the two positions of the right foot by the above difference. Add the height of the observer's eye from the ground to this quotient. The sum will be equal to the height of the object.

5. Altitude of High Objects

We shall now discuss the method of finding the altitude of objects. Such objects are generally cylindrical pillars, walls, buildings, hills etc.

Take a straight bamboo of at least 5 zaruns (units) of length. Make a circular mark round the bamboo at a distance of one zarun from one end of it. Measure the height of the eye of the observer from the ground. Measuring from the first mark, make another circular mark on the bamboo at a distance equal to the height of the observer from the ground. (As 4 zaruns are far greater than the height of a man, some portion of the bamboo may be left over).

Fix the bamboo to the ground so that the second mark on the bamboo just reaches the ground and the bamboo is at a right angle to the ground. The observer

(3) Produce the perpendicular bisector to meet the arc.
(4) Join one end of the arc to the extremity of the perpendicular and produce it backwards.
(5) At the extremity of the chord, draw a line perpendicular to this line.
(6) With the extremity of the centre and any radius (equal to or less than the chord) draw an arc cutting the chord, the perpendicular and the line in step 4.
(7) The ratio between the portion of the arc cut by the three lines in step 6 will be equal to the ratio between the original arc and the circumference of the circle.

14. To Find the Area of Solids Enclosed by Strength Lines:

All solids which are enclosed by straight lines are in some way connected with cones (with circular, triangular or any other rectilinear base).

To find the volume of a cone:

(1) Find the area of the base.
(2) Multiply the area by the height of the cone.
(3) 1/3 of this product will be the volume of the cone.

Now the area of the base can be found either by the formula for finding the area of a triangle or by dividing the base into triangles. (If it is a rectilinear figure having more than three sides) and then finding the areas of these triangles.

15. To Find the Side Opposite to the Angle

The method for finding a side opposite to an angle at the rectilinear base of a cone or any other solid, is given below.

(1) Place two rulers along the sides of the base so that some portion of the ruler remains projected above the corner where an angle is formed.
(2) The angle between the projected portion will be equal to the angle between the two sides of the base of the solid. (This angle can also be found by producing the two sides of the base in the opposite directions).
(3) Construct a triangle whose two sides are equal and the side of the base and the angle between them is equal to the angle between the two rulers.
(4) The diagonal of the base opposite to the above mentioned angle can be found by the method given earlier.

16. Area of the Base of a Right Circular Cylinder

The base of a right circular cylinder is a circle and its area can be found as follows:

(1) Measure the circumference of the base.
(2) Divide the circumference by the base.
(3) Take half of this quotient which will be equal to the radius of the circle.
(4) The area of the base will be equal to radius²/7.

17. Volume of a Right Circular Cylinder

The volume of a right circular cone is equal to 1/3rd of the volume of a right circular cylinder of the same base and the same height. Therefore the volume of the cone is found by:

(1) Finding the area of the base.
(2) Multiplying this area by the height.
(3) Taking 1/3rd of the above product, and
(4) The result will be the required volume of the cone.

19. Area of the Greatest Circle of the Sphere

The method for finding the diameter of the greatest circle has been explained earlier. The area of this circle can be found by squaring its diameter and subtracting from the sphere 1/7th and 1/14th parts of the square. This will come out to be 22/7 r².

20. Volume of a Sphere

(1) Find the area of the greatest circle in it (the diameter of the circle will be same as the diameter of the sphere). It will be 22/7 r², where r is the radius of the sphere.
(2) Multiple the area by 2/3rd of the diameter (or 4/3 r³).
(3) The product, 4/3 x 22 x r³ will be the volume of the sphere.

21. Calculating the Heights

The methods for calculating the altitudes of cones, towers, hills and other bodies have already been explained earlier. The surveyor is not to be bothered about their proof. He is simply to note the steps given in the methods.
In the above figure, the lengths of AB and AC are known. Which the length of BC is to be calculated.

Produce BA to D, making AD = 1 zarakh.

Produce CA to S, making AS = \( \frac{AC}{AB} \).

Now, \( \frac{AS}{AD} = \frac{AC}{AB} \) (AD = 1 zarakh)

or, \( \frac{AS}{AC} = \frac{AD}{AB} \) (by inverting two terms)

In \( \triangle \) ASD and BAC

\( \frac{AS}{AD} = \frac{AC}{AB} \) (proved)

\( \angle \) ASD = \( \angle \) BAC (vertically opposite angles)

\( \therefore \) ASD and BAC are similar,

\( \frac{BC}{SD} = \frac{AB}{AD} \)

or, \( BC = \frac{AB \times SD}{AD} \);

but AD = 1 zarakh.

\( \therefore \) BC = \( AB \times SD \)

9. Area of a Circle

In order to find the area of a circle:

1. Find the length of the diameter of the circle;
2. Find the square of the length of the diameter.
3. From this square subtract its 1/7th and 1/14th parts, and.

10. Area of the Sector of a Circle

The area of the sector of a circle can be found by multiplying the length of the arc by the sector of the radius by the circle.

11. Area of the Segment of a Circle

In order to find out the area of the segment of a circle:

1. Draw the chord joining the two ends of the arc.
2. Find the area of the sector by the above method.
3. Find the area of the triangle formed by the two radii and the chord.
4. From the area of the sector, subtract the area of the triangle.
5. The remainder will be the area of the segment.

12. The Formula for Finding the Height

Subtract the distance between the foot and the bamboo in the second position from their distance in the first position. Divide the distance between the two positions of the right foot by the above difference. Add the height of the observer's eye from the ground to this quotient. The sum will be equal to the height of the object.

13. Ratio between the Length of an Arc and the Circumference

The method of finding the ratio between the length of an arc and the circumference of a circle is as follows:

1. Draw the chord joining the ends of the arc.
2. Draw the perpendicular bisector of this chord.

We have discussed above all about the practical side of Surveying that might prove to be useful to the surveyor.

NOTES

1. According to the modern method of computation, the volume of a sphere = \( \frac{4}{3} \pi r^3 \) where 4 = radius of the sphere. The above formulas by Thabit al-Harithi give the volume of the sphere = \( \frac{4}{3} \pi r^3 \) in diameter \( \frac{4}{3} \pi r^3 \) or \( 8 \pi r^3 \).

2. Theodosius was a Greek geometer who had written a book on the surface and volume of sphere.

3. This formula for finding the area of a triangle was found by a mathematician of Alexandria, Hero.

In modern geometry, Hero's formula is given as \[ \sqrt{s(s-a)(s-b)(s-c)} \] where \( a, b, c \) are the sides of the triangle and \( s = \frac{a+b+c}{2} \).

4. In modern geometry, the above hypothesis can be explained as follows:

Let the diameter of the circle = 2r

Then the square of the diameter = 4r²

1/7th of this square = \( \frac{4}{7} \)

1/14th of this square = \( \frac{1}{7} \)

The area of the circle = \( \pi r^2 \) = \( \frac{14 \times 2}{7} \)

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5. Let O be the centre and AB a chord of the circle.

Let OD be the right bisector of the chord.

In the right angled \( \triangle \) BOC

\[ \text{OB}^2 = \text{OC}^2 + \text{BC}^2 \]

\[ = (OD - DC)^2 + \text{BC}^2 \]

\[ = OD^2 + DC^2 - 2OD \times DC + BC^2 \]

As \( OB = OD \) (radius of the same circle)
So \( \angle ADC = \angle BDC \) or \( \angle BDC = \frac{1}{4} \angle ADB \)

(angle at the centre) \( \triangle AOB = \angle ADB \)

(angle at the remaining part of the circumference),

\( \therefore \angle BDC = \frac{1}{4} \angle AOB \)

In the right angled \( \triangle BDC \), \( \angle BCD + \angle BDC = 1 \) rt. \( \angle \)

Also \( \angle ABK + \angle BDC = \angle DBK = 1 \) rt. \( \angle \)

\( \therefore \angle ABK = \angle BDC; \)

or \( \angle ABK = \frac{1}{4} \angle AOB \)

The ratio between area = the ratio between their \( \triangle S \)
at the centre.

So \( \triangle LK : \triangle KS = \angle ABK : \angle KBS \);

But \( \angle KBS = 1 \) rt. \( \angle \) and \( \angle ABK = \frac{1}{4} \angle ADB; \)

\( \therefore \triangle LK : \triangle KS = \frac{1}{4} \angle AOB = 1 \) rt. \( \angle \). \( \therefore \triangle LK : \triangle KS = \frac{1}{4} \angle AOB = 1 \) rt. \( \angle \)

Because \( \triangle ABK \) subtends \( \angle AOB \) at the centre;

\( \triangle ABK : \angle AOB = \frac{1}{4} \angle AOB = 1 \) rt. \( \angle \)

\( \therefore \triangle ABK : \angle AOB = \triangle LK : \triangle KS \)

A Treatise on Burning Glasses

The second of the several problems, which have been examined by the geometers in respect of which the ancients have long contended, and as a result of it, a variety of ingenious geometrical illustrations have been introduced, and which involve (several) principles of the science of physics, is the preparation of burning glasses by reflecting the rays of the sun. They (geometers and mathematicians) have tackled this problem differently.

They observe that simple plane glasses, (i.e., mirrors) reflect the ray and they further observe that the surfaces of the spherical mirrors do as well reflect the ray but it is reflected in different places according to their position. It is clear to them that the ray which is reflected from a plane glass to a single point is reflected from only one point; and that the ray which is reflected from all the points of one of the circles which happen to exist in that sphere (or globe). Respective proofs are to be found in their works.

A group of the geometers hold that (this can be achieved) if a large number of plane glasses are so jointly arranged that the ray is reflected from each one of them to one single point. Others think (that (this) can be relieved by arranging a large number of spherical mirrors in such a way that the rays shall converge at one single point (called focus), thus making the power of burning stronger. Those who are known to have actually prepared such mirrors are Archimedes, Archimidas (7) and others. Later they devoted themselves to investigating into the special features illustrating how the ray (of light) is reflected. They also examined the properties of the convex surfaces and found that the rays reflected from the various sides of the convex surface of a hollow body converge at one single point (focus). Thus it was known to them that the capacity to burn with this type of mirror is larger than with any other type of mirror, but unfortunately they could not prove it (mathematically), nor could they explain the method of their discovery. In view of the great benefits and general usefulness that this proposition possesses, we feel it necessary to explain it to a student who has love for learning the truth and is possessed of strong determination to utilize (this knowledge). We shall therefore explain it in this treatise and briefly describe the proofs of this truth. We shall also describe the method of preparing these (burning glasses). As an introduction to all this, we shall enumerate the principle which the geometers employ in the various types of these glasses. This will be a source of guidance to the student.

*Trans. from English by Dr. Rana M.N. Bundo Haq, M.A., Ph. D., Punjab Ph. D. (Civil); Head, Department of Arabic, University of the Punjab, Lahore.*
Theorem: The ray of the sun while issuing from the great celestial body and falling on the surface of different types of glasses (and also reflected therefrom) to all other objects, travels in a straight line. The angle of incidence formed by the ray falling on the surface of a plate glass is equal to the angle of reflection formed by the ray issuing from the surface of the glass. Similarly, the angle of incidence formed by the ray falling on the tangent at a point on the surface of a concave glass is equal to the angle of reflection formed by the ray issuing from that point on the surface of the glass. Therefore, I mean to say that the reflected ray enclosing the straight line which equally divides the two straight lines which are (originally) the ray of light.

By the straight lines which fall on the surface of the various types of glasses and which are reflected at the same equal angles either from the surface of the plate glass or from the surface of the tangent forming on the concave glass, I mean the lines which reflect in the form of the reflected ray, because such rays which issue to these lines are again reflected at these lines. By the surface of the tangent on the surface of the concave glass is meant the surface which exists between this line, and the surface of the concave glass (a single point common with both of them). By the term (surface) is meant the reflected line, and by the reflected line is meant the surface in which both of the lines.

A proof of the above is that every conic section has a directrix (or the fixed line). The directrix is cut at the length of the radius, because every line (or ray) parallel to the directrix that enters the conic section and touches the curve is reflected therefrom shall converge into one point.

Example: The curve ABH is the conic section and the line AZ is the directrix thereof (i.e., the fixed straight line). From AZ the line AX is cut equal to the radius. A line TB is drawn parallel to the line DA. The points B and X are joined. Then tangent KH is drawn.

But I say that if $\angle ATB = \angle XBH$, then

$\angle AXH$ is an acute $\angle$.

By way of analysis, let us suppose that $\angle ATB = \angle XBH$. Line TB is a parallel line AZ.

$\angle ATB = \angle ZBH$.

But $\angle BHX = \angle CBX$.

$\angle AXH = \angle DXB$.

hence $AX^2 = DX^2$.

Then we draw BC on the directrix;

$AX^2 = AX^2 + 4(AX \times AX)$.

But AX is $\frac{1}{2}$ of the radius;

$CA \times AX = (CA \times AX)$.

$AX^2 = AX^2 + 4LX^2$.

But $AX^2 = 2AX \times AX$.

hence $AX^2 = 2AX^2$.

Similarly all the lines drawn // to the directrix and converging on the fixed point (=focus) (make equal angles) Q.E.D.

Therefore by the Lemmas in the introduction, we suppose it to be 20.

Let line TB|| line HZ on account of $\angle XBH = \angle TBK$. Hence $\angle AXH = \angle XBH$, $\angle XBH = \angle XH$.

Thus the line XB is like the line XH.

Then we draw BZ; then BZ and AX are like AX and $\angle LAX = \angle LAX$, $\angle AXH = \angle ZAX = \angle AXH$. Suppose $AZ = AM$, then $\angle ZAM = \angle AXH = \angle AXH$.

Since the line TB is // to the line AZ.

$\angle AXH = \angle TBK$, and

$\angle XBH = \angle XH$, and

$\angle ZAX = \angle TBK$.

Q.E.D.

Therefore the line TB is on the conic section.

$\angle AXH = \angle XBH$, $\angle XBH = \angle XH$.

Thus the line XB is like the line XH.

But $AX^2 = AX^2$;

$BX^2 = 4AX^2$. Since the line TB is // to the line AZ.
We have seen that the rays which issue from the celestial body called the sun and fall on the surface of the various types of concave mirrors are parallel to the redirex. They converge at one single point. Now we shall describe how a conic mirror is prepared.

A sheet of steel of the best possible quality should be selected and pieces of the shape (ABCD) (Fig. 8) be cut out. They are to be put together at the line. They will assume the form of a conic figure. This has been explained by one of the geometrical theories, but they have not been precise and rigorous. We should therefore describe how these pieces are to be prepared. The only practical scheme for preparing these round and conic figures with accuracy is the use of compass. This will be of great use and ease, as to how the angles are to be adjusted, and where the right-angled corners are to be fixed, and which side should be at the top and which is the middle and so on. This is how the piece of the conic section is prepared.

But for the fear that we should go into details and treat the subject unnecessarily and add what is not relevant to it, we would have described it at this juncture. But in fact we have mentioned it where it was suitable.

When pieces of the shape ABCD are cut out for the conic cone, we should sharpen the edges and place one upon another. Then we should cut out another set of pieces to form a cone or cover the same. An iron file should be used to make it of the required shape. These pieces are arranged in such a manner as to form the shape of an egg cut into half. If all the pieces are cut in the shape of cones and assembled together to form the shape of a circular ring on one side, then an iron file should be used to hollow out and the conic sections of the cones. When we have finished with this, we should mount this piece on the lathe in such a way that the top cone to the centre (i.e., central axis) even though it is oval (in shape). Then the lathe is allowed to work as in other cases on the lathe. By this process the whole surface of the mirror is rubbed to remove roughness from the surface. The required shape is obtained. It is then polished and used.

This is the dissertation relating to burning glasses having conic hollow shapes. Now as regards the mode of using it as a caustic (i.e., burning) at a required distance, the distance is to be found out from the line of the redirex. Should we require the burning glass to be in oval shape, we shall have to adopt it and make it of steel on the pattern ABCD and we shall draw a straight line BC on one side. The required distance say, CZ can be estimated. We can then cut open pieces of the conic cone with the head tapering like AX to fit reaches at the point C. The redirex is BC; and the perpendicular is four times the distance ZC as we have already mentioned above. We have described how the existence of this point is known by the process of the conic. When a curve AX is described on the sheet, the line ZC is one-fourth of the perpendicular drawn on the redirex. It is evident that the mirror (or burning glass) which is prepared out of the curve AX would reflect all the rays that fall on its surface to one single point (focus). The distance between the focus and the surface of the glass is the length to be known.

And the burning glass which is prepared out of the curve AX, its (point) of causticity is focused at ZC. The distance between the point Z and the surface of the glass is the focal length. If the glass is of oval shape, as mentioned above, its (point) of causticity is at the required distance.

Should we have the burning glass of a round shape, we shall have to prepare the pieces like ABCD and draw a straight line like the line B.H. This will join H. We can use this with the (focal) length we require, so that it becomes the point of the conic. Then pieces are cut to make a burning glass (in the centre whereof would be its redirex, and its perpendicular is four times the line in Fig. 10).

We have also described this in our book on functions of conics. If we prepare a piece of the conic glass of this description like the curve AX we can use the line BC outside the surface of the glass, and also in respect of the other curve XM, the line is outside (the surface of the cone). The redirex thereof will meet at point; we can suppose ZT like the line XH.

Since the curve AX is the conic curve and its redirex is B and the perpendicular is four times the line TD which is equal to line H; therefore ZT is 1/4 of the perpendicular. Hence the mirror which is prepared of any type of piece with curve AXZ would reflect all the rays that would fall on the surface of the mirror to one single point. And since in the curve AX we measure its focal length from the top of the curve like the focal length and line XH the line BZ would be equal to the focal length and line XH and TD = AC. Therefore the point of focus (which is the point of causticity) is T.

The burning glass, which is of the shape as shown in Fig. 10, has the strongest property of burning, because the rays reflect from all the sides to one single point (called focus). This is what we intended to say in this treatise.

**FINIS**

All praise is due to Allah, and may Allah send peace in abundance on Muhammad the Prophet and on his posterity!

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1. The story of the burning of the Roman ships by the reflection of the sun, though very current in later times, is probably a
tongue. See Smith, Classical Dictionary (1855) p. 73.
2. Appollonius: *Conic Sections*. 

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