Handout of the workshop on abjad-numbers

History of Mathematics Research Group, University of Utrecht, Netherlands. October 2022,

1. The principle in the normal alphabet. We give the letters of the alphabet a numerical value as follows:

a=1	**b**=2	**j**=3	**d**=4	**h**=5	**w**=6	**z**=7	**H**=8	**t**=9
i=10	**k**=20	ℓ=30	**m**=40	**n**=50	**s** = 60	**e** = 70	**f** = 80	**c** = 90
q = 100	**r** = 200	**x** = 300						

To write 11 in abjad, we first write the letter for 10 and then the letter for 1, thus: **ia**, but not **ai**; "123" will be **qkj**, not **jkq** or **kjq** etc. There is no letter corresponding to zero, so "202" = **rb**.

Exercise 1: Translate the abjad-numbers: **nw, ℓd, fa, xmh, xit, qH, rc, e.**

Exercise 2: Write in abjad: 24, 258, 307, 130.

2. The Arabic alphabet. The same principle was used for Arabic letters. Note that every Arabic letter has at most four shapes: a shape when it is isolated, and three shapes in the beginning, middle or end of a word. We put
1 = alif, 2 = ba, 3 = jim, 4 = dal, 5 = ha, 6 = waw, 7 = zay,
8 = ḥa (sharp h), 9 = ṭa,
10 = ya, 20 = kaf, 30 = lam, 40 = mim, 50 = nun, 60 = sin, 70= ayn,
80 = fa, 90 = ṣad, 100 = qaf, 200 = ra, 300 = shin, etc.

The system is often called "abjad"; this artificial word is a way to memorize "1, 2, 3, 4 (alif + ba + jim + dal)

1. Table of the numbers 1-91 in *abjad*

		(0)	10	20	30	40	50	60	70	80	90
	name		i= ya	k= kaf	ℓ=lam	m= mim	n=nun	s=sin	e=ayn	f=fa	c= ṣad
(+0)			ى	ك	ل	م	ن	س	ع	ف	ص
+1	a=alif	ا	ىا	كا	لا	ما	نا	سا	عا	فا	صا
+2	b=ba	ب	ىب	كب	لب	مب	نب	سب	عب	فب	
+3	j= jim	ح	ىح	كح	لح	مح	نح	سح	عح	فح	
+4	d=dal	د	ىد	كد	لد	مد	ند	سد	عد	فد	
+5	h=ha	ه	ىه	كه	له	مه	نه	سه	عه	فه	
+6	w=waw	و	ىو	كو	لو	مو	نو	سو	عو	فو	
+7	z=zay	ز	ىز	كز	لز	مز	نز	سز	عز	فز	
+8	H=ḥa	ح	ىح	كح	لح	مح	نح	سح	عح	فح	
+9	t= ṭa	ط	ىط	كط	لط	مط	نط	سط	عط	فط	

Hundreds: qaf=ق =100, ra=ر=200, shin=ش=300. We first write the hundreds, then the teens, then the units. Example: 123=qkj = قكح.

Exercise 3: In the table on this page, locate the shapes of the following letters, when they are isolated:

1 = alif, 2 = ba, 3 = jim, 4 = dal, 5 = ha, 6 = waw, 7 = zay, 8 = ḥa, 9 = ṭa, 10 = ya, 20 = kaf, 30 = lam, 40 = mim, 50 = nun, 60 = sin, 70= ayn, 80 = fa, 90 = ṣad.

Exercise 4: Now look at the numbers "11" and "12" in the table. Try to find the beginning shape of ya and the end shapes of alif and ba. (If you know Arabic, do you find a difference with normal Arabic?)

Exercise 5: Same questions for the other numbers - what are the initial forms of kaf, lam, mim, nun, sin, ayn, fa, ṣad? What are the final shapes of jim, dal, ha, waw, zay, ḥa, ṭa?

Exercise 6: Your first exercise in writing Arabic: As you see, the places for the numbers "92" through "99" are empty in the table. Use your pencil to write these numbers in the table in the abjad system!

Exercise 7: Your first exercise in reading a medieval Arabic manuscript written in Istanbul: Try to read the red letters in the left, middle and right colums in a table for constructing astrolabes by the famous Ottoman astronomer Taqi al-Din ibn Ma^{c}ruf (Table A, written ca. 1577 CE).

Exercise 8: Your first exercise in reading an old Turkish astrolabe, made around 1706 CE. Try to read the abjad-numbers in the circular scale on the outside of the astrolabe (Photo 1). Many numbers are not clear on the photo - the best thing is to start with some numbers which you can read and then to find out the system. Then you can also read the remaining numbers.

Exercise 9: Now try to read as many abjad-numbers as you can on the plate in this astrolabe for cities with geographical latitude 41^o) (perhaps or Istanbul). See Photo 2. What do the numbers mean?

Degrees, minutes, seconds. Islamic astronomers used the abjad-system in their astronomical computations in the sexagesimal system. This is the same system that we still use in our computation of time (minutes, seconds) and angles (degrees, minutes, seconds). In such a system, one only needs the numbers from 1 to 59 for the minutes and seconds, plus a special sign for zero, usually a dot connected to an overbar.

For example, if P and Q are abjad numbers, the combination P Q in Arabic means $Q + \frac{P}{60}$ (note that the Arabic writes from right to left).

Exercise 10: Table A shows the construction of almucantars on the astrolabe. The red numbers are the altitudes of the almucantar. For each altitude, two numbers are indicated towards the elft

the first number is the radius of the almucantar, in two sexagesimal places

the second number is the distance of the centre of the almucantar to the centre of the astrolabe, also in two sexagesimal places.

The equator (the concentric circle in the astrolabe) is assumed to be 60 units. If you want explanations, ask the assistants.

Try to read as many numbers as you can. in Table A.

Note for 90 degrees (bottom left) is the zenith, which is one point. Therefore the radius is zero. There you can see 0 0 in the manuscript.

If there is time left, continue with Table B. (from the same work by Taqi al-Din as Table A, but now for a city on the equator). The numbers in the beginning (upper right) are very interesting!

The Arabic abjad system (table by Dr. Rob van Gent)

Arabic letter	letter form				abjad value		translit.		ArabTEX code
	alone	initial	medial	final	East	NWA	Eng.	Ger.	
'alif	ا	—	—	ـا	1	1	', ā	', ā	`A`
bā'	ب	بـ	ـبـ	ـب	2	2	b	b	`b`
tā'	ت	تـ	ـتـ	ـت	400	400	t	t	`t`
thā'	ث	ثـ	ـثـ	ـث	500	500	th [$\underline{\text{th}}$]	$\underline{\text{t}}$	`_t`
jīm	ج	جـ	ـجـ	ـج	3	3	j [$\underline{\text{dj}}$]	ǧ	`j / ^g`
ḥā'	ح	حـ	ـحـ	ـح	8	8	ḥ	ḥ	`.h`
khā'	خ	خـ	ـخـ	ـخ	600	600	kh [$\underline{\text{kh}}$]	$\underline{\text{h}}$	`x / _h`
dāl	د	—	—	ـد	4	4	d	d	`d`
dhāl	ذ	—	—	ـذ	700	700	dh [$\underline{\text{dh}}$]	$\underline{\text{d}}$	`_d`
rā'	ر	—	—	ـر	200	200	r	r	`r`
zāy	ز	—	—	ـز	7	7	z	z	`z`
sīn	س	سـ	ـسـ	ـس	60	300	s	s	`s`
shīn	ش	شـ	ـشـ	ـش	300	1000	sh [$\underline{\text{sh}}$]	š	`^s`
ṣād	ص	صـ	ـصـ	ـص	90	60	ṣ	ṣ	`.s`
ḍād	ض	ضـ	ـضـ	ـض	800	90	ḍ	ḍ	`.d`
ṭā'	ط	طـ	ـطـ	ـط	9	9	ṭ	ṭ	`.t`
ẓā'	ظ	ظـ	ـظـ	ـظ	900	800	ẓ	ẓ	`.z`
'ayn	ع	عـ	ـعـ	ـع	70	70	'	'	'
ghayn	غ	غـ	ـغـ	ـغ	1000	900	gh [$\underline{\text{gh}}$]	ġ	`.g`
fā'	ف	فـ	ـفـ	ـف	80	80	f	f	`f`
qāf	ق	قـ	ـقـ	ـق	100	100	q [$\underline{\text{k}}$]	q	`q`
kāf	ك	كـ	ـكـ	ـك	20	20	k	k	`k`
lām	ل	لـ	ـلـ	ـل	30	30	l	l	`l`
mīm	م	مـ	ـمـ	ـم	40	40	m	m	`m`
nūn	ن	نـ	ـنـ	ـن	50	50	n	n	`n`
hā'	ه	هـ	ـهـ	ـه	5	5	h	h	`h`
wāw	و	—	—	ـو	6	6	w, ū	w, ū	`w / U`
yā'	ي	يـ	ـيـ	ـي	10	10	y, ī	y, ī	`y / I`