The Istanbul manuscript Fitzih 34 contains an Arabic treatise on mensuration by the qâ‘i (judge) Abû Bâkr. This treatise has been listed in Krause 1 and Sezgin 2, but it has not been investigated until the present. The correct title of the treatise, Book of (Li’il in the Explanation of the Measurements (Kitâb al-tabâqat fi sharh al-misâhât)), has not been generally known, because it appears only in the middle of the preface. We know nothing about the identity of the author other than that he wrote the treatise before the end of the twelfth century A.D. when the extant manuscript was copied 3.

In 1968, E. Busard 4 published a Latin translation of the Arabic text, called the Liber in rei/ionum, also written by Abû Bâkr. Sezgin 1 raised the question whether there is any connection between the two texts. We will answer this question in this paper.

Section 2 of this paper is a survey of the mathematical part of the Book of (Li’il in the Explanation of the Measurements. Section 3 compares this text with the Liber in rei/ionum. In Section 4, the authors are compared. In Section 5, the two texts are analyzed, and a complete edition of the Book of (Li’il in the Explanation of the Measurements may not be far away.

In the preface of the treatise, a complete edition of the Book of (Li’il in the Explanation of the Measurements may not be far away.
Iconclude this introduction by a summary of the Preface of the Book of the Levels in Explanation of the Measurements. The text begins as follows: "Qai (judge) Abul Bakr said:" The author then says that some "brethren" (ikhwãn) suggested discussion of the measurement of figures, together with examples, in which the fundamental figures are treated systematically and in a simple and correct wording. In response to this request he wrote the present work, and he took care to make it concise, so that it is easily mastered. The work contains not only a large number of technical terms, but also "finer tricks and subtle curiosities," which will lead the student to other knowledge, which distinguishes a human being from an animal. The book has various uses, and the intelligent student will quickly master its contents. The science of measurement is an obscure science, because it is a proved science and its assertions are correct and verified. It is useful in connection with financial and legal matters, such as inheritances and testaments (several examples are mentioned). The student should master a section completely before proceeding to the next, because these sections are logically dependent.

The author then says, "I called it the Book of the Levels in Explanation of the Measurements," and then gives a table of contents which I do not render here because translations of all titles can be found in the next section of this paper. The Arabic text of the table of contents is to be found in Section 5 of this paper.

He then says that "each section contains the necessary definitions, proofs, synthesis, analysis and reasons," and adds that the book is based on writings of the scholars and tricks of the experts in the field, and in connection with the book of Euclid's "Book of Elements," which is known as "Euclid" (Kitãbal-Ustuqiát al-rûfbi-Uqlidis) in this connection. This concludes my summary of the Preface of the Book of the Levels in Explanation of the Measurements.
The solid is the convexity of a surface, and it is the most complete magnitude. The solid is the convexity of a surface, and it is the most complete magnitude.

These second level, including the circumstances of triangles, their properties, the derivation of the lines from each other, and the discussion of their measurements.

These first section of the knowledge of the kinds of figures and angles. The text mentions rectilinear and non-rectilinear figures; triangles, quadrilaterals, circular figures and figures consisting of arcs; equilateral, isosceles and scalene triangles, right-angled, obtuse-angled and acute-angled triangles. The sum of any two sides of a triangle is greater than the third side. Only one angle of a triangle can be right or obtuse. The next section on the derivation of the altitude.

These second section on giving examples of the three angles. The author presents figures of acute, right and obtuse angles.

These third section, on the knowledge of the right angle in another way. If in triangle $ABO$, $AB^2 + BG^2 = AG^2$, $B$ is a right angle. Example: $AB = 6$, $BO = 8$, $AG = 10$.

These fourth section, on the knowledge of shapes in the two right-angled triangles. The two kinds are the scalene right-angled triangle (such as $ABU$) and the isosceles right-angled triangle (such as $EI$) $EL = 15$, $KL = 13$ and $LI = 14$ as a kind of a right angle in another way. $AI$ in rectangle $ABVI$. $AV + BI = 16$ right angle with the knowledge of the right-angled triangle.

These fifth section, on the areas of both right-angled triangles. The area is half the product of the two sides containing the right angle. The author remarks that the scalene right-angled triangle is half a rectangle, and that the isosceles right-angled triangle is half of a square.

These sixth section on the knowledge of the circumstances of acute-angled triangles. The author distinguishes between equilateral, isosceles and scalene triangles. The area is half the product of the base and the altitude, as referred to the next section on the computation of the altitude.

These seventh section on the derivation of the altitude in the isosceles and scalene triangles, and the discussion of their areas. If $AI$ is the altitude in triangle $ABG$ and $AB = AD = 10$, $AD = 7$. The author remarks that the quantity is irrational so the answer is 8 someone how much is too much. It is said to him: "Your horse is lacking (by something how much is too much)."
The first section, on the knowledge of the square, and the discussion of its area. The author divides the quadrilaterals into (a) parallelograms and (b) other quadrilaterals. The category (a) is further divided into four kinds, to be discussed in 1.1, 1.2, 1.3, and 1.4. The square is equilateral and reflexangular, and its area is the product of two sides. He describes the example of a square with sides 10 and diagonal root 200. For "square" he uses the term murabba' rnuflaq, literally absolute quadrilateral. Apparently he avoided confusion that might arise because his term for quadrilateral, murabba', is often used for square. He also explains that parallel lines are lines which, if extended indefinitely, do not meet.

The second section, on the knowledge of the rectangle, and the discussion of its area. The rectangle is a quadrilateral with right angles and unequal sides. He discusses a rectangle with sides 6 and 8, and shows that the area is 48 and the diagonal is 10.

The third section, on the knowledge of the rhombus and its area. A rhombus is a parallelogram with equal sides and unequal angles. The area is half the product of the two diagonals. In order to find the area, it is necessary to know the side and at least one diagonal. The other diagonal can be found by means of the theorem of Pythagoras. This is illustrated by means of a rhombus with sides 1 and 2, and diagonals 1 and 2, respectively.

In the end of this section, the author describes a discovery of his own, stated in a rather unclear way:

"We inform you here of a subtlety that we have not found written in any place, and that we have not heard. It is as follows. If we imagine the square to be stretched out along the two opposite sides, so that it is transformed into the rectangle and the square is in the same figure. If we imagine it as stretched out as far as possible, its four sides coincide and the two remaining angles that were not stretched also coincide. The verification of this is as follows: If we return it in the imagination to the quadrilateral with right angles and the quadrilateral with equal sides, it is possible."

The fourth section, on the knowledge of the rhomboid and the discussion of its area. A rhomboid is a parallelogram which is neither equilateral nor equiangular, but its opposite sides and angles are equal. The author says that this is impossible. The conclusion and the discussion of the area. The rhomboid is a part of the quadrilateral with right angles. If we consider the quadrilateral of the same kind, we cannot transform it into the quadrilateral with right angles. If we transform it into this, then it is stretched, and its area is the product of two sides. If it is stretched, then the problem of finding the area of the rectangle cannot be solved. Therefore, it is impossible to transform it into the rhomboid.
The fourth section on the knowledge of the sphere, the author distinguishes four kinds of solids: the regular, the spherical, the cylindrical, and the quadrilateral. Among these, the author says that the sphere is the most perfect and the most beautiful, and that the quadrilateral is the least perfect and the least beautiful. The author then discusses the properties of the sphere and the quadrilateral, and he establishes the following theorems:

I. The area of a circle is 
\[ A = \pi r^2 \]

II. The area of a sphere is 
\[ A = 4\pi r^2 \]

III. The area of a quadrilateral is 
\[ A = \frac{1}{2} \times \text{base} \times \text{height} \]

The author then applies these results to a regular hexagon, showing that the area of a regular hexagon is 
\[ A = 6 \times \frac{1}{2} \times s \times h = 3s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular heptagon, showing that the area of a regular heptagon is 
\[ A = \frac{1}{2} \times 14s \times r = 7s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular octagon, showing that the area of a regular octagon is 
\[ A = \frac{1}{2} \times 16s \times r = 8s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular pentagon, showing that the area of a regular pentagon is 
\[ A = \frac{1}{2} \times 10s \times r = 5s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular hexagon, showing that the area of a regular hexagon is 
\[ A = \frac{1}{2} \times 6s \times r = 3s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular heptagon, showing that the area of a regular heptagon is 
\[ A = \frac{1}{2} \times 7s \times r = 7s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular octagon, showing that the area of a regular octagon is 
\[ A = \frac{1}{2} \times 8s \times r = 8s^2 \]

The author then discusses the properties of the regular polygons, showing that the area of a regular polygon is 
\[ A = \frac{1}{2} \times p \times r \]

where 
\[ p = ns \]
\[ r = \frac{1}{2} \times \text{circumradius} \]

The author then applies these results to a regular pentagon, showing that the area of a regular pentagon is 
\[ A = \frac{1}{2} \times 5s \times r = 5s^2 \]
The author distinguishes between the full (mumat) sphere and the hollow (mukhawwaf) sphere, that is the solid contained between two concentric spheres. He points out that there can be two kinds of measurement, namely the surface area and the volume. The surface area of a sphere with diameter $d$ is $\frac{4}{3}\pi$ units squared, and its volume is $\frac{4}{3}\pi$ units cubed. These formulas are based on $\pi = 3.14159265358979323846$. The second section on the knowledge of the surface area of a sphere, whose diameter is 10, is the topic at hand. He correctly says that the circumference is 60, but the volume is incorrect. The volume of a cylinder is the product of the circumference of its base and its height. He discusses a cylinder with height 10 and with a circular base of diameter 3. He causes the volume of a cylinder defined in a is the product of the circumference of the circular base and its diameter, is also the surface area of a cylinder with the base is a circle, but the discussion of the concept solid of which the base is a circle, but the calculations are correct, but the second rule is correct, but the correct formula is $V = \frac{4}{3}\pi r^3$ where $r$ is the radius of the sphere, and the volume of the sphere is $\frac{4}{3}\pi r^3$ for a sphere with radius $r$. The second section on the knowledge of the solid cylindrical figures and the description of the cylinder of which the bases are two equal circles, and the rotation of a pyramid. He mentions three kinds of solid cylindrical figures (ashkâlmujassamaustuwânulivva): a. the round solid cylinder, with two equal circular bases, and produced by the rotation of a rectangle around one of its sides. The example is the height of a cylinder with two equal circular bases, and the circumference of the base is a circle, but the calculations are correct, but the second rule is correct. b. the conical solid figure, which is the figure contained by a plane and one of its sides, which is the triangle in the triangle of the pyramid. c. the prismatic (munshī, "sawed-off") figure, contained by five planes, namely two parallel triangles and three rectangles. He says that the volume of an ashkāl is the product of the circumference of the circular base and its diameter, is also the surface area of a pyramid. The example is the height of a pyramid, which is the height of a pyramid. The second section on the knowledge of the solid cylindrical figures of which the two bases are equal circles, one of which is the circle of the volume is computed as $\frac{4}{3}\pi r^3$ for a sphere with radius $r$. The third section on the knowledge of the solid cylindrical figure, with square base and no more than 10 sides of the square base, and having the base is a square, and the dimensions of the solid cylindrical figure is also the surface area of the solid cylindrical figure, which is the product of the circumference of the circular base and its diameter, is also the surface area of a cylinder with the base is a circle, but the calculations are correct, but the second rule is correct. The fourth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The fifth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The sixth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The seventh section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The eighth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The ninth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The tenth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The eleventh section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid. The twelfth section is the knowledge of the solid cylindrical figures of which the base is a plane, and the volume is the product of the circumference of the base and its height. The example is the example of the pyramid.
The eighth section: the knowledge of the prismatical (manshrūr, sawed-off) solid. He says that a prism is always half of a square cylinder. Thus, he only considers right triangular prisms of which the base is a right-angled triangle. Let then introduce a prism with height 20 and base half of a square with side 10. Let the prism be tested that the volume of this solid is $\frac{1}{2}(120^2) = 1000$. This is the end of the Book of Liviu's intuition on the measurement of figures.

In this section, I will refer to problems in the Liber as well as in the Liber de mensurandis.

With the Liber de mensurandis

The Book of Liviu's intuition on the measurement of figures.

3. Comparison of the...
The Lilanc Man'swere written by a mathematician who was interested in algebra and theoretical problems. On the other hand, the Book of Levi displays no sign of such interest, and it contains mistakes, not only in the computation but also of a more fundamental nature (in 11:7 and IV:6). The author's own discovery (announced in 11:3) that a rhombus can be obtained by stretching a square is rather trivial, and would not have been of interest to the mathematicians of his time. From the preface we can conclude that most of the other mathematicians in the Book of Levi is derivative. The increase of its author's interest in philosophy and interminological matters, as can be seen in the preface and the first section, which is translated in Section 4 of this paper. These considerations suggest to me that the Abilakr who wrote the Lilanc Man was probably not the same person as the judge Abi Bakr who authored the Book of Levi.

We have no further light to the identity of these two figures. The Abilakr who wrote the Lilanc Man must have lived in the middle of the twelfth century, when Al-Din of Al-Karim translated his work into Latin. The book immediately preceding the text of the Book of Levi was written in the first decade of the year 587 of the Hijra (Jan. 30, 1191) in the city of Mosul, in Iraq, so that the judge Abi Bakr must have lived in the 12th century A.D. or before. The judge Abi Bakr refers in his preface to Euclid's Elements, but he must have used other sources, which I have not been able to identify. Perhaps a reader will be able to identify the source of his first section, on the philosophical discussion of a point (see the next section of this paper).
Acknowledgements

The Hermann-Wil1komrn Stiftung, Frankfurtam Main financially supported my visit to Istanbul in June 1986. Dr. Muammer Ulker, director of the Suleymaniye Library in Istanbul, gave permission to consult Arabic manuscripts, and the staff members of the Suleymaniye Library were very kind and helpful. Prof. F. 8ezgin, Frankfurtam Main, showed me microfilm of Arabic manuscripts, and Prof. D.R. King, Frankfurtam Main, gave helpful advice.

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