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The qibla table in the Ashrafī Zīj

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Die ungelösten Probleme halten den Geist lebendig und nicht die gelösten.
(Erwin Guido Kolbenheyer)

Introduction

In 1975 D. A. King drew attention to an unpublished table for finding the qibla, that is the direction of Mecca, in the Ashrafī Zīj. This work is a Persian astronomical handbook with tables that was compiled around 1305 by a certain Muḥammad ibn Abī ᶜAbdallāh Sanjār al-Kamālī (see Matvievskaya and Rozenfeld vol. 2, p. 440, no. 396a) and it survives in a unique manuscript in the Bibliothèque Nationale in Paris (pers. supp. 1488). The work was identified by Kennedy (1956, no. 4, p. 2), and further arguments confirming the identification can be found in a later study by the same author (1977).

The qibla table contains one entry for every degree of longitude difference and every degree of latitude difference between 1° and 30°, so there are 30 x 30 = 900 entries. To find the qibla for a locality with given geographical coordinates, one first computes the longitude difference $\Delta\lambda$ and the latitude difference $\Delta\phi$ from Mecca, and one then enters the table with $\Delta\lambda$ (horizontally) and $\Delta\phi$ (vertically). The corresponding entry $f(\Delta\lambda, \Delta\phi)$ in the table is the angle in degrees and minutes between the qibla and the southernmost point on the horizon. The table can only be used for localities north of Mecca.

The computation of the tables is not explained in the text of the Ashrafī Zīj. King (1986, p. 138) published selected values of the table and he stated that "the Ashrafī qibla table clearly contains numerous copyists' mistakes and appears to be based on an approximate formula which I have been as yet unable to determine." It is a pleasure to present here an edited version of the whole table and a partial explanation of its mathematical structure. It turns out that a subset of the table (including all values for arguments that are integer multiples of 3°) was computed by an exact method, but in a careless way. Most other entries for $\Delta\phi \geq 3°$ were found by linear interpolation. I have not been able to explain the structure of the columns for $\Delta\phi = 1°$ and 2°.

From the ninth century onwards, the Islamic mathematicians were able to compute the direction of Mecca in a mathematically exact way (see King, 1986). The computation is very complicated and the tables that have come down from that period are based on simple and not-so-simple approximate methods. Thus far, the oldest known qibla tables based on an exact method were the astonishingly accurate tables of the Syrian astronomer al-Khalīlī, who lived around 1365 (the tables were

published in King, 1975). Elsewhere King (1986, pp. 133-136) mentions earlier tables by Ibn al-Haytham (ca. 1000) and al-Khāzinī (12th century), but these have not survived. Because the Ashrafī Zīj was compiled around 1305, the qibla table in it is older than the tables of al-Khalīlī. We shall see below that the author made many random computational errors, so his qibla table is considerably less accurate than the tables of al-Khalīlī.

The author of the Ashrafī Zīj uses a latitude of Mecca $\phi_M = 21^\circ 40'$ in his text (fol. 104a-b), but we will see below that the qibla table was computed on the basis of a different value, namely $\phi_M = 21^\circ$. It is therefore likely that the author of the Ashrafī Zīj copied the qibla table from an earlier source. Kennedy and Kennedy (1987, p. 226) list more than 15 sources in which $\phi_M = 21^\circ$ occurs. Many of these sources are related to al-Khwārizmī (9th century) and the astronomers of Caliph al-Maᶜmūn, but $\phi_M = 21^\circ$ was also used by a few later astronomers such as Ibn Yūnus (ca. 1000) and Muḥyī al-Dīn al-Maghribī (13th century). Hence it is impossible to date of the qibla table on the basis of the latitude of Mecca that was used.

Cracking the table

Plates 1 and 2 display the qibla table of the Ashrafī Zīj table in the unique Paris manuscript, (fols. 130v-131r).

The entries $f(\Delta\lambda, \Delta\phi)$ in the table are given in degrees and minutes of arc, in the Arabic abjad numerical notation. This notation resembles the sexagesimal notation in Greek astronomical works such as the Almagest of Ptolemy. Here we only describe the system for the numbers 1 to 89. The numbers 1.10, 20, 30, 40, 50, 60, 70 and 80 are denoted by letters of the Arabic alphabet: alif (1), bā' (2), jīm (3), dāl (4), hā' (5), wāw (6), ~~zayn~~ (7), ḥā' (8), ṭā' (9), yā' (10), kāf (20), lām (30), mīm (40), nūn (50), sīn (60), ᶜayn (70), ~~ṣād~~ (80). The other numbers are written as combinations of these letters: for example 11: yā'-alif (10+1), etc.. The number combinations are written as two-letter words with the usual ligatures between the Arabic letters. The diacritical marks of yā' and zayn are almost never written in sexagesimal abjad-numbers.

Many numbers are very easily confused in the abjad-system, so trivial scribal errors (which change the numerical value) are frequent. For example, the initial forms of yā' (10) and nūn (50) differ only by a dot, and therefore 11 (yā'-alif) is very easily confused with 51 (nūn-alif), etc. Similary, initial yā' (10) resembles initial lām (30), hence 12 can be confused with 32, etc. etc. (confusion between 11 and 31 is less likely because of the distinctive shape of the ligature lām-alif (31)). Almost all numbers in the qibla table are written unambigiously, that is to say the scribe knew what number he wanted to write. Because the abjad-numbers are so easily confused, we can almost never exclude the possibility that a given entry is a scribal error. Thus "cracking" the table really means: detecting the scribal errors in the manuscript.

Fortunately the table has a mathematical property that makes it possible to identify many scribal errors. It turns out that for $\Delta\phi \geq 3^\circ$, the entries f($\Delta\lambda$, 3n+1) and f($\Delta\lambda$, 3n+2) were found by careless linear interpolation between f($\Delta\lambda$, 3n) and f($\Delta\lambda$, 3n+3), for $1 \leq n \leq 9$ (except for $3^\circ \leq \Delta\phi \leq 6^\circ$, $\Delta\lambda = 3^\circ, 6^\circ, 9^\circ$). Linear interpolation was also used in columns in the same way, at least for $\Delta\phi \geq 15^\circ$, and also for $3^\circ \leq \Delta\phi \leq 15^\circ$, $\Delta\lambda \geq 15^\circ$. The author first interpolated in the columns and then in the rows. On the basis of these insights many trivial scribal errors can be corrected and a large part of the original table can be reconstructed. We can even identify some computational errors made by the author in the process of linear interpolation.

Tables 1-5 are the result of my attempts to reconstruct the original form of the qibla table in the Ashrafī Zīj. I have used the semicolon to separate the integer from the fractional part, thus 42;35 means 42°35′. All scribal errors are indicated by an asterisk; the restored values are printed in Tables 1-5, and the numbers in the manuscript are in the apparatus to Tables 1-5 in the appendix to this paper. The scribal errors are trivial, except in cases indicated by a question mark. In the apparatus, a duplex such as [3, 14] means the entry in the third column and the 14th row (i.e. $\Delta\phi = 3^\circ$, $\Delta\lambda = 14^\circ$). I have indicated the mathematical structure of the table as follows. Entries in boldface were computed independently (we will show below that they were computed by an exact method). Entries in normal print were obtained by linear interpolation (horizontal or vertical) between the nearest entries in boldface in the same row or column. It is important to note that the entries in normal print and in boldface could be checked or corrected on the basis of the mathematical structure of the table; hence we can be reasonably certain that these entries are free from scribal errors. In the cases where I could not check or corrected an entry in this way, I have printed the entry as it stands in the manuscript using italics. These unchecked and uncorrected entries include the entire columns for $\Delta\phi = 1^\circ$ and $\Delta\phi = 2^\circ$ and the entries for $\Delta\lambda = 3^\circ, 6^\circ, 9^\circ$ with $3^\circ \leq \Delta\phi \leq 5^\circ$.

Entries followed by the letter c (for computational error) are the result of an easily identifyable (computational) error made by the author of the table. Most of these computational errors occured when the author started to work on linear interpolation and misread a number which he had computed himself. The number that caused the errors is indicated by the symbol &, and further information can be found in the apparatus for the number indicated by &.

Entries followed by the letter d (for displaced) are found in an incorrect position in the manuscript. I have rendered these numbers in their correct position. Further explanations can be found in the apparatus. A question mark without a number means that we cannot reconstruct the entry in the original table because the scribe wrote an entry that belonged in a different position (see the apparatus).

Further analysis of the table

From now on we shall only be concerned with the entries in Tables 1-5 which were computed directly, that is to say, not by linear interpolation. These entries

were computed by an exact method. This can be seen, for example, by comparing the diagonal entries f(3n, 3n) for integer n, with the qibla tables published by King (1986, pp. 93, 108-9, 120-121), which were computed by means of the correct method and the different approximation methods used in the medieval Islamic world. These tables depend on the value ϕ_M of the latitude of Mecca that was used, but all historically attested values are between 21° and 22°, and the effect of variation of ϕ_M on individual entries in the qibla tables is only a few minutes of arc.

An exact method for determining the qibla is a method which is mathematically equivalent to the formula

$$\tan q = \sin \Delta\lambda / (\sin \phi \cos \Delta\lambda - \cos \phi \tan \phi_M).$$

Here ϕ is the latitude of the locality, ϕ_M is the latitude of Mecca, and $q=f(\Delta\lambda, \Delta\phi)$ is the angle between the qibla and the southernmost point on the horizon.

In the case of the qibla table in the Ashrafī Zīj, the underlying value of ϕ_M can be determined without much difficulty. To investigate which value was used, I have recomputed tables for the historically attested values $\phi_M=21°$, 21°15′, 21°20′, 21°30′ and 21°40′ (Kennedy and Kennedy, p. 226) and compared them with the entries f(3m, 3n) in the table for integer n, m. The results can be summarized as follows. For $n \leq 5$ (i.e. $\Delta\phi \leq 15°$), the f(3m, 3n) differ considerably and in a random way from recomputed values for all plausible ϕ_M. For $n \geq 6$ ($\Delta\phi \geq 18°$) the agreement is much better. I have therefore compared the set S of 50 entries f(3m, 3n) for $6 \leq n \leq 10$, $1 \leq m \leq 10$ with recomputed values. If the author made rounding errors but no other computational errors, we can assume that the difference between an entry in the table and an exactly recomputed value is at most 3 minutes of arc. This assumption is motivated by the fact that the errors in approximately 95% of the entries in al-Khalīlī's qibla table are at most 3 minutes of arc. Let $h(\phi_M)$ be the number of entries in S which differ at most 3 minutes of arc from recomputed values for the value ϕ_M. Then h(21°)=34, h(21°15′)=36, h(21°20′)=37, h(21°30′)=21, h(21°40′)=10. Therefore the values 21°30′ and 21°40′ can be excluded.

I then discarded the 12 entries in S which differ by more than 3 minutes from all recomputed values for $\phi_M=21°$, 21°15′ and 21°20′, and I considered the errors in the remaining 38 entries (error = entry in the table minus recomputed value). The sum of these errrors is +67′ for $\phi_M=21°20′$, +43′ for $\phi_M=21°15′$ and -26′ for $\phi_M=21°$. In the case of $\phi_M=21°20′$, almost all errors are positive, whereas for $\phi_M=21°$ posisitve and negative errors occur (see tables 6 and 7). Thus the agreement for $\phi_M=21°$ is better than for $\phi_M=21°15°$ and 21°20′. There are no historically attested values of ϕ_M between 21° and 21°15′ (see Kennedy and Kennedy, 1987, p. 226), and we further note that $\phi_M=21°$ occurs much more frequently than $\phi_M=21°15′$, which the Kennedys found in only one source, namely an anonymous Egyptian work on timekeeping. We conclude that our qibla table was computed for $\phi_M=21°$.

In tables 6 and 7, the directly computed entries (that is, the boldface entries in Tables 1-5) have been collected and compared with recomputed values for

$\phi_M = 21^\circ$. The errors in minutes of arc (entries in the table minus recomputed values) are displayed in small print. Most errors in Al-Khalīlī's qibla tables are 1 or 2 minutes of arc at most, so the qibla table in the Ashrafī Zīj is far less accurate.

The columns for $\Delta\phi = 1^\circ$ and 2° present a problem, because the values seem to differ both dramatically and systematically from recomputed values. I have not found a method to detect scribal errors in these columns, and I must therefore leave this problem to the reader (see the quote at the beginning of this paper).

Bibliography

KENNEDY, E. S.: A Survey of Islamic Astronomical Tables. Transactions of the American Philosophical Society N.S., 46 1956, 123-177

KENNEDY, E. S.: The Astronomical Tables of Ibn al-Aʿlam. Journal for the History of Arabic Science 1 1977, 13-23

KENNEDY M. H. and KENNEDY E. S.: Geographical coordinates of localities from Islamic sources, Frankfurt a. M.: Institut für Geschichte der Arabisch-Islamischen Wissenschaften 1987

KING, D. A.: al-Khalīlī's Qibla table. Journal of Near Eastern Studies 34, 81-122, 1975. Reprinted in idem: Islamic Mathematical Astronomy, London: Variorum 1986, 2nd ed. 1993

KING, D. A.: The Earliest Islamic Mathematical Methods and Tables for Finding the Direction of Mecca, Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften 3 (1986), 82-149. Reprinted in idem, Astronomy in the Service of Islam, Aldershot: Variorum 1993

MATVIEVSKAYA, G. P., ROZENFELD, B. A.: Mathematiki i astronomi musulmanskogo srednevekovya i ikh trudy (VIII-XVIII vv.), 3 vols., Moscow: Nauka 1983

$\Delta\phi$	1	2	3	4	5	6
$\Delta\lambda$						
1	*42;58**	*23;12*	**16;07***	14;04*	12;01	**9;59**
2	*61;46*	*43;11*	**30;12**	25;52*	21;32	**17;12***
3	*70;52*	*61;12*	*42;35*	*35;12*	*29;17*	**25;17**
4	*70;14*	*65;20*	**53;12***	45;57*	38;43	**31;23**
5	*77;04*	*67;36*	**59;12***	52;00	44;45*	**37;32***
6	*78;11*	*69;32*	*63;14*	*55;56*	*48;34*	**43;20**
7	*78;57*	*71;12*	**66;24***	60;18	54;15*	**48;12**
8	*79;43*	*73;12*	**69;31**	63;46*	58;13*	**52;39**
9	*80;19*	*75;32*	*72;22*	*65;17*	*60;02*	**55;56***
10	*80;59*	*78;05*	**75;02**	69;42*	64;22	**59;02**
11	*81;49*	*79;38*	**76;59**	71;44	66;29	**61;14**
12	*82;46*	*80;11*	**77;01***	72;31	68;03	**63;30***
13	*83;39*	*81;10*	**78;22**	74;31	70;40	**66;50***
14	*84;19*	*82;15*	**79;24**	75;47?*	72;13*	**68;58***
15	*85;19*	*84;39*	**80;43**	76;23*	73;02*	**69;40**
16	*88;18*	*85;39*	80;43d	77;31	74;13*	71;06*
17	*88;56*	*86;40*	82;02cd	78;52*	75;41	72;30
18	*89;18*	*87;09*	**83;24d**	80;10	76;58*	**73;46**
19	*89;16*	*87;36*	84;03d	81;01	77;59	74;57*
20	*89;30*	*88;05*	84;43d	81;51?*	78;59	76;04
21	*89;54*	*88;31*	**85;24**	82;42	80;00?*	**77;18**
22	*88;55*	*88;56*	85;56	83;16	80;36	77;56*
23	*88;37*	*89;18*	86;27	83;50*	81;12	78;34
24	*87;57*	*89;35*	**86;59**	84;23	81;46	**79;10**
25	*87;45*	*89;52*	87;34	85;04	82;38*	80;13
26	*87;34*	*89;52*	88;10	85;52	83;34*	81;16*
27	*86;54*	*89;55*	**88;46**	86;38*	84;18*	**82;19**
28	*86;31*	*88;53*	89;06	87;10*	85;14?*	83;17
29	*85;54*	*88;50*	89;27	87;40	85;59*	84;16
30	*85;10*	*88;42*	**89;47**	88;16*	86;45*	**85;14**

Table 1: Partially reconstructed qibla table for $1^\circ \leq \Delta\phi \leq 6^\circ$.

$\Delta\phi$	6	7	8	9	10	11	12
$\Delta\lambda$							
1	**9;59**	8;47	7;35	**6;23**	5;54	5;20	**4;49**
2	**17;12***	15;12	13;12	**11;14***	10;28	9;45	**8;59***
3	**25;17**	22;37*	19;57*	**17;18***	15;56	14;34	**13;11**
4	**31;23**	28;35	25;47*	**23;02**	21;09	19;17*	**17;23**
5	**37;32***	34;34	31;36*	**28;39***	26;21*	24;03*	**21;45***
6	**43;20**	40;10	37;00*	**33;50**	31;05	28;21	**25;38**
7	**48;12**	44;36*	40;?*	**37;23**	34;35	31;47*	**29;00**
8	**52;39**	48;45	44;51*	**40;57***	38;18c*	35;39c	**32;39&**
9	**55;56***	52;07*	48;18*	**44;31**	41;40	38;49	**35;57***
10	**59;02**	55;12	51;22	**47;32***	44;38*	41;49	**38;59**
11	**61;14**	57;42*	54;11*	**50;42**	47;47	44;52	**41;57***
12	**63;30***	60;09	56;46	**53;23**	50;28	47;33*	**44;38**
13	**66;50***	63;04	59;18	**55;30**	52;51*	50;10	**47;30?***
14	**66;58***	65;16	61;38	**57;59**	55;13*	52;27	**49;40***
15	**69;40**	67;24	63;06	**59;59?***	56;15	54;29	**51;47**
16	71;06*	67;57	64;48*	61;39	58;59*	56;19	53;39
17	72;30	69;28	66;26	63;24	60;46*	58;08*	55;30
18	**73;46**	70;54*	68;02	**65;10**	62;34	59;58*	**57;22**
19	74;57*	72;10	69;24	66;37?*	64;07*	61;37*	59;07
20	76;04	73;26	70;49*	68;04	65;39*	63;15*	60;51
21	**77;18**	74;42	72;06	**69;30**	67;12	64;53*	**62;35**
22	77;56*	75;29	73;02	70;35	68;16	65;54*	63;30
23	78;34	76;16	73;58*	71;40	69;15	66;50*	64;25*
24	**79;10**	77;02	74;54	**72;46**	70;18*	67;49	**65;20**
25	80;13	77;54*	75;37*	73;18	71;11	69;04	66;58*
26	81;16*	78;47	76;18	73;49*	72;05	70;21	68;37
27	**82;19**	79;40	77;01	**74;22**	73;01	71;39	**70;16**
28	83;17	80;48	78;18	75;48	74;18	72;48	71;18
29	84;16	81;55	79;35	77;14	75;37	73;58	72;20*
30	**85;14**	83;03	81;51	**78;40**	76;54	75;08	**73;22***

Tabelle 2: Reconstructed qibla table for $6^\circ \leq \Delta\phi \leq 12^\circ$.

$\Delta\phi$	12	13	14	15	16	17	18
$\Delta\lambda$							
1	**4;49**	4;30	4;11	**3;52**	3;34	3;17	**3;00**
2	**8;59***	8;15	7;30	**6;45**	6;30	6;17	**5;59**
3	**13;11**	12;19	11;27	**10;33**	10;03	9;31	**8;57***
4	**17;23**	16;19*	15;08	**13;56**	13;14	12;34	**11;55**
5	**21;45***	20;15	18;47*	**17;18**	16;28*	15;38*	**14;49***
6	**25;38**	24;04	22;30	**20;56***	19;54*	18;47	**17;43**
7	**29;00**	27;12*	25;24	**23;37**	22;33*	21;29	20;25
8	**32;39***	30;46	28;52*	**26;58**	25;41	24;24	23;07
9	**35;37***	33;25	30;53*	**28;21**	27;29	26;39	**25;49**
10	**38;59**	36;30	34;01	31;32	30;27	29;22	28;17
11	**41;57***	39;36	37;15	34;54*	33;30	32;07	30;44
12	**44;38**	42;29	40;20	**38;13**	36;33	34;53	**33;12**
13	**47;30?***	45;08	42;48*	40;27	38;46	37;05	35;25
14	**49;40***	47;23	45;06	42;49	41;05	39;22	37;38
15	**51;47**	49;36	47;24	**45;12**	43;25	41;38*	**39;50&**
16	53;39	51;30	49;22	47;13	45;17c	43;20c	41;24c
17	55;30	53;26c	51;22c	49;13*&	47;22c	45;32c	43;41c
18	**57;22**	55;20	53;16	**51;12**	49;29	47;44	**45;58***
19	59;07	57;04	55;00	52;57*	51;14*	49;30*	47;47*
20	60;51	58;47	56;44	54;41	53;00?*	51;17*	49;37*
21	**62;35**	60;32	58;28	**56;25**	54;47	53;09	**51;31***
22	63;30	61;36	59;42	57;48*	56;12	54;35	52;59
23	64;25*	62;40	60;55	59;10	57;32	55;59*	54;24d
24	**65;20**	63;43	62;08	**60;30***	58;55	57;21*	**55;47d**
25	66;58	65;20c	63;45c	62;07c*	60;29c	58;51c	57;13*
26	68;37	67;00c	65;23c	63:46c	62;04c	60;22c	58;39
27	**70;16**	68;29	66;42	**64;54&**	63;38c	61;51c	**60;04**
28	71;18	69;33*	67;49	66;02*	64;28	62;53*	61;18*
29	72;20*	70;37*	68;54	67;11	65;38*	64;05	62;32
30	**73;22***	71;42	70;02	**68;22**	66;50?*	65;18*	**63;47**

Table 3: Reconstructed qibla table for $12^\circ \leq \Delta\phi \leq 18^\circ$.

$\Delta\phi$	18	19	20	21	22	23	24
$\Delta\lambda$							
1	**3;00**	2;53	2;46	2;39	2;32	2;24*	2;17
2	**5;59**	5;46	5;32	5;18	5;04	4;49	4;34
3	**8;57***	8;37	8;17	**7;55***	7;34	7;13*	**6;51**
4	**11;55**	11;21	10;52	10;18	9;54	9;31	9;06
5	**14;49***	14;06	13;24	12;41	12;14	11;48	11;21
6	**17;43**	16;50*	15;57*	**15;04**	14;36	14;06	**13;37**
7	20;25	19;28	18;31	17;35*	16;57*	16;22	15;47
8	23;07	22;06	21;06	20;05	19;22	18;40	17;55
9	**25;49**	?	23;40	**22;35**	21;46*	20;57*	**20;08**
10	28;17	26;10	25;02*	24;55	24;01	23;05	22;12
11	30;44	29;34	28;25	27;15	26;15	25;16	24;16
12	**33;12**	32;00	30;47	**29;34***	28;30	27;25*	**26;20**
13	35;25	34;06	32;46	31;26	30;23	29;20	28;17
14	37;38	36;11*	34;46	33;18	32;17	31;15	30;14
15	**39;50&**	37;48*c	36;28c	**35;09**	34;09*	33;09	**32;09**
16	41;24c	40;00*c	39;37c	37;13	36;08	35;03	33;58*
17	43;41c	42;13*	40;45*c	39;17	38;07	36;58*	35;48
18	**45;58***	44;26	42;54*	**41;22**	40;07*	38;52*	**37;37***
19	47;47*	46;12?*	44;37*	43;02	41;40	40;34	39;18*
20	49;37*	?	46;?*	44;51*	43;31	42;14	40;58
21	**51;31***	49;52	48;14	**46;34**	45;17	43;58*	**42;39**
22	52;59	51;23	49;46	48;09	46;50*	45;31	44;12
23	54;24d	52;50*d	?	49;42	48;24	47;05	45;46
24	**55;47d**	54;16*d	?	**51;15**	49;57*	48;38*	**47;19***
25	57;13*	?	54;12	52;40	51;19*	49;57	48;34*
26	58;39	57;09d	55;39	54;08	52;45	51;17	49;47
27	**60;04**	58;34d	57;02	**55;35**	54;03	52;31	**51;00**
28	61;18*	59;50*	58;23*	56;56*	55;28	54;01	52;32
29	62;32	61;07?*	59;41*	58;16	56;53*	55;30	54;08*
30	**63;47**	62;20	60;59	**59;36**	58;18	57;00*	**55;42**

Table 4: Reconstructed qibla table for $18^\circ \leq \Delta\phi \leq 24^\circ$.

$\Delta\phi$	24	25	26	27	28	29	30
$\Delta\lambda$							
1	2;17	2;13	2;08	2;03	2;00	1;56	1;52
2	4;34	4;25	4;15	4;06	3;58*	3;49*	3;42
3	**6;51**	6;37	6;23	**6;09**	5;58	5;47	**5;36***
4	9;06	8;48	8;29	8;11	7;56	7;42	7;27
5	11;21	10;58	10;35	10;13	9;55*	9;39	9;18
6	**13;37**	13;09	12;42	**12;15**	11;53	11;31	**11;09**
7	15;47	15;17	14;42	14;13	13;47	13;22	12;56
8	17;55	?	16;46	16;11	15;40	15;14	14;37*
9	**20;08**	19;28	18;49	**18;09**	17;37*	17;06*	**16;34***
10	22;12	21;28	20;40	20;01	19;24*	18;46	18;08
11	24;16	23;30	22;44*	21;57	21;11	20;26*	19;42
12	**26;20**	25;29	24;38*	**23;48***	22;57	22;06*	**21;16***
13	28;17	27;24	26;31	25;38	24;49	24;00*	23;10
14	30;14	29;19	28;22*	27;28	26;40*	25;52	25;03
15	**32;09**	31;05c	30;04c&	**29;18**	28;31	27;44	**26;56***
16	33;58*	33;00	32;01	31;02	30;13*	29;24	28;36
17	35;48	34;47*	33;46	32;45	31;55*	31;05*	30;15*
18	**37;37***	36;35	35;32	**34;29**	33;37*	32;45	**31;53***
19	39;18*	38;14*	37;09	36;02	35;11	34;18*	33;25
20	40;58	39;52	38;45	37;39	36;45	35;51	34;57
21	**42;39**	41;31	40;23	**39;15**	38;20	37;24	**36;29**
22	44;12	43;03	41;54	40;45	39;50	38;54	37;59
23	45;46	44;35	43;25	42;15*	41;19	40;24	39;28
24	**47;19***	46;08	44;56	**43;46***	42;51*	41;54	**40;58***
25	48;34*	47;27	46;20	45;13*	44;21	43;29	42;36*
26	49;47	48;46	47;44	46;42	45;52	45;03	44;14
27	**51;00**	50;04	49;08*	**48;11***	47;24*	46;38	**45;51***
28	52;32	51;31*	50;33*	49;32	48;40	47;49	46;57*
29	54;08*	53;03	51;59*	50;55	49;58*	49;01	48;04
30	**55;42**	54;33	53;26	**52;17***	51;16*	50;14	**49;12**

Table 5: Reconstructed qibla table for $24^\circ \leq \Delta\phi \leq 30^\circ$.

$\Delta\phi$	3	6	9	12	15	18
$\Delta\lambda$						
1	16;07 -72	9;59 +67	6;23 +26	4;49 +20	3;52 +12	3;00 -1
2	30;12 -109	17;12 -9	11;14 -33	8;59 +4	6;45 -26	5;59 -3
3	42;35 -44	25;17 +6	17;18 -7	13;11 -5	10;33 -10	8;57 -3
4	53;12 +89	31;23 -48	23;02 +17	17;23 -6	13;56 -16	11;55 -2
5	59;12 +73	37;32 -48	28;39 +54	21;45 +12	17;18 -17	14;49 -2
6	63;14 +29	43;20 -21	33;50 +87	25;38 +11	20;56 +3	17;43 +1
7	66;24 -5	48;12 -6	37;23 +44	29;00 -8	23;37 -27	
8	69;31 +2	52;39 +21	40;57 +24	32;39 +1	26;58 -10	
9	72;22 +26	55;56 +9	44;31 +25	35;57 +2	28;21 -104	25;49
10	75;02 +64	59;02 +13	47;32 +11	38;59 -1		
11	76;59 +76	61;14 -16	50;42 +24	41;57 +3		
12	77;01 -12	63;30 -21	53;23 +23	44;38 +2	38;13 +2	33;12 -2
13	78;22 -10	66;50 +52	55;30 +1	47;30 +22		
14	79;24 -18	68;58 +67	57;59 +15	49;40 +9		
15	80;43 -1	69;40 +7	59;59 +10	51;47 +3	45;12 -1	39;50 -4
18	83;24 +4	73;46 -2	65;10 +1	57;22 -14	51;12	45;58 +8
21	85;24 +2	77;18 +10	69;30 +3	62;35 +5	56;25 +2	51;31 +25
24	86;59 -5	79;10 -42	72;46 -14	65;20 -79	60;30 -24	55;47
27	88;46 +15	82;19 +9	74;22 -99	70;16 +3	64;54 +3	60;04 +6
30	89;47 -2	85;14 +64	78;40 +2	73;22 +2	68;22 +2	63;47 +3

Table 6: Selected entries with errors (entries minus recomputed values for $\phi_M=21^\circ$) in minutes of arc. $3^\circ \le \Delta\phi \le 18^\circ$.

$\Delta\phi$	18	21	24	27	30
$\Delta\lambda$					
3	8;57 -3	7;55 +8	6;51 -1	6;09	5;36
6	17;43 +1	15;04 -18	13;37 +1	12;15 +1	11;09 +1
9	25;49	22;35 -2	20;08 +1	18;09	16;34
12	33;12 -2	29;34 +11	26;20 +1	23;48 -4	21;16 -35
15	39;50 -4	35;09 -29	32;09 -1	29;18 -1	26;56 -1
18	45;38 -12	41;22 +1	37;37	34;29	31;53 +2
21	51;31 +25	46;34 +1	42;39 -1	39;15 -7	36;29 -2
24	55;47	51;15 -2	47;19 -3	43;46 -11	40;58
27	60;04 +6	55;35 -1	51;00 -42	48;11 -4	45;51 +39
30	63;47 +3	59;36 +4	55;42 -1	52;17	49;12 -1

Table 7: Selected entries with errors (entries minus recomputed values for $\phi_M=21^\circ$) in minutes of arc. $18^\circ \le \Delta\phi \le 30^\circ$.

Apparatus for tables 1-5

A notation such as [3, 12] refers to the entry in column 3, row 12.

Table 1:

[1,1] 42;18.

[3,1] 16;04, [3,3] 12;35, [3,4] 53;52, [3,5] 59;02, [3,7] 67;24, [3,12] 77;40, [3,16] 82;02, [3,17] 83;24, [3,18] 84;03, [3,19] 84;43, [3,20] 85;24.

These errors can be explained as follows. The author used the entry 80;43, [3,15] by mistake as [3,16], and he then obtained [3,17] by careless linear interpolation between [3,16] and [3,18]. The scribe thought that the two identical numbers [3,15] and [3,16] were a mistake, and he therefore wrote in row 16 the entry [3,17] etc. After putting [3,21] in this way in row 20 he realized that [3,21] was in row 21 in his original, so he wrote [3,21] once again in its correct place.

[4,1] 14;07, [4,2] 25;12, [4,4] 45;57, [4,8] 64;46, [4,10] 69;12, [4,14] 75;30, [4,15] 76;13, [4,17] 78;12, [4,20] 81;14, [4,23] 83;55, [4,27] 86;18, [4,28] 87;05, [4,30] 88;36.

[5,5] 44;15, [5,7] 54;45, [5,8] 59;13, [5,14] 72;48, [5,15] 73;12, [5,16] 75;33, [5,18] 76;13, [5;19] 77;19, [5,20] 78;19, [5,21] 80;18, [5,25] 82;28, [5,26] 83;54, [5,27] 85;18, [5,28] 85;59, [5,29] 86;39, [5,30] 87;45.

[6,2] 17;52, [6,5] 37;12, [6,9] 55;16, [6,12] 64;01, [6,13] 67;30, [6,14] 68;28, [6,16] 71;40, [6,19] 74;17, [6,22] 77;46, [6,26] 81;46.

Table 2:

[7,3] 22;22, [7,7] 44;12, [7,9] 52;30, [7,11] 57;52, [7,18] 70;14, [7,25] 77;14.

[8,3] 19;30, [8,4] 25;12, [8,5] 31;06, [8,6] 34;00, [8,7] 40;51 (last sexagesimal copied from [8,8]) [8,8]: 44;11, [8,9] 48;38, [8,11] 54;31, [8,16] 64;58, [8,20] 70;19, [8,23] 74;18, [8,25] 77;14.

[9,2] 11;44, [9,3] 17;38, [9,5] 28;19, [9,8] 40;17, [9,10] 47;12, [9,15] 59;33, [9,19] 66;20, [9,26] 73;59.

[10,5] 26;51, [10,8] 38;48, [10,10] 44;48, [10,13] 52;11, [10,16] 58;19, [10,17] 60;26, [10,19] 64;30, [10,20] 65;59, [10,24] 70;13.

[11,4] 59;18, [11,5] 24;23, [11,7] 32;47, [11,12] 47;38 [11,17] 58;38, [11,18] 59,18, [11,19] 60;37, [11,20] 64;15, [11,21] 63;13, [11,22] 64;54, [11,23] 66;02.

[12,2] 8;49, [12,5] 21;25, [12,8] 32;59 (the author used this value to obtain [11,8] and [10,8] by linear interpolation), [12,9] 35;17, [12,11] 41;17, [12,13] 47;25, [12,14] 49;47, [12,23] 64;22, [12,25] 66;18, [12,29] 72;27, [12,30] 73;32.

Table 3:

[13,4] 16:59, [13,7] 27;52, [13,28] 69;13, [13,29] 70;34.

[14,5] 18;40, [14,8] 28;32, [14,9] 30;13, [14,13] 42;58.

[15,6] 20;56, [15,11] 34;14, [15,17] 49;18 (the author used this value to obtain [14,17] and [13,17] by linear interpolation), [15,19] 52;17, [15,22] 57;18, [15,24] 60;50, [15,27]: (the author misread 64;54 as 65;24, and using this number he found (by linear interpolation) [16,27], [17,27], [15,25] and [15,26], and hence (again by interpolation) [13,25], [13,26], [14,25], [14,26], [16,25], [16,26], [17,25], [17,26]), [15,28] 66;12.

[16,5] 16;23, [16,6] 19;14, [16,7] 22;13, [16,19] 51;37, [16,20] 53,45, [16,29] 65;28, [16,30] 66;25.

[17,5] 15;28, [17,15] 41;58, [17,19] 49;07, [17,20] 50;51 (sexagesimals interchanged and "17" read as "50"), [17,23] written twice as 55;19 in column 17, and as 55;59 in row 23, column 18. [17,24] written twice, as 57;21 in column 17 and 56;21 in row 24, column 18. [17,28] 62;13, [17,30] 65;13.

[18,3] 8;54, [18,5] 14;41, [18,15]: (The author misread 39;50 as 39;07 and he used this number to find [19,15], [20,15], [18,16], [18,17] and hence [16,16], [17,16], [16,17], [17,17], [19,16], [19,17], [20,16] and [20,17] by linear interpolation), [18,18] 45;38, [18,19] 47;07, [18,20] 48;17, [18,21] 50;31, [18,23] and [18,24] (see apparatus for [20,23], [20,24]), [18,25] 57;33, [18,28] 61;38.

Table 4:

[19,6] 17;06, [19,9] 25;19 (the scribe copied [18,9] here), [19,14] 36;41, [19,15] 36;48, [19,16] 40;44, [19,17] 42;18, [19,19] 45;00, [19,20] 46;44 (perhaps the scribe copied [20,20] here), [19,23] 52;30 (written in col. 20, row 13), [19,24] 53;16 (written in col. 20, row 14, see the apparatus for [20,23] and [20,24]), [19,25]: the eye of the scribe skipped this entry, and in rows 25-27 he wrote the entries [19,26] - [19-28] respectively (thus the entry [19,28] occurs twice, in rows 27 and 28). [19,28] 59;04 (the same in rows 27 and 28), [19,29] 60;50.

[20,6] 16;17, [20,10] 25;42, [20,17] 40;42, [20,18] 44;14, [20,19] 44;17, [20,20] 46;41, [20,23] and [20,24]: (These entries in the original table were left out by mistake. When the scribe copied rows 23 and 24 (i.e. one horizontal segment of the table between two parallel lines), he wrote [17,23] and [17,24] again in col. 18 by mistake. He then wrote [18,23] and [18,24] in column 19 and [19,23] and [19,24] in column 20.), [20,28] 58;13, [20,29] 59;11.

[21,3] 7;15, [21,7] 17;55, [21,12] 29;31, [21,20] 44;11, [21,28] 56;36.

[22,7] 16;17, [22,9] 21;56, [22,15] 34;01, [22,18] 40;04, [22,22] 46;30, [22,24] 49;47, [22,25] 51;39, [22,29] 56;33.

[23,1] 2;27, [23,3] 7;18, [23,9] 21;17, [23,12] 27;35, [23,17] 36;38, [23,18] 38;12, [23,21] 43;18, [23,24] 48;58, [23,30] 57;13.

[24,16] 33;38, [24,18] 37;34, [24,19] 39;38, [24,24] 47;29, [24,25] 48;37, [24,29] 54;03.

Table 5:

[25,8] 17;15 (the scribe copied [24,8] here), [25,17] 37;47, [25,19] 38;34, [25,28] 51;21.

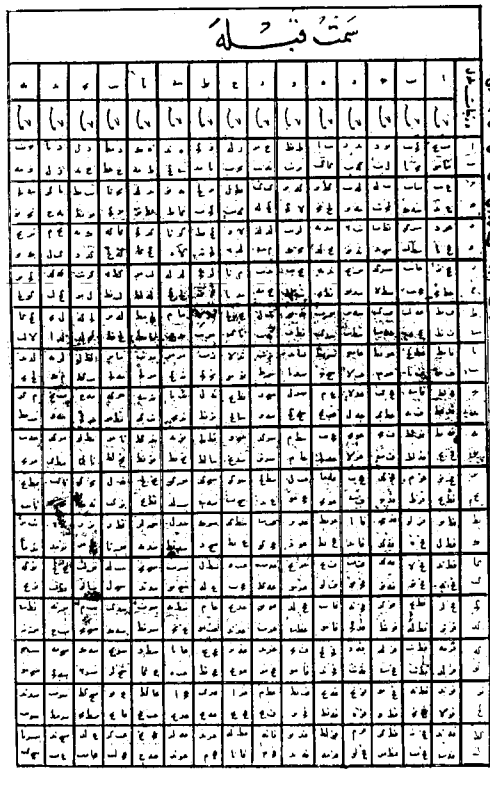
[26,11] 22;38, [26,12] 24;44 (the last sexagesimals of [26,11] and [26,12] were exchanged) [26,14] 28;32, (It seems that [26,15] was found by extending the linear interpolation between [30,15] and [27,15]), [26,27] 49;03, [26,28] 50;13, [26,29] 51;49.

[27,12] 23;43, [27,23] 42;25, [27,24] 43;16, [27,25] 45;33, [27,27] 43;41, [27,30] 52;57.

[28,2] 3;53, [28,5] 9;15, [28,9] 17;34, [28,10] 39;24, [28,14] 27;40, [28,16] 30;08, [28,17] 31;15, [28,18] 33;17, [28,24] 42;41, [28,27] 47;34, [28,29] 49;38, [28,30] 51;07.

[29,2] 3;19, [29,9] 17;16, [29,11] 20;06, [29,12] 22;30, [29,13] 24;05, [29,17] 31;55 (cf. [28,17]), [29,19] 34;38.

[30,3] 15;36, [30,8] 14;27, [30,9] 16;27, [30,12] 21;56, [30,15] 26;26, [30,17] 30;12, [30,18] 31,33, [30,24] 40;38, [30,25] 42;26, [30,27] 45;11, [30,28] 46;17.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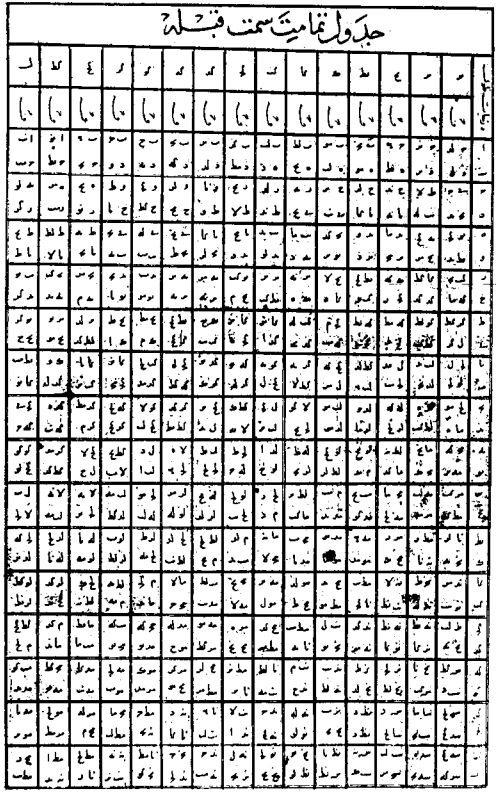
جدول تمامیت سمت قبله