The Mathematical Structure of Two Islamic Astrological Tables for 'Casting the Rays'

by

JAN P. HOGENDIJK*

1. Introduction and summary

In medieval Islamic astrology the sun, the moon and each planet were believed to cast seven rays, meeting the ecliptic in seven astrologically significant points.[1] The seven rays are, in order of increasing ecliptical longitude of their points of intersection with the ecliptic: the left sextile ray, the left quartile ray, the left trine ray, the ray to the diametrically opposite point, the right trine ray, the right quartile ray and the right sextile ray. The medieval Muslim astrologers generally believed that the sextile, quartile and trine rays were determined by the sides of a regular hexagon, a square and an equilateral triangle respectively. There were however two different methods for positioning these regular polygons.

According to the simplest theory, the polygons have to be inscribed in the ecliptic in such a way that the planet is at one angular point (Figure 1). The rays are cast along the sides of the polygons. Thus if the celestial longitude of the planet is λ, the sextile, quartile and trine rays are cast to the points with longitude $\lambda \pm 60°$, $\lambda \pm 90°$, $\lambda \pm 120°$. Al-Ṣūfī (903–983) and Al-Bīrūnī (972–1048) prescribe that unimportant modifications be made in case the planet has non-zero latitude.[2]

In the second theory it is assumed that the rays of the planets are related to the apparent daily rotation of the universe, so that the

*Mathematical Institute, State University of Utrecht, Budapestlaan 6, NL-3508 TA Utrecht, The Netherlands.

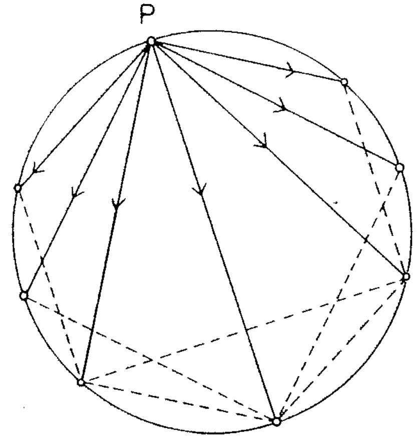

Figure 1.

regular polygons have to be positioned on the celestial equator. In the *Masudic Canon,* Al-Bīrūnī gives a rather vague description of this theory, which can be summarized as follows.[3] In Figure 2, *NESW* is the local horizon and *N, E, S* and *W* are the cardinal points. *EMW* is the celestial equator, the planet is at point *P,* and semicircle *NPS* intersects the celestial equator at P^*. The regular polygons have to be inscribed in the equator in such a way that P^* is at an angular point. Then the semicircles through *N, S* and the other angular points (such as Q^* for the left sextile ray in Figure 2; arc $P^*Q^* = 60°$) determine the positons of the rays; thus *PQ* in Figure 2 is the left sextile ray, etc. Compare also Figure 7 below.

The exact computation of the rays according to this second method is complicated. Al-Bīrūnī describes an approximate computation, which has been treated by Kennedy and Krikorian,[4] and which we will discuss below (Section 2) in connection with some geometrical preliminaries. Al-Bīrūnī says that the astrologers also used instruments and tables for the computation.[5] No instruments for this purpose are known to be extant, but two sets of tables have come down to us. In this paper we will discuss these tables, and give some new insight in their mathematical structure.

The first set of tables was computed by Muḥammad ibn Mūsā al-Khwārizmī,[6] whose modern fame is mainly based on his treatises on

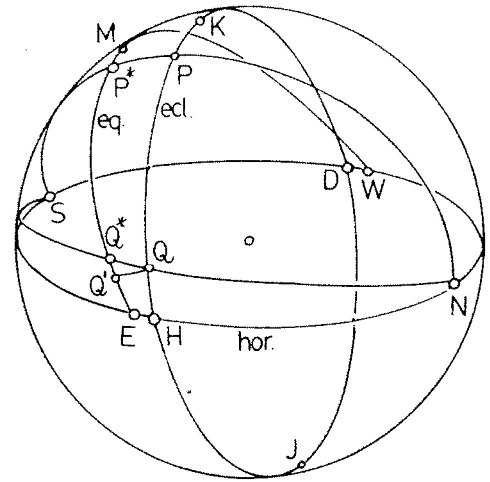

Figure 2.

arithmetic and algebra (although his astronomical work is far more substantial). These tables, which we will call the Khwārizmī tables, are presented in the *Book on astrology that renders (all others) superfluous* (*al-mughnī fī aḥkām al-nujūm*) of the Christian astrologer Ibn Hibintā (early 10[th] century),[7] who attributes them to Al-Khwārizmī,[8] and they also occur (in Latin transcription) in the *Toledan Tables* (12[th] century).[9] The Khwārizmī tables were published by Kennedy and Krikorian in 1972.[10] They involve a function of two variables, which has been explained arithmetically by Toomer and geometrically by Kennedy and Krikorian,[11] and three functions of a single variable, which have not been understood by modern historians until now. We describe the Khwārizmī tables and discuss the significance of the two functions in Section 3 below.

The Khwārizmī tables were originally contained in the famous Zīj (astronomical handbook with tables) of Al-Khwārizmī. This work has only come down to us in a garbled Latin translation of a version which had been revised by the Andalusian astronomer Maslama ibn Aḥmad al-Majrīṭī,[12] who died around 1007. In this process the original Khwārizmī tables for casting the rays were replaced by a set of tables for the same purpose, computed for a geographical latitude $\varphi = 38;30°$,[13] probably for the city of Cordoba.[14] Hence this set of tables was certainly not by Al-Khwārizmī, and we will therefore call it the Majrīṭī tables, even though his authorship is not completely certain. The Majrīṭī tables are of enormous length, they extend over 24 pages of Suter's 1914 edition of the Latin version of Al-Khwārizmī's Zīj.[15] The Majrīṭī tables have been studied by Suter, Neugebauer and Kennedy and Krikorian,[16] but the underlying mathematics has remained a mystery until the present. In Section 4 we explain the mathematical structure of the Majrīṭī tables. It turns out, not surprisingly, that these tables can be considered as an improvement of the Khwārizmī tables. As we will see, Al-Majrīṭī solved a quite difficult trigonometrical problem, and he then went through a complicated and laborious computation. Thus the Majrīṭī tables are of considerable interest for the history of applied mathematics in the middle ages.

2. *Preliminaries*

Right and oblique ascensions

Figures 3 and 4 display the celestial sphere for observers on the terrestrial equator (3) and on the Northern hemisphere, south of the Arctic Circle (4). *ESDWNH* is the local horizon and the cardinal directions are *E, S, W, N, EVMW* is the celestial equator, *V* is the vernal point. *DKVH* is part of the ecliptic, *SKMZCN* is the meridian, *C* is the celestial north pole and *Z* is the zenith. The intersection *H* of the ecliptic with the Eastern horizon is called the horoscopus or ascendant. Suppose *H* has celestial longitude λ, and write ε for the obliquity of the ecliptic and φ for the geographical latitude ($\varphi = 0°$ in Figure 3), then $\lambda = \text{arc}\,VH$, $\varepsilon = \angle EVH$, $\varphi = \text{arc}\,CN$.

Definitions:
For $\varphi = 0°$ (Figure 3), arc *VE* is called the right ascension of arc *VH*. Notation: $VE = A_0(\lambda)$.

For $0° < \varphi < 90° - \varepsilon$ (Figure 4), arc *VE* is called the oblique ascension of arc *VH*. Notation: $VE = A_\varphi(\lambda)$. The oblique ascension $A_\varphi(\lambda)$ can be defined in an analogous way for southern geographical latitudes, that is, for $-(90° - \varepsilon) < \varphi < 0°$. The functions $A_0(\lambda)$ and $A_\varphi(\lambda)$ are defined for all values $0° \leq \lambda < 360°$ and they assume values between 0° and 360°.

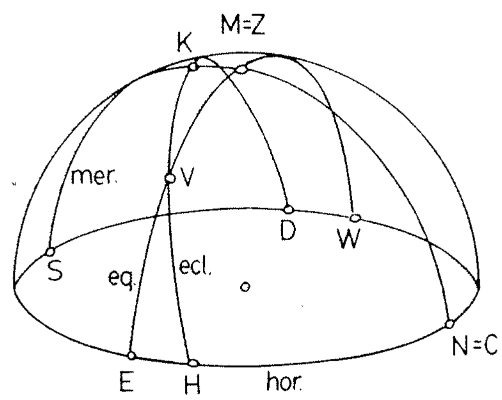

Figure 3.

The declination δ of the horoscopus is its distance to the celestial equator, reckoned positive if *H* is in the northern half of the celestial sphere, and negative if *H* is in the southern half, note that $-\varepsilon \leqslant \delta \leqslant \varepsilon$. In Figure 3, $\delta = EH$. For given λ and ε, δ can be computed from

$$\sin\delta = \sin\lambda \sin\varepsilon. \tag{2.1}$$

Then the right ascension can be found from

$$\sin A_0(\lambda) = \tan\delta / \tan\varepsilon, \tag{2.2}$$

if we keep in mind that $A_0(\lambda)$ is in the same quadrant as λ.
Further $A_\varphi(\lambda) = A_0(\lambda) - \eta$ with

$$\sin\eta = \tan\delta \tan\varphi \tag{2.3}$$

and $-90° < \eta < 90°$; compare figure 4, in which $HF = \delta$, $FE = \eta$. Note that $\angle VEN = 90° + \varphi$.

Other ways of computing the oblique ascension are possible, but the method presented here is essentially that used by many medieval astronomers. Tables of right and oblique ascensions (for various latitudes) occur in the *Almagest* of Ptolemy and in many Islamic astrono-

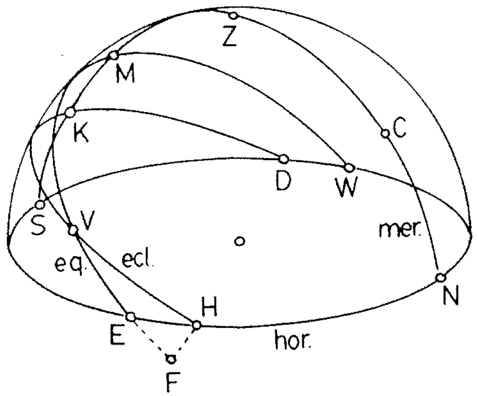

Figure 4.

mical handbooks; further details can be found in publications of Kennedy, King and Pedersen.[17]

Definition: the time-degree is the interval of time in which the celestial sphere revolves by one degree.

A sidereal day consists of 360 time-degrees, and one time-degree is a little less that 4 minutes.

For the observer of Figure 4, arc VH rises on the Eastern horizon in $A_\varphi(\lambda)$ time-degrees.

If X and Y are two arbitrary points on the celestial equator with longitudes λ and μ, then arc XY (i.e. the arc extending from X in the direction of increasing celestial longitude towards Y) rises in $A_\varphi(\mu) - A_\varphi(\lambda)$ (modulo 360) time-degrees.

Oblique descensions and incident horizons

We now ask the question: how long does it take XY to pass over an arbitrary semicircle through N and S (other than the Eastern horizon) such as NPS in Figure 5.

If the semicircle is half of the meridian (above or below the horizon), the passage takes place in $A_0(\mu) - A_0(\lambda)$ (modulo 360°) time-degrees.

If the semicircle is the Western horizon, the passage takes $A_{-\varphi}(\mu) - A_{-\varphi}(\lambda)$ (modulo 360°) time-degrees. To see this, rotate the celestial sphere over 180° around the axis through the zenith. The expression $A_{-\varphi}(\mu) - A_{-\varphi}(\lambda)$ (modulo 360°) is called the oblique descension of arc XY for geographical latitude φ.

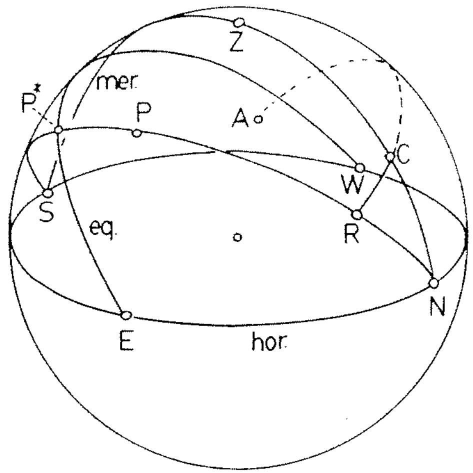

Figure 5.

To settle the question for an arbitrary semicircle NPS with P in the Eastern half of the celestial sphere (Figure 5) we draw arc CR perpendicular to semicircle NPS. Then the pole A of circle NPS is on the great circle to which arc CR belongs. Let NPS intersect the celestial equator at P^*.

Because C is the pole of the celestial equator and A is the pole of circle NPS, the intersection P^* of the equator and NPS is the pole of CA. Therefore arc $P^*R = 90°$. This means that mathematically, NP^*S can be treated as "horizon" for a geographical latitude CR, with north point R and east point P^*. Put arc $CR = \xi$, then arc XY passes over NPS in $A_\xi(\mu) - A_\xi(\lambda)$ (modulo 360°) time-degrees.

Finally, if P is in the western half of the celestial sphere, one can show in the same way that arc XY passes over NPS in $A_\xi(\mu) - A_\xi(\lambda)$ time-degrees, but in this case the latitude ξ is negative, $\xi = -|CR|$.

Some Arabic astrologers called a circle such as NPS in Figure 5 "ufq ḥādith",[18] for which we will use Professor Kennedy's translation "incident horizon". They called arc CR the "latitude" of the incident horizon NPS.[18]

The following two formulas can easily be verified (Figure 4). Let λ_H and λ_K be the celestial longitudes of the horoscopus H and the intersection K of the ecliptic with the meridian above the horizon. Because arc $EW = 180°$ and arc $EM = 90°$, we have

$$A_{-\varphi}(\lambda_H - 180°) = A_\varphi(\lambda_H) - 180° \text{ (modulo } 360°) \qquad (2.4)$$

$$A_0(\lambda_K) = A_\varphi(\lambda_H) - 90° \text{ (modulo } 360°). \qquad (2.5)$$

The identity (2.4) shows that $A_{-\varphi}$ can be found from a table of A_φ. Therefore oblique descensions were never tabulated separately.

The standard computation of the rays

Many Arabic sources[19] describe what I call the standard computation of the astrological rays. This procedure has also been discussed by Nallino and by Kennedy and Krikorian.[20] It is based on the above-mentioned theory that the regular polygons which determine the rays have to be located on the celestial equator. I begin with introductory remarks, referring to Figure 2.

The basic idea is

$$P^*Q^* = A_\xi(PQ) \qquad (2.6)$$

where ξ is the latitude of the incident horizon NPS.[21]

Formula (2.6) is correct only when the incident horizon NQS has the same latitude as NPS, or when Q is the vernal or autumnal point. In all other cases (2.6) is at best an approximation. This can be seen in Figure 2, where $A_\xi(PQ) = P^*Q'$, with Q' defined such that $\angle PP^*W = \angle QQ'W = 90+\xi$.

The approximation (2.6) can be used if tables of oblique ascensions for all latitudes are available (which was often the case). In the standard computation a linear interpolation is made, so that the entire computation can be done by means of a table of right ascensions and a table of oblique ascensions for one's own geographical latitude.

The standard computation is as follows in modern notation. Let λ_P be the given celestial longitude of the planet P, and λ_Q the required longitude of the point where the ray hits the ecliptic. Choose for $t = P^*Q^*$ one of the values $\pm 60°$, $\pm 90°$ or $\pm 120°$. Let the ecliptic intersect the Eastern horizon at H, the Western horizon at D, and the meridian above the horizon at K and under the horizon at J (see Figure 2). One assumes that the longitude λ_H of the horoscopus has been previously determined; then λ_K can be found by (2.5) and clearly $\lambda_D = \lambda_H \pm 180°$, $\lambda_J = \lambda_K \pm 180°$.

Several cases have to be considered.

1. If P is on the meridian (i.e. $\lambda_P = \lambda_K$ or $\lambda_P = \lambda_J$), then $\lambda_Q = \lambda_1$ with $A_0(\lambda_1) = A_0(\lambda_P)+t$.
2. If P is on the Eastern horizon ($\lambda_P = \lambda_H$), then $\lambda_Q = \lambda_2$ with $A_\varphi(\lambda_2) = A_\varphi(\lambda_P)+t$.
3. If P is on the Western horizon ($\lambda_P = \lambda_D$), then $\lambda_Q = \lambda_3$ with $A_{-\varphi}(\lambda_3) = A_{-\varphi}(\lambda_P)+t$. One can find λ_3 from a table of A_φ by (2.4).
4. If P is in the Eastern hemisphere, then $\lambda_Q = \lambda_1 + k(\lambda_2 - \lambda_1)$ with $k = (A_0(\lambda_P) - A_0(\lambda_K))/(A_0(\lambda_H) - A_0(\lambda_K))$ if P is above the horizon (as in Figure 2), and $k = (A_0(\lambda_J) - A_0(\lambda_P))/(A_0(\lambda_J) - A_0(\lambda_H))$ if P is under the horizon. (If the vernal point is between P and K, $A_0(\lambda_P) < A_0(\lambda_K)$; in this case $(A_0(\lambda_P) - A_0(\lambda_K))$ has to be replaced by $(A_0(\lambda_P) - A_0(\lambda_K)) + 360°$; what is intended is always "the right ascension of arc PK". Similar modifications have to be made in the other parts of the formulas if the vernal point is between H and K, etc.).
5. If P is in the Western quadrant, substitute λ_3 for λ_2 and D for H.

In the *Great Introduction to Astrology* of Abū Maᶜshar (A.D. 787–886) the standard method is attributed to Ptolemy,[22] and this has caused some confusion because no trace of the method occurs in Ptolemy's astrological work, the *Tetrabiblos*. Ptolemy mentions the casting of rays but he gives no details about the computation. Nallino[23] explained the attribution by the fact that Ptolemy uses a similar method in a problem related to the astrological theory of progressions (Arabic: *tasyīr*).[24] In this problem he computes from λ_Q, λ_P, and λ_H the time-interval it takes the moving point Q to reach the fixed great circle NPS by means of the daily rotation of the universe (Figure 2). Nallino's argument is supported by the fact that Ptolemy uses a similar interpolation coefficient k. In Ptolemy's solution, k appears in a natural way as a consequence of the assumption (actually stated by Ptolemy) that semicircle *NPS* is approximately an hour line.[25]

3. *The Khwārizmī tables*

We now describe the Khwārizmī tables following Kennedy and Krikorian.[26] The ecliptical signs will be numbered Aries = 1, Taurus = 2, Gemini = 3, Cancer = 4, Leo = 5, Virgo = 6, Libra = 7, Scorpio = 8, Sagittarius = 9, Capricornus = 10, Aquarius = 11, Pisces = 12. Each sign is divided into three equal 10°-intervals, called decans or faces (Arabic: *wajh*, Latin: *facies*). A notation such as [3,2] will be used for the second decan of sign 3 (that is the interval of points on the ecliptic with celestial longitude between 70° and 80°).

The Khwārizmī tables consist of 432 values of a function F and 36 values of three functions S, Q and T. F is used in all computations, while S is for sextile rays, Q for quartile rays and T for trine rays. Table 1 contains for every decan d and sign m a number $F(d,m)$ tabulated in degrees and minutes of arc. Following Kennedy and Krikorian, the columns in the manuscript have been rendered as rows, but for the rest the arrangement of the manuscript has been maintained. Therefore the row for d = [3,2] begins not with $F([3,2],1)$ but with $F([3,2],3)$. For sake of clarity, the sign number m appears in Table 1 above each group of three numbers $F([n,1],m)$, $F([n,2],m)$, $F([n,3],m)$.

Table 2 contains for each decan d a number $T(d)$ in degrees and minutes of arc. The Khwārizmī tables also contain values $S(d)$ and $Q(d)$ but these have not been rendered here because always

$$S(d):Q(d):T(d) = 60:90:120 \tag{3.1}$$

as noted by Toomer.[27]

By means of the Khwārizmī tables one can compute the astrological rays from the longitude of the ascendant λ_H and the longitude of the planet λ_P. Al-Khwārizmī's instructions for the use of the tables have been preserved in the *Book on astrology that makes (all others) superfluous* of Ibn Hibintā, and they have been translated by Kennedy and Krikorian.[28] I paraphrase these instructions by means of a numerical example.

d	$F(d,m)$ m = 1	m = 2	m = 3	m = 4	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10	m = 11	m = 12
[1,1]	20;05	25;41	31;24	32;16	27;47	22;40	20;05	25;41	31;24	32;16	27;47	22;40
[1,2]	21;15	27;02	31;59	32;16	27;52	22;40	19;52	25;59	31;59	32;16	27;24	22;40
[1,3]	22;24	28;23	32;33	32;16	27;56	22;40	19;39	26;17	32;33	32;16	27;01	22;40
	2	3	4	5	6	7	8	9	10	11	12	1
[2,1]	23;34	30;32	33;08	29;54	25;15	22;40	23;34	30;32	33;08	29;54	25;15	22;40
[2,2]	25;36	31;41	33;17	29;54	25;15	22;31	24;01	31;41	33;49	29;54	25;15	23;27
[2,3]	27;38	32;51	33;26	29;54	25;15	22;23	24;29	32;51	34;29	29;54	25;15	24;13
	3	4	5	6	7	8	9	10	11	12	1	2
[3,1]	29;40	34;00	32;01	27;50	25;15	25;41	29;40	34;00	32;01	27;50	25;15	25;41
[3,2]	31;24	34;18	31;56	27;50	25;11	25;59	31;24	35;21	32;24	27;50	25;38	27;02
[3,3]	33;08	34;37	31;52	27;50	25;06	26;17	33;08	36;43	32;47	27;50	26;01	28;23
	4	5	6	7	8	9	10	11	12	1	2	3
[4,1]	34;52	34;07	30;25	27;50	27;47	30;32	34;52	34;07	30;25	27;50	27;47	30;32
[4,2]	35;19	33;59	30;25	27;50	27;56	31;41	36;54	34;54	30;25	27;50	28;28	31;41
[4,3]	35;47	33;50	30;25	27;50	28;06	32;51	38;56	35;40	30;25	27;50	29;09	32;51
	5	6	7	8	9	10	11	12	1	2	3	4
[5,1]	36;14	33;00	30;25	29;54	31;24	34;00	36;14	33;00	30;25	29;54	31;24	34;00
[5,2]	36;01	33;00	30;29	29;54	31;59	35;21	37;24	33;00	30;02	29;54	31;59	34;18
[5,3]	35;48	33;00	30;34	29;54	32;33	36;43	38;33	33;00	29;39	29;54	32;33	34;37
	6	7	8	9	10	11	12	1	2	3	4	5
[6,1]	35;35	33;00	32;01	32;16	33;08	34;07	35;35	33;00	32;01	32;16	33;08	34;07
[6,2]	35;35	33;09	31;52	32;16	33;49	34;54	35;35	32;13	31;20	32;16	33;17	33;59
[6,3]	35;35	33;17	31;42	32;16	34;29	35;40	35;35	31;27	30;39	32;16	33;26	33;50
	7	8	9	10	11	12	1	2	3	4	5	6
[7,1]	35;35	34;07	33;08	32;16	32;01	33;00	35;35	34;07	33;08	32;16	32;01	33;00
[7,2]	35;48	33;49	32;33	32;16	32;24	33;00	34;25	32;46	32;33	32;16	31;56	33;00
[7,3]	36;01	33;31	31;59	32;16	32;47	33;00	33;16	31;25	31;59	32;16	31;52	33;00
	8	9	10	11	12	1	2	3	4	5	6	7
[8,1]	36;14	34;00	31;24	29;54	30;25	33;00	36;14	34;00	31;24	29;54	30;25	33;00
[8,2]	35;47	32;51	30;43	29;54	30;25	32;13	34;12	32;51	31;15	29;54	30;25	33;09
[8,3]	35;19	31;41	30;03	29;54	30;25	31;27	32;10	31;41	31;06	29;54	30;25	33;17
	9	10	11	12	1	2	3	4	5	6	7	8
[9,1]	34;52	30;32	27;47	27;50	30;25	34;07	34;52	30;32	27;47	27;50	30;25	34;07
[9,2]	33;08	29;11	27;24	27;50	30;02	32;46	33;08	30;14	27;52	27;50	30;29	33;49
[9,3]	31;24	27;49	27;01	27;50	29;39	31;25	31;24	29;55	27;56	27;50	30;34	33;31
	10	11	12	1	2	3	4	5	6	7	8	9
[10,1]	29;40	25;41	25;15	27;50	32;01	34;00	29;40	25;41	25;15	27;50	32;01	34;00
[10,2]	27;38	24;54	25;15	27;50	31;20	32;51	29;13	25;49	25;15	27;50	31;52	32;51
[10,3]	25;36	24;08	25;15	27;50	30;39	31;41	28;45	25;58	25;15	27;50	31;42	31;41
	11	12	1	2	3	4	5	6	7	8	9	10
[11,1]	23;34	22;40	25;15	29;54	33;08	30;32	23;34	22;40	25;15	29;54	33;08	30;32
[11,2]	22;24	22;40	25;38	29;54	23;33	30;14	23;47	22;40	25;11	29;54	32;33	29;17
[11,3]	21;15	22;40	26;01	29;54	31;59	29;55	24;00	22;40	25;06	29;54	31;59	27;49
	12	1	2	3	4	5	6	7	8	9	10	11
[12,1]	20;05	22;40	27;47	32;16	31;24	25;41	20;05	22;40	27;47	32;16	31;24	25;41
[12,2]	20;05	23;27	28;28	32;16	31;15	25;49	20;05	22;31	27;56	32;16	30;43	24;54
[12,3]	20;05	24;13	29;09	32;16	31;06	25;58	20;05	22;23	28;06	32;16	30;03	24;08

Table 1. Al-Khwārizmī's table of $F(d,m)$.

d	T(d)	d	T(d)	d	T(d)	d	T(d)	d	T(d)	d	T(d)
[1,1]	106;35	[3,1]	116;18	[5,1]	129;58	[7,1]	133;25	[9,1]	123;42	[11,1]	110;02
[1,2]	107;44	[3,2]	118;46	[5,2]	131;07	[7,2]	132;16	[9,2]	121;14	[11,2]	108;53
[1,3]	108;53	[3,3]	121;14	[5,3]	132;16	[7,3]	131;07	[9,3]	118;46	[11,3]	107;44
[2,1]	110;02	[4,1]	123;42	[6,1]	133;25	[8,1]	129;58	[10,1]	116;18	[12,1]	106;35
[2,2]	112;07	[4,2]	125;47	[6,2]	133;25	[8,2]	127;53	[10,2]	114;13	[12,2]	106;35
[2,3]	114;13	[4,3]	127;53	[6,3]	133;25	[8,3]	125;47	[10,3]	112;07	[12,3]	106;35

Table 2. Al-Khwārizmī's table for $T(d)$.

Suppose $\lambda_H = 77°$, $\lambda_P = 133°$. We wish to compute the left trine ray. Because 77° is in [3,2], we read in Table 2 $T([3,2]) = 118;46°$, and we then consider the row for [3,2] in Table 1. Because P is in sign 5 (i.e. Leo, the interval [120°, 150°]), we read off $F([3,2],5) = 31;56°$. Since P is in degree 13 of the sign, we compute (30−13)/30 times 31;56° = 18;6°. We then perform a number of subtractions:

$T([3,2]) - 18;6° = 100;40°$,
$100;40° - F([3,2],6) = 72;50°$
$72;50 - F([3,2],7) = 47;39°$
$47;39° - F([3,2],8) = 21;40°$, which is less than $F([3,2],9) = 31;24°$.
Finally we compute $30(21;40°)/31;24° = 20;42°$.
The left trine ray is cast to the point 20;42° Sagittarius.

Thus if the planet is in degree k of sign m (i.e. if its ecliptical longitude is $30(m-1)+k°$), we subtract $(30-k)/30$ times $F(d,m)$ and then $F(d,m+1)$, $F(d,m+2)$, $\ldots F(d,m+j-1)$ from $T(d)$ until we obtain a remainder r with $r < F(d,m+j)$. We then compute $x = 30r/F(d,m+j)$. The left trine ray falls in degree x of sign $m+j$. (Here and further on, we reckon sign numbers modulo 12: thus sign 7+8 is the same as sign 3.)

The computation of the numbers in Tables 1 and 2 is not explained in any known medieval source. Toomer and Kennedy have shown that Table 1 was computed for $\varphi = 33°$, $\varepsilon = 23;51°$,[29] in a way to be mentioned below. Note that medieval sources[30] give values near 33° for the latitude of Baghdād, where Al-Khwārizmī worked, and that Al-Khwārizmī uses $\varepsilon = 23;51°$ elsewhere.[31]

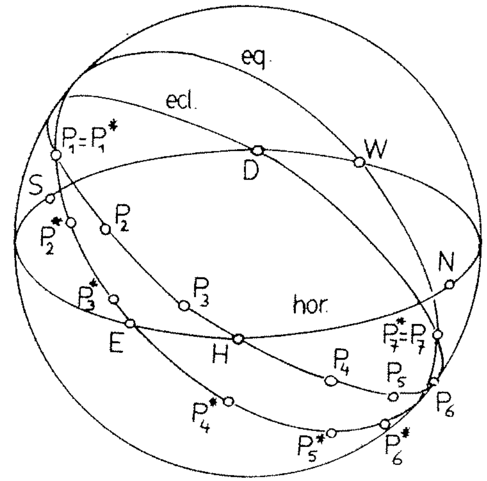

Figure 6.

The following explanation of Table 1 is inspired by Kennedy and Krikorian.[32] In Figure 6, N and S are the North and South points of the horizon, $P_1, \ldots, P_{12}$ are the beginnings of the zodiacal signs, and the great circle through N, S and P_i intersects the celestial equator at P_i^*. Figure 6 displays the P_i and P_i^* for $i = 1,2\ldots,7$ (the great circles $NP_iP_i^*S$ are not shown). Consider a fixed sign m, and call $P_m^*P_{m+1}^*$ the projection of sign m. Geometrically, each $F(d,m)$ in Table 1 is an approximation of the projection of sign m, at the time when decan d is rising at the Eastern horizon.

If decan $[m,1]$ is rising, sign m is at the Eastern horizon, so that its projection is approximately its oblique ascension for $\varphi = 33°$. This explains why in Table 1

$$F([m,1],m) = A_\varphi(m.30°) - A_\varphi((m-1).30°). \quad (3.2)$$

If decan $[m+3,1]$ or decan $[m+9,1]$ is rising, sign m is approximately in the meridian, so that its projection is approximately its right ascension. Thus in Table 1 in the fourth column

$$F([m+3,1],m) = F([m+9,1],m) = A_0(m.30°) - A_0((m-1).30°). \quad (3.3)$$

If decan $[m+6,1]$ is rising, sign m is setting on the Western horizon, so that its projection is approximately its oblique descension. Therefore in Table 1 in the seventh column

$$F([m+6,1],m) = A_{-\varphi}(m.30°) - A_{-\varphi}((m-1).30°). \qquad (3.4)$$

As a consequence of (3.2), (3.3), (3.4) and (2.4) we have the linear relation

$$F([m+3,1],m) = (1/2)(F([m,1],m) + F([m+6,1],m)). \qquad (3.5)$$

For $i = 1,2,4,5$, the values $F([m+i,1],m)$ in the table were computed by linear interpolation:

$$F([m+i,1],m) = (1-i/6)F([m,1],m) + (i/6)F([m+6,1],m). \qquad (3.6)$$

Note that (3.5) is the case $i = 3$ of (3.6). The values in the table also satisfy

$$F([m+6+i,1],m) = F([m+6-i,1],m). \qquad (3.7)$$

We now consider the second and third decans of sign m. In the first column in Table 1 linear interpolaton was used:

$$F([m,2],m) = (2/3)F([m,1],m) + (1/3)F([m+1,1],m+1) \qquad (3.8)$$

$$F([m,3],m) = (1/3)F([m,1],m) + (2/3)F([m+1,1],m+1). \qquad (3.9)$$

One would expect the same to be true for the other columns. But as a matter of fact, for the fourth column we have for $k = 2,3$,

$$F([m+3,k],m) = F([m+3,1],m). \qquad (3.10)$$

For $k = 2,3$ the numbers in the seventh column are related to those in the fourth and the first column by

$$F([m+3,k],m) = (1/2)(F([m,k],m) + F([m+6,k],m) \qquad (3.11)$$

and the numbers in the remaining columns are defined by

$$F([m+6+i,k],m) = F([m+6-i,k],m). \qquad (3.12)$$

The identities (3.11) and (3.12) are analogous to (3.5) and (3.7), but (3.10) is odd. As a consequence of this the numbers in the seventh column vary irregularly. This concludes the explanation of Table 1.

Like Kennedy and Krikorian[33] I am unable to understand why the compiler of the table used (3.10). For if the $F([m+3,k],m)$ in the fourth column had been obtained by linear interpolation between $F([m+3,1],m)$ and $F([m+4,1],m+1)$, the six right columns of the table would be identical to the six left columns, so that the right half of the table could be dispensed with (compare also Section 4 of this paper).

We now turn to Table 2 and the functions S, Q and T. We first explain their arithmetical structure. Let

$$\Phi(d) = F(d,m) + F(d,m+1) + \ldots + F(d,m+11).$$

Note that the number $F(d,m)$ is in general not the exact value, but only an approximation of the projection $P_m{}^*P_{m+1}{}^*$ of sign m. This can be seen from the fact that always

$$P_m{}^*P_{m+1}{}^* + P_{m+1}{}^*P_{m+2}\ldots + P_{m+11}{}^*P_m{}^* = 360° \qquad (3.13)$$

whereas in general

$$\Phi(d) \neq 360°. \qquad (3.14)$$

It can be empirically verified that the functions S, Q and T in the Khwārizmī tables satisfy[34]

$$S(d):Q(d):T(d):\Phi(d) \approx 60:90:120:360. \qquad (3.15)$$

Note that (3.15) is a generalisation of (3.1).

The functions S, Q and T were probably introduced for the following reason. Figure 7 shows a planet at A, which casts a left trine ray to B, a planet at B casting a left trine ray to C, and a planet C casting a left trine ray to D. Then the projections A^*, B^* and C^* form an equilateral triangle, and therefore $D^* = A^*$, whence $D = A$. On the

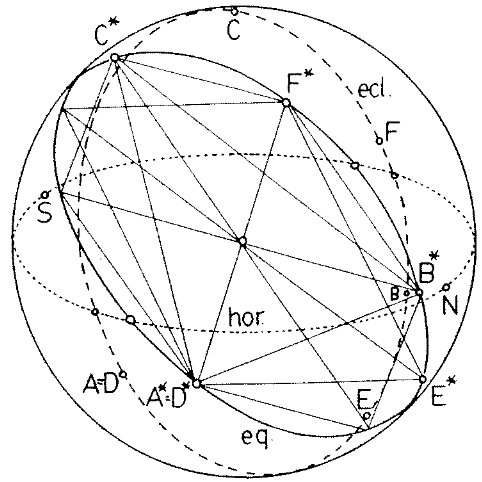

Figure 7.

other hand, one can also compute B, C and D by means of Tables 1 and 2. The computation of D from A can be characterized thus: we continuously subtract numbers $F(d,m)$ in Table 1 from $T(d) + T(d) + T(d)$, as suggested by the numerical example, until we obtain remainder zero. The fact that D and A coincide means that we obtain remainder zero after subtracting an entire row of numbers $F(d,m)$, $m = 1,2,\ldots,12$ in Table 1. Therefore $\Phi(d)$ must be equal to $T(d) + T(d) + T(d)$. A similar argument shows $\Phi(d) = 4Q(d)$ and $\Phi(d) = 6S(d)$. Therefore Al-Khwārizmī introduced his T, Q and S because he wanted any succession of three trine rays (or four quartile rays, or six sextile rays) beginning in A to return to the same point A.

If d is the first decan of a sign, we have $F(d,m+i) = F(d,m+i+6)$ for all i so that

$$F(d,m+i) + F(d,m+i+1) + \ldots + F(d,m+i+5) = \Phi(d)/2. \quad (3.16)$$

This means that if A casts a quartile ray at E and E casts a quartile ray at F, then A and F are diametrically opposite points, which makes astrological sense (compare Figures 1 and 7); a similar property holds for a succession of three sextile rays. No such nice properties are obtained if d is the second or third decan of a sign. This astrological

flaw is another consequence of the fact that Al-Khwārizmī (or whoever else compiled Table 1 in its present form) defined the numbers $F([m,2],m+3)$ and $F([m,3],m+3)$ in the fourth column of Table 1 by (3.10) and not by linear interpolation.

4. *The Majrīṭī tables*

The purpose of the Majrīṭī tables is the computation of the astrological rays from the longitude of the horoscopus λ_H and the longitude of the planet λ_P. As mentioned above, the tables were computed for a geographical latitude $\varphi = 38;30°$.[35] The Majrīṭī tables are a collection of 72 subtables. Each subtable is for a fixed λ_H, and there is a subtable for every multiple of 5° of λ_H (Table 3, taken from Suter 1914, p. 207 is for $\lambda_H = 20°$). The heading of the subtable for λ_H contains three numbers $s(\lambda_H)$, $q(\lambda_H)$ and $t(\lambda_H)$ for the sextile, quartile and trine rays; in Table 3 we find $s(20°) = 49;57°$, $q(20°) = 74;56°$ and $t(20°) = 99;54°$. Each subtable also contains 36 numbers $f(\lambda_H, \lambda_P)$ in six columns. Each column is for λ_P in the two diametrically opposite signs whose names appear above the column. In other words

$$f(\lambda_H, \lambda_P) = f(\lambda_H, \lambda_P + 180°). \quad (4.1)$$

If λ_H is in sign n (that is, $30n - 30° < \lambda_H \leq 30n°$), the first column is for λ_P in signs n and $n+6$, the second column is for λ_P in signs $n+1$ and $n+7$, and so on. For later use we call $m(\lambda_H)$ the maximal number in the subtable for λ_H; $m(\lambda_H) = f(\lambda_H, 30(n+5)°)$, $m(20°) = 149;32°$ according to Table 3, but we will see below that this number is a scribal error.

In the heading of each of the six columns the Latin text has the word "horoscopus". This is a translation of the Arabic *maṭāli*ᶜ, and should therefore be interpreted as "ascensions";[36] for typographical reasons, the abbreviation Asc. has been used in Table 3.

The accompanying text[37] explains how the tables are to be used. The procedure is as follows in modern notation. Suppose we wish to compute the position λ_T of the left trine ray. Using the subtable for λ_H we compute

$$a = f(\lambda_H, \lambda_P) + t(\lambda_H). \quad (4.2)$$

Twenty degrees Aries

Sextile 49;57° Quartile 74;56° Trine 99;54°

	Aries Libra Asc.	Taurus Scorpio Asc.	Gemini Sagitt. Asc.	Cancer Capric. Asc.	Leo Aquarius Asc.	Virgo Pisces Asc.
5	2;58	21;43	45;24	75;54	108;21	133;19
10	5;45	25;28	49;49	81;24	112;54	136;52
15	9;02	29;10	54;54	86;56	117;24	140;12
20	12;10	32;35	59;57	92;32	121;39	143;19
25	15;22	36;32	65;00	98;04	125;49	146;22
30	18;50	40;29	70;34	103;39	129;54	149;32

Table 3. Al-Majrīṭī's table for $\lambda_H = 20°$.

If $a > m(\lambda_H)$ we put $a' = a - m(\lambda_H)$, if $a \leq m(\lambda_H)$ we let $a' = a$.

We then find (by means of linear interpolation) a number λ_T such that $f(\lambda_H, \lambda_T) = a'$ and $\lambda_P < \lambda_T < \lambda_P + 180°$.

Examples for $\lambda_H = 20°$ (Table 3):

1. $\lambda_P = 40°$, then $a = 25;28° + 99;54° = 125;22° = a'$, $\lambda_T = 144;28°$.
2. $\lambda_P = 155°$, then $a = 133;19° + 99;54° = 233;13°$, $a' = 83;41°$, $\lambda_T = 282;4°$.

The same method is used for the left quartile and sextile rays. The text says that the right trine, quartile and sextile rays are diametrically opposite the left sextile, quartile and trine rays respectively.

The extant manuscripts do not contain information about the geometrical significance and the computation of the numbers in the Majrīṭī tables. Investigations by means of a personal computer led to the following insights in the structure of the functions f, q, s and t. For didactical reasons we will first state the conclusions, and present the numerical evidence afterwards.[38]

We first discuss the arithmetical structure of f, referring to Figure 6. Assume that the horoscopus is in sign n (i.e. $\lambda_H = 30(n-1)° + 5k$ for some k, $k = 1,2,3,4,5$ or 6, in Figure 6 $n = 3$, $k = 2$, $\lambda_H = 70°$). Let $\xi(i)$ be the latitude of the incident horizon NP_iS (see Section 2), that is the semicircle through N, S and the beginning of the i^{th} sign. Recall that $\xi(i) > 0$ if P_i is in the Eastern hemisphere, and $\xi(i) < 0$ for P_i in the Western hemisphere. Then the last number of the first column is

$$f(\lambda_H, 30n°) = A_{\xi(n)}(30n°) - A_{\xi(n)}(30(n-1)°) \quad (4.3)$$

and the last numbers in the $j+1^{th}$ column ($j = 1,2,3,4,5$) are defined by

$$f(\lambda_H, 30(n+j)°) = f(\lambda_H, 30(n+j-1)°) + A_{\xi(n+j)}(30(n+j)°) - A_{\xi(n+j)}(30(n+j-1)°). \quad (4.4)$$

The other $f(\lambda_H, \lambda_P)$ were obtained by very careless linear interpolation. We will not discuss them further.

Put $m(\lambda_H) = f(\lambda_H, 30(n+5)°)$ as above. The functions s, q and t are determined by

$$s(\lambda_H):q(\lambda_H):t(\lambda_H):m(\lambda_H) = 60:90:120:180. \quad (4.5)$$

Example: in Table 3 18;50° is the oblique ascension of the sign Aries for the latitude of the incident horizon through the beginning of Aries, 21;39° is the oblique ascension of the sign Taurus for the latitude of the incident horizon through the beginning of Taurus, the sum 40;29° = 18;50°+21;39° appears as the last number of the second column, and so on.

We now turn to the geometrical idea behind the arithmetical structure. Let P be any point on the ecliptic. As in section 3 we call the projection of P (on the celestial equator) the intersection P^* of the great semicircle NPS and the celestial equator (compare Figures 2, 5, 6). If λ_H is in sign n, then $f(\lambda_H, \lambda_P)$ is an approximation of the projection of the arc of the ecliptic between the beginning of the n^{th} sign and P. Here we assume $\lambda_H < \lambda_P \leq \lambda_H + 180°$; the case $\lambda_P > \lambda_H + 180°$ will be discussed below. Thus in Table 3, 40;29° is an approximation of the projection of the signs Aries plus Taurus on the celestial equator at the time when the horoscopus is 20° Aries. As in the standard computation (Section 2) and the Khwārizmī tables (Section 3), the basic idea is that these projections can be approximated as ascensions; this explains the words "ascensions" in the headings of the columns of Table 3. The projection of the i^{th} sign is approximated by the oblique ascension of the i^{th} sign for the latitude of the incident horizon through the beginning of the sign. This is to say that in the notation of Figure 6,

$$P_i^*P_{i+1}^* \approx A_{\xi(i)}(30i^\circ) - A_{\xi(i)}(30(i-1)^\circ). \tag{4.6}$$

This formula explains (4.3) and (4.4). The fact that (4.6) is only an approximation has as a consequence that arc $P_n^*P_{n+6}^*$, which is always equal to 180°, is "approximated" by the number $m(\lambda_H)$, which can be very different from 180°. Thus in Table 3, the projection of the arc between 0° Aries and 0° Libra is approximated by $m(\lambda_H) = 149;32°$.

We now discuss the reason why Al-Majrīṭī did not tabulate the $f(\lambda_H,\lambda_P)$ beyond $m(\lambda_H)$. Apparently he knew at least one of the following two identities.

$$P_i^*P_{i+1}^* = P_{i+6}^*P_{i+7}^* \tag{4.7}$$

$$A_{\xi(i)}(30i^\circ) - A_{\xi(i)}(30(i-1)^\circ) = A_{\xi(i+6)}(30(i+6)^\circ) - A_{\xi(i+6)}(30(i+5)^\circ). \tag{4.8}$$

Formula (4.7) is true because N, P_i, P_i^*, S, P_{i+6}, P_{i+6}^* are on one great circle, and so are N, P_{i+1}, P_{i+1}^*, S, P_{i+7}, P_{i+7}^*.

Therefore the approximation of $P_n^*P_{n+k+6}^*$ can always be obtained by adding to the approximation of $P_n^*P_{n+6}^*$ (that is $m(\lambda_H)$) the approximation of $P_{n+6}^*P_{n+6+k}^*$, which is equal to the approximation of $P_n^*P_{n+k}^*$ (by (4.7) or (4.8)). In Table 3 the projection of the first seven signs (0° Aries to 30° Libra) is equal to $m(\lambda_H) = 149;32°$ plus the projection of the sign Libra, which is equal to the projection of the sign Aries (i.e. 18;50°). Therefore there is no need to tabulate more than six columns.

The functions s, q and t in the Majrīṭī tables can be explained in a similar way as the functions S, Q and T of Al-Khwārizmī. Al-Majrīṭī wanted to make sure that a succession of three sextile rays (or of a trine and a sextile ray, or of two quartile rays) as in Figure 7 always arrives at the point diametrically opposite the planet. Because in general $m(\lambda_H) \neq 180°$, he had to introduce the functions s, q and t satisfying (4.5).

The Majrīṭī tables were compiled in order to replace the Khwārizmī tables. The two sets of tables are based on the same astrological doctrine, and two basic ideas are the same, namely that the "projections" on the celestial equator are approximated by ascensions, and that three functions (S, Q and T for Al-Khwārizmī, s, q and t for Al-Majrīṭī) have to be introduced in order to get astrologically meaningful results, such as: if A casts a trine ray at B, and B casts a trine ray at C, then C must cast a trine ray at A, etc. However, Al-Majrīṭī improved on Al-Khwārizmī in three respects.

1. Al-Khwārizmī approximated the quantities $A_{\xi(i)}(30i^\circ) - A_{\xi(i)}(30(i-1)^\circ)$ but Al-Majrīṭī computed these quantities exactly.
2. The $F(d,n)$,$F(d,n+1)$ etc. in the Khwārizmī tables are approximations of the projections of the individual signs n, $n+1$, etc., but Al-Majrīṭī computed approximations of the sums of signs (i.e. of sign n plus sign $n+1$, etc.). An astrologer working with the Majrīṭī tables had to make only one addition (and possibly one subtraction of $m(\lambda_H)$), whereas anyone using the Khwārizmī tables would have had to subtract quantities $F(d,m)$ several times.
3. The Majrīṭī subtables contain six columns, but the Khwārizmī tables have 12 columns, as a consequence of (3.10). This mathematical advantage has its astrological counterpart: suppose one computes a succession of two quartile rays (or of three sextile rays, or of a sextile and a trine ray) from a point A on the ecliptic, as shown by Figure 7. If one uses the Majrīṭī tables, the last ray in the row is cast at point F diametrically opposite A. This is not always the case if the Khwārizmī tables are used.

Before we describe our recomputation and compare the recomputed values with the text, one more problem has to be discussed. Neugebauer pointed out that the numbers in the text vary irregularly, and that they contain numerous errors.[39] In general it is not possible to distinguish between a scribal and a computational error. However, almost all scribal errors in the values $s(\lambda_H)$, $q(\lambda_H)$, $t(\lambda_H)$ and $m(\lambda_H)$ can be removed thanks to the relation (4.5). For example, in Table 3 (taken from Suter) we have $s(20°) = 49;57°$, $q(20°) = 74;56°$, $t(20°) = 99;54°$, $m(20°) = 149;32°$, which cannot all be correct. Because $s(20°):q(20°):t(20°) = 60:90:120$, these three values must be correct, and thus it turns out that 149;32° must be a scribal error for 149;52°. The scribal error is easy to explain, because in the Arabic *abjad*-notation

x	$\lambda_H = x$	$\lambda_H = 60+x$	$\lambda_H = 120+x$	$\lambda_H = 180+x$	$\lambda_H = 240+x$	$\lambda_H = 300+x$
5	147;09	163;59	199;18	206;50	178;00	151;38
10	147;32	167;45	201;50	205;08	175;21	148;45
15	148;36	170;07	203;55	204;49	173;18	148;25
20	149;52	172;43	205;51	202;30	169;05	146;42
25	151;26	174;17	206;02	201;05	166;53	146;27
30	152;30	178;55	208;00	200;39	163;33	146;05
35	152;43	183;33	208;26	197;55?	159;27	144;58
40	154;43	185;41	208;37	195;03	158;25	145;55
45	154;53?	189;39	208;57	193;10	154;38	145;55
50	157;28	193;15?	208;30	186;55	154;28	145;55
55	160;31	195;36?	207;21	182;03	152;58	146;15
60	163;50	197;47?	207;40	183;00	151;43	146;31

Table 4. Reconstructed values of $m(\lambda_H)$.

32 (*lām-bāʾ*) and 52 (*nūn-bāʾ*) are easily confused. In this way one can reconstruct almost all the values of $m(\lambda_H)$ which Al-Majrīṭī computed.[40] These restored values have been collected in Table 4. Question marks indicate values $m(\lambda_H)$ that could not be checked, because the corresponding $s(\lambda_H)$, $q(\lambda_H)$, and $t(\lambda_H)$ are missing in all manuscripts. The irregularities in Table 4 must be due to computational errors made by Al-Majrīṭī.

I now give the details of my computation of the Majrīṭī tables, referring to Figure 8. Let $\lambda_H = 30(n-1) + 5k°$ for fixed k, $1 \leq k \leq 6$, and let P be the point of the ecliptic such that $\lambda_P = 30(n+j-1)°$ for fixed j, $1 \leq j \leq 6$. Points N, E and S are the North, East and South points of the horizon, V is the vernal point, H is the horoscopus, C is the celestial North pole, and arc CR is perpendicular to the incident horizon NPS. Recall that $\varphi = \text{arc}\, CN$ is the geographical latitude, $\varepsilon = <HVE$ is the obliquity of the ecliptic.

The rising amplitude ω of the horoscopus and the angle ψ between ecliptic and horizon can be computed from λ_H and $A_\varphi(\lambda_H)$ by the sine rule in triangle VHE.

$$\sin\omega/\sin\varepsilon = \sin A_\varphi(\lambda_H)/\sin\psi = \sin\lambda_H/\cos\varphi. \qquad (4.9)$$

In triangle PHN we know $PH = \lambda_P - \lambda_H$, $<PHN = \psi$ and $HN =$

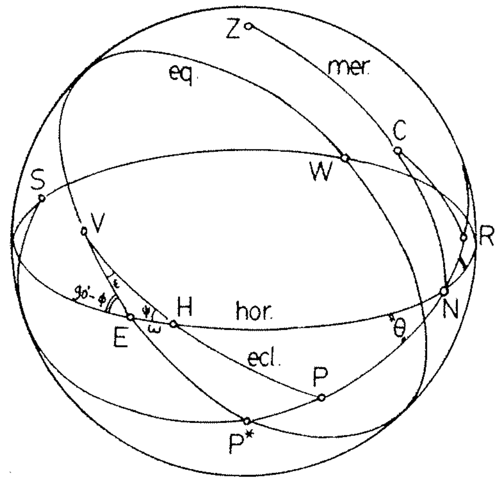

Figure 8.

$90° - \omega$. Therefore $<PNH = \theta$ can be determined by the cotangent rule:

$$\cos\omega\,\cot(\lambda_P - \lambda_H) - \cos\psi\,\sin\omega = \sin\psi\,\cot\theta. \qquad (4.10)$$

The latitude ξ of the incident horizon NPS can be found by the sine rule in $\triangle CRN$:

$$\sin\xi = \cos\theta\,\sin\varphi \qquad (4.11)$$

recall that $\xi = \pm CR$, the plus sign being used if R is in the Eastern hemisphere, and the minus sign if R is in the Western hemisphere.

The oblique ascensions in (4.3) and (4.4) can now be obtained by means of formulas (2.1)–(2.3), if we replace φ by ξ.

There are of course many equivalent ways to compute ξ, and it would be of interest to know what method Al-Majrīṭī used because the problem was rather difficult for his time. The crucial step in our argument is the computation of θ in triangle HNP by means of (4.10). This problem is mathematically equivalent to the determination of the qibla, that is the direction of Mecca, from the latitude of Mecca, the latitude of the locality, and the difference in longitude between Mecca and the locality (to see this, identify H with the north pole of the

λ_H	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
	1	2	3	4	5	6
5	18;24 (+1)	40;46 (-1)	70;39 (-4)	103;09 (-11)	128;03 (-14)	147;08 (+1)
10	18;26 (-1)	40;41 (-6)	70;26 (-1)	103;11 (-11)	128;30 (-65)	147;48(-16)
15	18;30 (-5)	40;40 (-1)	70;16 (+13)	103;17 (+1)	129;03 (+47)	148;35 (+1)
20	18;36 (+14)	40;42 (-13)	70;11 (+23)	103;26 (+13)	129;41 (+13)	149;31 (+21)
25	18;43 (-3)	40;47 (-32)	70;09 (+31)	103;39 (+26)	130;26 (+30)	150;34 (+52)
30	18;52 (-2)	40;55 (+10)	70;12 (+18)	103;56 (+19)	131;16 (+19)	151;45 (+35)
	2	3	4	5	6	7
35	22;04 (-4)	51;15 (-60)	85;13 (-10)	113;09 (-16)	134;03 (-20)	153;06 (-23)
40	22;07 (+13)	51;13 (+16)	85;25 (-5)	113;59 (-1)	135;19 (-6)	154;37 (+6)
45	22;11 (-6)	51;14 (-6)	85;39 (-81)	114;53 (-82)	136;44 (-73)	156;17 (-84)
50	22;18 (+7)	51;18 (+15)	85;56 (-28)	115;52 (-29)	138;16 (-23)	158;09 (-41)
55	22;26 (-1)	51;25 (+18)	86;14 (+11)	116;55 (+31)	139;57 (+44)	160;11 (+20)
60	22;37 (+18)	51;36 (+24)	86;36 (+29)	118;02 (+58)	141;46 (+89)	162;25 (+85)
	3	4	5	6	7	8
65	28;59 (+4)	64;08 (-10)	96;22 (-9)	120;51 (+3)	142;00 (-1)	164;51 (-52)
70	29;01 (-1)	64;19 (-9)	97;19 (+3)	122;39 (+8)	144;19 (+13)	167;27 (+18)
75	29;04 (-239)	64;28 (-7)	98;15 (-7)	124;29 (+6)	146;46 (-6)	170;13 (-6)
80	29;09 (+11)	64;38 (-48)	99;10 (+25)	126;21 (+32)	149;19 (-41)	173;09 (-26)
85	29;15 (+12)	64;47 (-65)	100;01 (-41)	128;13 (-53)	151;56 (-66)	176;11 (-114)
90	29;22 (+1)	64;55 (-9)	100;48 (-15)	130;03 (-17)	154;35 (-26)	179;18 (-23)
	4	5	6	7	8	9
95	35;32 (+8)	71;59 (+21)	102;18 (+30)	127;41 (+32)	152;56 (+52)	182;27 (+66)
100	35;29 (-4)	72;24 (+1)	103;46 (+7)	130;06 (-8)	155;54 (+4)	185;35 (+6)
105	35;24 (-6)	72;41 (+18)	105;05 (+24)	132;22 (+47)	158;48 (+56)	188;40 (+59)
110	35;18 (*)	72;50 (*)	106;13 (*)	134;29 (*)	161;33 (*)	191;37 (+98)
115	35;09 (*)	72;51 (*)	107;08 (*)	136;24 (*)	164;08 (*)	194;26 (+70)
120	35;00 (*)	72;45 (*)	107;49 (*)	138;05 (*)	166;31 (*)	197;02 (+47)
	5	6	7	8	9	10
125	37;42 (+8)	73;27 (+56)	104;42 (-17)	133;50 (0)	164;36 (+14)	199;26 (-8)
130	37;34 (+16)	73;52 (+18)	106;04 (+6)	135;56 (+6)	166;56 (-166)	201;34 (+16)
135	37;21 (+19)	74;05 (+35)	107;10 (+7)	137;45 (+40)	169;02 (+33)	203;26 (+29)
140	37;03 (+10)	74;06 (+27)	108;01 (+22)	139;19 (+24)	170;51 (+37)	205;03 (+48)
145	36;42 (+51)	73;56 (+57)	108;36 (-7)	140;37 (+5)	172;24 (-4)	206;22 (-20)
150	36;18 (+2)	73;35 (+20)	108;56 (+14)	141;38 (+22)	173;41 (+34)	207;25 (+35)
	6	7	8	9	10	11
155	37;14 (+11)	73;09 (+5)	106;32 (+20)	138;50 (+21)	172;20 (+6)	208;11 (+15)
160	37;04 (+1)	73;29 (-9)	107;30 (+10)	140;04 (+8)	173;19 (-7)	208;41 (-4)
165	36;49 (-6)	73;36 (-20)	108;14 (-10)	141;04 (-5)	174;04 (-2)	208;55 (+2)
170	36;27 (-1)	73;31 (-5)	108;44 (-3)	141;49 (-8)	174;35 (-16)	208;53 (-23)
175	36;01 (-3)	73;15 (-17)	109;00 (-22)	142;20 (-44)	174;51 (-65)	208;35 (-74)
180	35;31 (-1)	72;48 (-3)	109;02 (-2)	142;37 (-27)	174;53 (-18)	208;02 (-22)

Table 5. Recomputed values $f(\lambda_H, 30(n+j-1)^\circ)$, and the differences (values in the Latin text minus recomputed values). The sign number $n+j-1$ appears above each group of 6 values.

earth, P with Mecca, and N with the locality). Thus the computation of θ belonged to a class of problems that were studied in medieval Islamic science (but there is of course no evidence that Al-Majrīṭī recognized any relationship with the determination of the qibla, also no treatment of the *qibla* problem by him is known to us). I know of only two medieval texts where computation of ξ is described. Al-Bīrūnī's *Masudic Canon* contains a computation of ξ by means of the

λ_H	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
	7	8	9	10	11	12
185	37;14 (-16)	73;54 (-51)	107;44 (-56)	139;44 (-4)	172;17 (-22)	207;14 (-24)
190	37;04 (-9)	74;06 (+7)	108;10 (-57)	139;56 (-102)	171;52 (-62)	206;10 (-62)
195	36;47 (-11)	74;08 (-5)	108;26 (-11)	139;57 (-11)	171;15 (+1)	204;52 (-3)
200	36;25 (-10)	73;58 (-11)	108;29 (-74)	139;46 (-31)	170;26 (-41)	203;18 (-48)
205	35;56 (-16)	73;38 (-18)	108;21 (-26)	139;23 (-28)	169;25 (-30)	201;30 (-25)
210	35;21 (+12)	73;06 (+22)	108;01 (+32)	138;48 (+58)	168;11 (+65)	199;28 (+71)
	8	9	10	11	12	1
215	37;42 (*)	72;48 (*)	103;21 (*)	132;06 (*)	162;32 (*)	197;12 (+43)
220	37;33 (+7)	72;47 (+1)	103;07 (+11)	131;14 (+12)	160;47 (+26)	194;42 (+21)
225	37;17 (+13)	72;39 (+21)	102;46 (+39)	130;14 (+66)	158;55 (+75)	192;00 (+70)
230	36;54 (-44)	72;22 (-82)	102;17 (-82)	129;08 (-83)	156;55 (-68)	189;07 (-132)
235	36;26 (-51)	71;57 (-112)	101;40 (-111)	127;55 (-177)	154;49 (-176)	186;04 (-241)
240	35;51 (+11)	71;24 (+18)	100;55 (+27)	126;36 (+14)	152;37 (+8)	182;53 (+7)
	9	10	11	12	1	2
245	35;32 (-12)	64;54 (-9)	90;01 (-16)	115;11 (-16)	144;27 (-32)	179;38 (-98)
250	35;28 (-16)	64;41 (-1)	89;17 (-7)	113;58 (-18)	141;54 (-3)	176;20 (-59)
255	35;21 (+7)	64;28 (+10)	88;35 (+13)	112;09 (+19)	139;26 (+11)	173;03 (+15)
260	35;11 (-6)	64;13 (+2)	87;54 (-54)	110;44 (-49)	137;03 (-53)	169;50 (-45)
265	34;59 (-9)	63;58 (+2)	87;16 (-8)	109;24 (-24)	134;48 (+15)	166;43 (+10)
270	34;44 (-19)	63;43 (-28)	86;40 (-7)	108;11 (-8)	132;42 (-14)	163;45 (-12)
	10	11	12	1	2	3
275	28;59 (+3)	51;40 (0)	72;37 (-9)	96;19 (-19)	126;31 (-84)	160;59 (-92)
280	29;02 (+7)	51;29 (+22)	71;56 (+1)	94;54 (-7)	124;17 (-5)	158;27 (-2)
285	29;07 (+3)	51;23 (+5)	71;24 (+4)	93;41 (-105)	122;17 (-109)	156;08 (-90)
290	29;14 (-4)	51;23 (+37)	71;00 (+30)	92;41 (+24)	120;33 (+22)	154;05 (+23)
295	29;22 (+6)	51;27 (-7)	70;45 (+10)	91;53 (+3)	119;04 (+29)	152;17 (+41)
300	29;32 (+16)	51;35 (+13)	70;37 (+21)	91;17 (+41)	117;50 (+50)	150;44 (+59)
	11	12	1	2	3	4
305	22;04 (+14)	40;54 (+14)	61;08 (+35)	87;08 (+35)	119;42 (+61)	149;25 (+133)
310	22;08 (+7)	40;48 (+17)	60;40 (+15)	86;08 (+22)	118;25 (+25)	148;19 (+26)
315	22;14 (+66)	40;46 (+49)	60;20 (+40)	85;20 (+50)	117;19 (+51)	147;26 (+59)
320	22;22 (-17)	40;49 (-4)	60;07 (-17)	84;41 (-11)	116;24 (-4)	146;44 (-2)
325	22;32 (+18)	40;57 (+27)	60;00 (+10)	84;12 (+13)	115;40 (+15)	146;13 (+14)
330	22;44 (+6)	41;07 (+13)	60;00 (+10)	83;51 (+19)	115;04 (+16)	145;52 (+13)
	12	1	2	3	4	5
335	18;24 (+6)	37;07 (-7)	60;41 (-3)	91;40 (-12)	122;41 (-23)	145;39 (-41)
340	18;27 (+3)	37;02 (+8)	60;20 (+20)	91;06 (+24)	122;22 (+18)	145;35 (+20)
345	18;31 (+4)	37;01 (-6)	60;04 (+11)	90;38 (+17)	122;09 (+11)	145;39 (+16)
350	18;37 (-2)	37;03 (-3)	59;54 (+21)	90;16 (+19)	122;02 (+13)	145;51 (+4)
355	18;45 (0)	37;09 (-9)	59;48 (+2)	90;00 (+5)	122;01 (-1)	146;09 (+6)
360	18;54 (+6)	37;17 (+8)	59;47 (+8)	89;49 (-4)	122;05 (-4)	146;35 (-4)

Table 6. Recomputed values $f(\lambda_H, 30(n+j-1)^\circ)$, and the differences (values in the Latin text minus recomputed values). The sign number $n+j-1$ appears above each group of 6 values.

azimuth and altitude of P.[41] Al-Majrīṭī probably did not use this method, because the preliminary determination of the azimuth and altitude of P involves much unnecessary work. My second source is the anonymous 14th century *Shāmil Zīj*, where the solution of Al-Bīrūnī is repeated.[42] Whatever Al-Majrīṭī's method was, it must have involved a lot of numerical work.

The recomputed values $f(\lambda_H, 30(n+k)^\circ)$ are displayed in Tables 5 and 6. Each row in Tables 5 and 6 contains from left to right an argument $\lambda_H = 30n+5k°$ ($k = 1,2,3,4,5,6$) and the recomputed values $f(\lambda_H, 30(n+j-1)°)$ for $j = 1,2,3,4,5,6$ in degrees and minutes of arc. The numbers in parentheses are the differences in minutes, with the understanding that the recomputed value plus the difference is the value in the Latin text. For the sixth column, the recomputed value plus the difference is the restored entry in Table 4. Tables 5 and 6 are based on parameters $\varphi = 38;30°$, $\varepsilon = 23;51°$. Recomputed values for $\varphi = 38;30°$, $\varepsilon = 23;35°$ differ from the text in a systematical way, hence Al-Majrīṭī must have used the Ptolemaic value $\varepsilon = 23;51°$, just as Al-Khwārizmī did. Because the differences in Tables 5 and 6 change sign frequently, these differences can in all probability be attributed to random computational errors. The length of the computation is one likely cause of these errors, but a second cause is surely a certain carelessness on the part of Al-Majrīṭī.[43] Note, however, that the average difference of the recomputed value of $m(\lambda_H)$ (sixth column) minus the corresponding value in Table 4 is only −2 minutes.

By means of Tables 5 and 6 one can identify some further scribal and computational errors in the Latin text. The enormous difference in the first entry for $\lambda_H = 75°$ and the absence of such differences in the subsequent columns show that the value 25;05 in the text must be a scribal error for 29;05. For $\lambda_H = 230°$ the differences −82, −82 and −83 in the second, third and fourth column suggest that Al-Majrīṭī added two correct numbers to an incorrect number in the second column.

In spite of the computational errors, the Majrīṭī tables are an impressive piece of work, and an example of the positive influence of astrology on mathematics in the Islamic middle ages.

Acknowledgement

This paper has been much improved thanks to comments made by Professors E. S. Kennedy (Princeton), D. A. King (Frankfurt) and H. J. M. Bos (Utrecht) on an earlier version. They are not involved in any remaining inadequacies.

REFERENCES

Abū Ma'shar,
1985: *The great introduction to the science of astrology* (in Arabic), Frankfurt (ed. F. Sezgin, Publications of the Institute for the History of Arabic-Islamic Sciences, Series C, Facsimile editions, Vol. 21).

Al-Bīrūnī,
1956: *Al-Qānūnu'l-Mas'ūdī* (in Arabic), Hyderabad (Osmania Oriental Publications Bureau), 3 vols.

Al-Bīrūnī,
1976: = Abū Raikhan Beruni (973–1048), *Izbrannye Proizvedeniya* V, part 2 = Kanon Mas'uda, knigi VI–XI, transl. B. A. Rosenfeld, A. Ahmedov (in Russian), Tashkent.

A. Bouché-Leclercq,
1899: *L'Astrologie grecque*, Paris.

J. Drecker,
1925: *Die Theorie der Sonnenuhren*, Leipzig (= Band 1, Lieferung E of: E. von Bassermann-Jordan (ed.). *Die Geschichte der Zeitmessung und der Uhren*).

DSB,
1972–1980: = C. G. Gillispie (ed.), Dictionary of Scientific Biography, New York, 14 vols. plus suppl. plus indices.

GAS see F. Sezgin.

Ibn Hibintā,
1987: *The complete book on astrology* (= *al-Mughnī fī aḥkām al-nujūm*; in Arabic), ed. F. Sezgin in collaboration with M. Amawi, A. Jokosha, E. Neubauer. Frankfurt (Publications of the Institute for the history of Arabic-Islamic Sciences, Series C, Facsimile editions, Vols. 41,1 and 41,2).

E. S. Kennedy,
1956: A survey of Islamic astronomical tables. *Transactions of the American Philosophical Society, New Series* 46, pp. 121–177.

E. S. Kennedy, M. H. Kennedy,
1987: *Geographical coordinates of localities from Islamic sources*, Frankfurt (F. Sezgin ed., Veröffentlichungen des Institutes für Geschichte der Arabisch-Islamischen Wissenschaften, Reihe A, Band 2).

E. S. Kennedy, H. Krikorian,
1972: The astrological doctrine of projecting the rays, *Al-Abhath* 25, pp. 3–14. Reprinted in E. S. Kennedy, colleagues and former students, *Studies in the Islamic exact sciences*, Beirut 1983, pp. 372–383.

MAMS, see G. P. Matvievskaya, B. A. Rosenfeld.

G. P. Matvievskaya, B. A. Rosenfeld,
1985: *Matematiki i Astronomi Musulmanskogo Srednevekovya i ikh trudy (VIII–XVII vv.)* (Mathematicians and astronomers of the Islamic middle ages and their works) (in Russian), Moscow 1983, 3 vols.

A. Nallino,
1903: *Al-Battānī sive Albatenii opus astronomicum*, Vol. 1, Milano (Reprint Frankfurt 1969).

O. Neugebauer,
1962: *The astronomical tables of Al-Khwārizmī*, Copenhagen (Historisk-filosofiske skrifter udgivet af det Kongelige Danske Videnskabernes Selskab 4 (no. 2)).
O. Pedersen,
1974: *A survey of the Almagest*, Odense.
Ptolemy,
1980: *Tetrabiblos*, edited and translated by F. E. Robbins, Cambridge Mass. (Loeb Classical Library 435).
F. Sezgin,
1974–1979: *Geschichte des arabischen Schrifttums*, Band V: Mathematik, Leiden 1974, Band VI, Astronomie, Leiden 1978; Band VII: Astrologie – Meteorologie und Verwandtes, Leiden 1979.
H. Suter,
1914: *Die astronomischen Tafeln des Muḥammad ibn Mūsā al-Khwārizmī in der Bearbeitung des Maslama ibn Aḥmed al-Madjrīṭī und der latein. Übersetzung des Athelard von Bath auf Grund der Vorarbeiten von A. Bjornbø und R. Besthorn herausgegeben und kommentiert*, Kopenhagen (D. Kgl. Danske Vidensk. Selskab Skrifter 7. Række. Historisk og filosofisk Afd. III.1). Reprinted in H. Suter, *Beiträge zur Geschichte der Mathematik und Astronomie im Islam. Nachdruck seiner Schriften aus den Jahren 1892–1922,* Frankfurt 1986, 2 vols. (Veröffentlichungen des Institutes für Geschichte der arabisch-islamischen Wissenschaften Reihe B, Abt. Mathematik, Band 1,1 und 1,2).
G. J. Toomer,
1968: A survey of the Toledan tables, *Osiris* 15, 5–174.
J. Vernet, M. A. Catalá,
1965: Las obras mathemáticas de Maslama de Madrid, *Al-Andalus* 30, pp. 15–45, reprinted in J. Vernet, *Estudios sobre historia de la ciencia medieval,* Barcelona 1979, pp. 241–271.
Yaḥyā ibn Abī Manṣūr,
1986: The verified astronomical tables for the Caliph al-Ma'mūn (in Arabic), Frankfurt (F. Sezgin ed., Publications of the Institute for the History of Arabic-Islamic Sciences, Series C, Facsimile editions, Vol. 28).

NOTES

1. On the astrological doctrine of "casting the rays" see Bouché-Leclercq 1899, pp. 247–251 (antiquity); and Nallino 1903, pp. 307–313, Suter 1912, pp. 98–102, Kennedy and Krikorian 1972 (Islamic middle ages).
2. See Al-Bīrūnī 1956 vol. 3, pp. 1385–1392, Russian translation in Al-Bīrūnī 1976 pp. 470–474, modern summary in Kennedy and Krikorian 1972, pp. 6–7.
3. See Al-Bīrūnī 1956 vol. 3, pp. 1378–1379 line 7, Russian translation in Al-Bīrūnī 1976, pp. 465–466.
4. Kennedy and Krikorian 1972, p. 5.
5. See Al-Bīrūnī 1956 vol. 3, p. 1385, Russian translation in Al-Bīrūnī 1976, p. 470.
6. On Al-Khwārizmī see GAS V, pp. 228–241, GAS VI, pp. 140–143, MAMS II, pp. 40–45 and the article by G. J. Toomer in DSB VII, pp. 358–365.

7. On Ibn Hibintā see GAS VII, pp. 162–164. A facsimile of the Khwārizmī tables is in Ibn Hibintā 1987, Vol. 2. pp. 69–74.
8. Noted by Nallino 1903, p. 312 and Suter 1914, p. 100.
9. The tables were described in Toomer 1968, pp. 147–151, and they were identified by Kennedy and Krikorian 1972, p. 4.
10. Kennedy and Krikorian 1972, pp. 8, 12.
11. Toomer 1968, pp. 148–151, Kennedy and Krikorian 1972, pp. 9–12.
12. On Maslama ibn Aḥmad al-Majrīṭī see GAS V, pp. 334–335, GAS VI, p. 226–227, MAMS II, pp. 194–195 (al-Majrīṭī means: from Madrid). On his revision of the Zīj of al-Khwārizmī see Suter 1914, pp. vii–x.
13. In transcriptions of sexagesimal numbers we separate the integer from the fractional part by means of a semicolon, and we use a comma to separate the first from the second sexagesimal, thus: 23;51,20°. The latitude $\varphi = 38;30°$ is mentioned in the heading of the first table, see Suter 1914, p. 206. Professor King points out that Al-Majrīṭī also used $\varphi = 38;30°$ in a table for astrolabe construction in some notes of his on Ptolemy's *Planisphaerium* in ms. Paris, Bibliothèque Nationale 4821, f. 81r, published in Vernet and Catalá.
14. Compare Suter 1914, p. 100. On the latitude of Cordoba in medieval sources, see Kennedy and Kennedy 1987, pp. 95, 697. The correct value is 37;53°.
15. Suter 1914, pp. 206–229 (reprinted in Suter 1986 vol. 2. pp. 704–727), compare Table 3 below.
16. Suter 1914, p. 100, Neugebauer 1962, pp. 130–131, Kennedy and Krikorian 1972, pp. 3, 8–9.
17. See Kennedy 1956, p. 140, the forthcoming article "Maṭāliᶜ" by D. A. King in the *Encyclopaedia of Islam*, Leiden (Brill), 2nd edition, and Pedersen 1974, pp. 99–101, 110–115.
18. The term is found for example in the *Shāmil Zīj*, ms. Paris, Bibliothèque Nationale, Fonds Arabe 2528, f. 9a line 23 (see GAS V, 324 no. 9). The *Shāmil Zīj* consists essentially of the 13th century *Athīrī Zīj* of Athīr al-Dīn al-Abharī (including Al-Abharī's preface, but without mention of his name) plus a few astrological sections taken mainly from the *Masudic Canon* of Al-Bīrūnī. The passage occurs in such an astrological section.
19. For a list see Kennedy and Krikorian, pp. 3–4. The description in the *Mumtaḥan Zīj* has now been printed in Yaḥyā ibn Abī Manṣūr 1986, pp. 191–194 (note that the editor of the facsimile has rearranged the folios without comment). The standard method is also to be found in Abū Maᶜshar 1985, pp. 408–410.
20. Nallino 1903, p. 311 (based on Abū Maᶜshar); Kennedy and Krikorian 1972, p. 5, based on Al-Bīrūnī 1956 vol. 3, pp. 1379–1384. Instead of "add e to whichever is smaller, P_0 or P_φ" in Kennedy and Krikorian 1972, p. 5 read "add e to P_0 if $P_0 < P_\varphi$, subtract e from P_0 if $P_0 > P_\varphi$".
21. Al-Bīrūnī uses the term "intermediate ascension" (*maṭāliᶜ mutawassiṭa*) for A_ξ in 1956 vol. 3, p. 1384 line 13.
22. Abū Maᶜshar 1985, p. 407 line 3.
23. Nallino 1903, p. 311.
24. Ptolemy 1980. pp. 290–305, see the next footnote.
25. Ptolemy 1980, p. 290 lines 12–16, translation on p. 291, lines 12–16. On hour lines see Drecker 1925, pp. 12–20. Ptolemy does not express any opinion as to the question whether the hour lines are great circles not passing through N and S.

Because Ptolemy's text in the *Tetrabiblos* (Ptolemy 1980 pp. 290–305) is not very clear, I explain his computation in modern notations for the case where P and Q are above the horizon in the Eastern half of the celestial sphere. Similar formulas hold for other positions of P and Q. Assume that φ, ε, λ_P, λ_Q and λ_K (or λ_H, cf. (2.5)) are known and that one has to compute the time-interval u it takes Q to reach the great circle NPS in Figure 2 by means of the daily rotation. In general $u \neq P^*Q^*$.

Ptolemy first computes $x(P)$, that is the distance from P to the meridian in seasonal hours:

$$x(P) = (A_0(\lambda_P) - A_0(\lambda_K))/h(\lambda_P),$$

where $h(\lambda_P)$ is the length of one seasonal hour when the sun has longitude λ_P. One seasonal hour is one-sixth of the period between sunrise and noon. Therefore

$$h(\lambda_P) = (1/6)\cdot(90^\circ + A_0(\lambda_P) - A_\varphi(\lambda_P)). \quad (2.7)$$

Similarly for point Q

$$x(Q) = (A_0(\lambda_Q) - A_0(\lambda_K))/h(\lambda_Q).$$

Suppose that Q arrives after a rotation of u time-degrees at point P' on semicircle NPS.

Then it takes Q $x(Q)-x(P')$ seasonal hours to reach NPS, so that $u = (x(Q)-x(P'))\cdot h(\lambda_Q)$.

Semicircle NPS is almost an hour line, that is to say that $x(P) \approx x(P')$. Therefore

$$u \approx (x(Q)-x(P))\cdot h(\lambda_Q),$$

or

$$u \approx (A_0(\lambda_Q)-A_0(\lambda_K)) - (h(\lambda_Q)/h(\lambda_P))\cdot(A_0(\lambda_P)-A_0(\lambda_K)). \quad (2.8)$$

Ptolemy restates (2.8) in the following form (1980 pp. 300–305):

$$u \approx (A_0(\lambda_Q)-A_0(\lambda_P)) + k\cdot\{A_\varphi(\lambda_Q)-A_\varphi(\lambda_P) - (A_0(\lambda_Q)-A_0(\lambda_P))\}$$

with

$$k = \{A_0(\lambda_P)-A_0(\lambda_K)\}/\{A_0(\lambda_P)-A_0(\lambda_R)\}. \quad (2.9)$$

R is the point of intersection of the ecliptic and the meridian if we turn the celestial sphere such that P is on the Eastern horizon. Compare Ptolemy 1980, pp. 300–303.

26. Kennedy and Krikorian 1972, pp. 7–12, for the Arabic text see Ibn Hibintā 1987 Vol. 2, pp. 66–68.
27. Toomer 1968, p. 147.
28. Kennedy and Krikorian 1972, pp. 13–14.
29. Toomer 1968, pp. 148–149, Kennedy and Krikorian 1972, pp. 9–10. The correct values of the oblique ascensions for the first six signs are as follows for $\varepsilon = 23;51^\circ$, $\varphi = 33^\circ$: 20;08°, 23;33°, 29;38°, 34;56°, 36;15°, 35;32°. Table 1 is based on the values 20;05°, 23;34°, 29;40°, 34;52°, 36;14°, 35;35°, see the first column.
30. On the latitude of Baghdād in medieval Islamic sources see Kennedy and Kennedy 1987, p. 55.
31. Suter 1914, pp. 19, 58; Neugebauer 1962, p. 47.
32. Kennedy and Krikorian 1972, p. 11.
33. Kennedy and Krikorian 1972, p. 10.
34. Variations of a few minutes of arc occur as a consequence of rounding errors and due to linear interpolation. For each sign m, the values $T([m,2])$, $T([m,3])$ etc. seem to have been computed by linear interpolation between $T([m,1])$ and $T([m+1,1])$.
35. See footnotes 13 and 14 above.
36. The Latin translator always rendered *maṭāliʿ* (meaning: ascensions) as *horoscopus* or *horoscopi*, and in the first table of the Majrīṭī tables, two manuscripts have the variant reading *matale* for *horoscopus* (see Suter 1914, p. 206 note 10 and the index on p. 244 under *matale*). The standard Arabic term for horoscopus (meaning: the point of the ecliptic which is instantaneously rising) is *ṭāliʿ*.
37. Suter 1914, pp. 30–31, translated by Neugebauer 1962, pp. 79–80. Neugebauer uses the term "aspect": if a planet casts a sextile ray to another planet, the two planets form a sextile aspect, etc.
38. Compare the recomputation below, p. 194.
39. Neugebauer 1962, p. 80.
40. The following corrections have to be made to Suter's 1914 edition (Suter's reading is in parentheses). Scribal errors that were already corrected by Neugebauer (1962, p. 130) are indicated by the letter N.

We write λ for λ_H and we omit the degree signs and the arguments of the functions s, q, t, m.

$\lambda = 10^\circ$ s = 49;11 N (49;3) t = 98;22 N (98;32), m = 147;32 (147;22).
$\lambda = 20^\circ$ m = 149;52 (149;32).
$\lambda = 30^\circ$ m = 152;20 (152;43).
$\lambda = 35^\circ$ t = 101;48 (101;70).
$\lambda = 40^\circ$ s = 51;34 N (51;24).
$\lambda = 45^\circ$ the values for s, q and t in the text are for $\lambda = 60^\circ$.
$\lambda = 50^\circ$ t = 104;59 (104;09).
$\lambda = 60^\circ$ q = 81;55 (81;45).
$\lambda = 95^\circ$ t = 122;22 N (122;12).
$\lambda = 105^\circ$ s = 63;13 (63;53), t = 127;26 (127;46).
$\lambda = 110^\circ$, 115°, 120°: the entries for f in the text are in fact for $\lambda = 95^\circ$, 100°, 105°. The values for m in Table 4 have been reconstructed from s, q and t.
$\lambda = 115^\circ$ s = 65;12° (65;42).
$\lambda = 130^\circ$ q = 100;55 (100;57).
$\lambda = 175^\circ$ s = 69;7 (69;50), t = 138;14 (138;26).
$\lambda = 180^\circ$ s = 69;13 (69;53).
$\lambda = 215^\circ$ s = 65;58 (65;28); the table for f is in fact for $\lambda = 220^\circ$. The value for m in Table 4 has been reconstructed from s, q and t.
$\lambda = 220^\circ$ the table for f in the text is in fact for $\lambda = 225^\circ$.
$\lambda = 225^\circ$ the table for f in the text is in fact for $\lambda = 230^\circ$.
$\lambda = 230^\circ$ q = 93;27 N (94;27), t = 124;37 (124;24), m = 186;55 (187;55), note that the correct value for m occurs in the previous table (with heading $\lambda = 225^\circ$).
$\lambda = 235^\circ$ s = 60;41 N (60;40).
$\lambda = 240^\circ$ s = 61;0 (60;0), q = 91;30 (90;30), t = 122;0 (119;22).
$\lambda = 265^\circ$ m = 166;53 (166;13).
$\lambda = 275^\circ$ t = 106;18 (107;18).
$\lambda = 290^\circ$ m = 154;28 (154;48).