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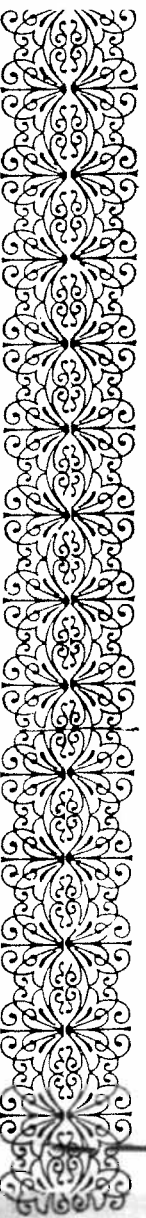
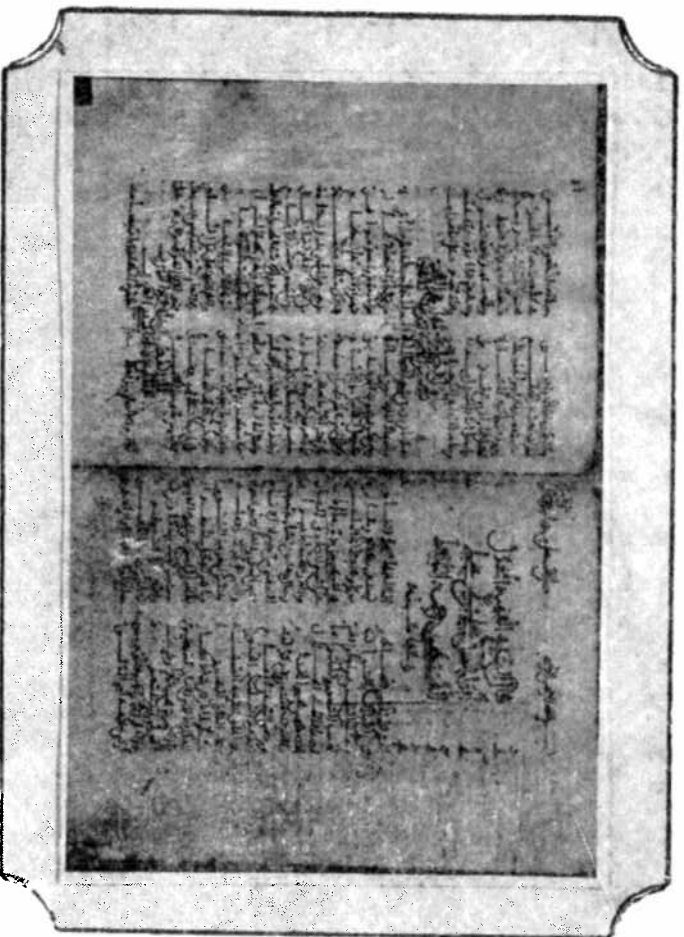
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Rearranging the Arabic Mathematical and Astronomical Manuscript Bankipore 2468

J. P. HOGENDIJK*

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At the request of Dr. J. J. Wilkern, keeper of the Oriental manuscripts in the Library of the University of Leiden, the staff of the Oriental Public Library in Patna kindly sent a microfilm of MS Bankipore 2468. Professor G. J. Toomer, Providence, lent me his excellent photographs of the manuscript during my visit to Providence in 1981, which was financially supported by the Netherlands Organization for the Advancement of Pure Research (Z.W.O.). The Chester Beatty Library, Dublin, sent a microfilm of Arabic MS 3652 to Leiden, and Professor Sezgin, Frankfurt Main showed me his copies of MS Istanbul, Resit 1191. Professor E. S. Kennedy and Dr. R. Lorch, Aleppo, Dr. H. J. M. Bos, and Dr. Kruck, Utrecht and Professor G. Saliba, New York, made helpful suggestions. Miss S. M. McNab, Utrecht, made linguistic improvements. The support of the above-mentioned persons and institutions is gratefully acknowledged.

1. Introduction

The Arabic manuscript Bankipore 2468 (now 2519)¹ in the Khuda Bakhsh Oriental Public Library in Patna (India) consists of a valuable collection of over forty treatises by medieval Islamic mathematicians and astronomers. The greater part of the manuscript was written in 631-2 H./1234 A.D. in Mosul.

Somewhere in its history the manuscript fell apart and several leaves were lost. It was rebound in an incorrect order; as a consequence the leaves of several treatises of Al-Birūnī² (362 H./972 A.D.-440 H./1048 A.D.) and Ibrāhīm ibn Sinān³ (296 H./909 A.D.-335 H./946 A.D.) are displaced.

The Osmania Oriental Publications Bureau in Hyderabad printed the disarranged parts of the manuscript in the *Rasā'il al-Birūnī* (1367 H./1948 A.D.)⁴ and the *Rasā'il ibn Sinān* (same year),⁵ following in most cases the incorrect

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1. See Maulvi Abdul Hamid, *Catalogue of the Arabic and Persian Manuscripts in the Oriental Public Library at Bankipore*, vol. 22, Arabic Mss, Science (Patna, 1937), pp. 60-92.

2. On the life of Al-Birūnī see the article by E. S. Kennedy in *DSB* II, 147-158. The mathematical and astronomical works of Al-Birūnī have been listed in *GAS* 5, 375-383 and 6, 261-267. and D. J. Boilot, "L'enquête d'al-Birūnī: Essai bibliographique", *Mélanges Institut Dominicain d'Etudes Orientales du Caire*, 2 (1955), 161-256. Complete bibliographical references to *GAS* and *DSB* are in section 3.

3. On Ibrāhīm ibn Sinān see *GAS* 5, pp. 292-295 and 6, pp. 193-195.

4. Complete references are in section 3.

order of the leaves in the manuscript. Thus the printed text is not in the correct order in the *Rasā'il al-Birūnī* in "*Istihraj al-Awtār*" (the Extraction of Chords) and "*Jfrād al-Maqāl fi Amr al-Ḥiṭā'*" (the Exhaustive Treatise on Shadows) and in the *Rasā'il ibn Sinān* in "*Kiāb fi Harakāt al-Shams*" (Book on the Movements of the Sun) and "*Al-Handasa wa-ʿilm al-Nujūm*" (Geometry and Astronomy). The parts of the manuscript that were printed as "*Istihraj al-Awtār*" and "*Al-Handasa wa ʿilm al-Nujūm*" contain fragments of three works which do not exist elsewhere. These can be identified as

1. "Treatise on the Solution and the Division of the (solar) Equation" (*Maqāla fi'l-taḥlīl wa'l-taqṭi' fi'l-ta'ḍīl*) by Al-Birūnī,
2. the "Exquisite Problems" (*Al-masā'il al-mukhāra*) by Ibrāhīm ibn Sinān,
3. "Treatise on (the fact) that the necessities of the infinite subdivisibility of magnitudes are related to the matter of the two lines which approach each other but do not meet in the distance" (*Maqāla fi anna lawāzim tajāzū' al-maqādir ilā lā nihāya qarība min amr al-khātayn alladhayn yaqrubān wa-lā yalaqayān fi'l-istib'ād*), a work by Al-Birūnī on parallels.

Most of what has been mentioned above was already described in 1960 in a remarkable article by Saidan.⁶ Relying completely on the printed texts in the *Rasā'il al-Birūnī* and the *Rasā'il ibn Sinān*, Saidan attempted to re-establish the correct order of the treatises. He correctly identified the fragments of 1. and 2., without, however, giving detailed evidence for the identification. The fragment of 3. was identified correctly by Saidan in 1973 and also by Bulgakov and Ahmedov in 1977.⁷ It should be noted that the fragment of 1. was also identified correctly by Hermelink in 1956.⁷ Unfortunately these results have not yet been incorporated in F. Sezgin's *Geschichte des Arabischen Schrifttums*.

Following the suggestion made by Saidan in his 1960 paper I have studied the manuscript Bankipore 2468. This paper contains the result of my research. It appears that the manuscript can be divided into five disconnected parts.

5. A.S. Saidan. "The *Rasā'il* of Birūnī and Ibn Sinān, A Rearrangement", *Islamic Culture*, 34 (1960), 173-175.
6. A. S. Saidan, "The Trigonometry of al-Birūnī", *Al-Birūnī Commemoration Volume*, ed. Harkim Muhammad Said (Karachi: Hamdard Academy, 1973), p. 690, and P. G. Bulgakov, A.A. Ahmedov, "Bernūnī Al-Kindī o teorīi paraleln'kh" (in Russian), *Obščestvennye nauki i Učebnye iskanie* (1977), 30-36.
7. H. Hermelink. "Al-Birūnī: Lehrbücher. Vier Abhandlungen aus der mathematisch-astronomischen Sammelhandschrift Bankipore Nr. 2468", *Zentralblatt für Mathematik und ihre Grenzgebiete* 54 (1956) 2.

In section 2 I shall discuss this division and shall investigate what happened to the manuscript when it was rebound.

Section 3 consists of a list of all the treatises and fragments in the manuscript (with the exception of the last leaf), in the correct order and with bibliographical references. Because the numbering on the manuscript corresponds to the present incorrect order of the leaves, I have devised a "restored" numbering corresponding to the original correct order of the leaves. All treatises will be listed in dual numbering. Saidan's statements on the order of the disarranged treatises appear to be correct, apart from a very few exceptions.

M. Dimirdāsh used MS Bankipore 2468 for his edition of the Arabic text of the "Extraction of Chords" of Al-Birūnī. However, Dimirdāsh did not fully realize to what extent the manuscript had become disarranged; thus his edition also contains parts of 1. and 2. Detailed references will be given in section 3.

Sections 4-6 deal with the three fragments in the manuscript. In section 4 I shall attempt to prove by means of references in other works of Al-Birūnī that one fragment is part of 1. So Hermelink and Saidan correctly identified this fragment.

Section 5 deals with a second fragment, which was identified by Saidan as part of the "Exquisite Problems" of Ibrāhīm ibn Sinān. I shall attempt to prove that this identification is correct by means of a passage in a work of Al-Sijzi⁸ in which Al-Sijzi refers to the "Exquisite Problems" of Ibrāhīm ibn Sinān. I give English translations of the reference and of the passage in the "Exquisite Problems" to which reference is made. These translations may also give the reader an impression of the contents of the "Exquisite Problems".

Section 6 consists of a brief discussion of the reasons why the third fragment probably is the above-mentioned work 3. of Al-Birūnī.

2. The manuscript and the correct order of its leaves

The first 324 leaves of the manuscript are numbered 1-326 in eastern Arabic numbers.⁹ There are no leaves numbered 218 and 219, but there is no interruption.

8. On Al-Sijzi see *GAS* 5, pp. 329-334.

9. The manuscript has apparently been rebound again in recent times. Professor Toomer's photographs show the effects of a second rebinding; at the time that these were taken the leaves of the manuscript were in the following order: 1-262, 264, 266, 263, 265, 267-304, 308, 306, 307, 305, 309-313, 315, 314, 317, 316, 318-326, last leaf.

Thus on the photograph f. 262b appears next to f. 264a, f. 264b appears next to f. 266a, et cetera.

No effects of the second rebinding are visible on the film which the Oriental Public Library sent to Leiden in 1980; on this film f. 262b is next to f. 263a etc.

Somebody attempted to change the numbers 263, 264, 265, and 266 into 264, 265, 266, 267 respectively. I refer to the original numbers (these are still legible).

tion in the text between f. 217b and f. 220a.¹⁰ The leaves were numbered after being rebound in the wrong order, so the numbers are of no help in establishing the correct order of the treatises. The last leaf of the manuscript is not numbered.

The manuscript can be divided into five continuous parts A, B, C, D and E. These parts consist of the following leaves in the following order:

A = 40—124, 323, 1, 131—195, 125—130, 196—217, 220—298, 326.

B = 325, 322, 299—305, 309—316.

C = 324, 321, 318, 320, 319, 306—308, 2—39.

D = 317.

E = the last leaf of the manuscript.

A contains 40 complete treatises. These are numbered 1-40 in the margin of the manuscript in eastern Arabic numbers. A list of the 40 treatises is in section 3.

All treatises in A were written in Mosul. The copyist wrote on f. 188b that he finished the first 30 treatises in Muharram 632 H./Sept.-Oct. 1234 A.D. The remaining treatises 31-40 are dated separately: nos. 31 and 32 (f. 189a-193b) were written in Safar 632 H./Oct.-Nov. 1234 A.D., nos. 33 and 34 (194a-195, 125-130, 196-244b) in Dhū'l-Hijja 631/Aug.-Sept. 1234, no. 35 (245a-266b) in Dhū'l-Qa'da 631/July-Aug. 1234, nos. 36-38 (267a-280a) in Muharram 632/Sept.-Oct. 1234, no. 39 (280a-282a) in Safar 632/Oct.-Nov. 1234, and no. 40 (282b-298, 326a) at the end of Dhū'l-Qa'da 631/Sept. 1234. So the order of the treatises in A and their marginal numbers do not correspond to the order in which they were copied. But it is likely that the same copyist who wrote the treatises also numbered them, because the numbers in the margin are written in exactly the same way as the numbers in the text.

At the top of some of the leaves of A one can make out Arabic letters whose numerical values indicate the numbers of gatherings. These letters and their numerical values are rendered below, because they show what happened to A when it was rebound incorrectly.

The tops of many pages of the manuscript are damaged. But one can read a fragment of a ا (3), a fragment of a ب (4), a clear ج (6), a fragment of a د (7), and a ه (9) on ff. 54, 62, 78, 86, 102 respectively; و (10), ز (11), ح (13), ط (14), ث (15), ث (16), ث (17) and ث (19) on ff. 110, 118, 138, 146, 154, 162, 170, 186 respectively; and ذ (20), ذ (21), ذ (22), ذ (23), ذ (24), ذ (25), ذ (26), ذ (27), ذ (28) and ذ (29) on ff. 194, 196, 204, 212, 221, 229, 237, 245, 253 and 261 respectively.

10. See E. S. Kennedy, *The Exhaustive Treatise on Shadows by Abū al-Rayḡān Muḥammad b. Ahmad al-Birūnī* (Aleppo: JHAS, 1976), vol. 1 (translation), p. 174 note 4.

These numbers correspond to a division of the first part of A into 29 gatherings, almost all consisting of 8 leaves, in the following way:

40-45 (?), 46-53, 54-61, 62-69, 70-77, 78-85, 86-93, 94-101, 102-109, 110-117 (nos. 1-10); 118-124 + 323, 1 + 131-137, 138-145, 146-153, 154-161, 162-169, 170-177, 178-185, 186-193, 194-195 + 125-130 (nos. 11-20); 196-203, 204-211, 212-217 + 220, 221-228, 229-236, 237-244, 245-252, 253-260, 261-268 (or 266?) (nos. 21-29).

We can now describe the effect of the rebinding on A as follows: The gatherings 11, 12 and 20 fell apart and the pieces were rebound in the wrong order.

The situation regarding the rest of A is more obscure. At the top of f. 267a there is a clear ج (36). This may be a scribal error resulting from the fact that the treatise numbered 36 also begins on f. 267a. F. 269 has a د (30) with perhaps another (illegible) letter attached to it, f. 282 has clearly و (32). I have not found other numbers indicating gatherings of A, which does of course not imply that such numbers never existed. The text on ff. 267a-282a and 282b-298 + 326 is continuous.

B, C and D are undated fragments of texts, written in the same hand as A.¹¹ I have not found any letter indicating a gathering, nor any marginal number in B, C and D. It is therefore conceivable that B, C and D are remainders of what was originally a separate manuscript to which A did not belong. The contents of B, C and D will be listed in section 3.

E is a fragment of a treatise on stellar constellations written in Persian in another hand and obviously at a later date. I shall not discuss it further.

At the very beginning of the manuscript there is an index, which was also compiled at a later date. This index must have been added after the manuscript had been rebound, because it corresponds to its present state.

3. The contents of A, B, C and D

This section is a list of the treatises and fragments in A, B, C and D.

The notation "A 1 *Ca* 4 (p 63) f 40a-40b rest la-3b" means that the relevant treatise is part of A, that it is numbered 1 in the margin of the manuscript, 4 in the catalogue¹ (on page 63) and the secondary literature, that it is on ff. 40a-42b in the numbering of the manuscript but on ff. la-3b in the "restored" numbering of A. I have devised this "restored" numbering in such a way that it corresponds to the correct order of the leaves.

11. Another manuscript written by the scribe of A, B, C and D is MS Berlin. Ahlwardt 5658, now Tübingen. Or. Quart. 71, containing the *Stellar Constellations* (Sunar al-Kawākib) by 'Abdarrahmān al-Sufī. See GAS 6, 214 and the facsimile of the colophon of this manuscript (dated 630 H., Mosul), plate 10 in:

Abū'l-Ḥusayn 'Abdūr-Raḥmān as-Sūfī, Sunaru'l-Kawākib or *Uranometry*, edited from the oldest extant Mss. and based on the Uigh Beg Royal Codex, Hyderabad (Dār'at al-Ma'sūfī al-'Uthmāniyya), 1373 H./1954 A.D.

The title of every treatise will be rendered in Arabic, exactly as it occurs in the manuscript, and also in English translation. I have made some explanatory additions in brackets. Arabic names will be rendered in abbreviated form in the translations; thus, for example, Abū'l-Rayhān Muḥammad ibn Ahmad al-Bīrūnī will be abbreviated to ... Al-Bīrūnī.

Reference will be made to such modern editions, translations and other relevant publications as are known to me. The cyrillic alphabet will be transcribed according to the system used in the *Mathematical Reviews* and the *Zentralblatt für Mathematik*.

Practically the whole manuscript has been printed (in Arabic) by the Osmania Oriental Publication Bureau (Dār'irat al-Mā'arif al-'Uthmāniyya) in Hyderabad, in several volumes. These volumes will be abbreviated as indicated below. The notation "XY p, q:r" always refers to line r of page q of the p-th text in volume XY.

RB = Rasā'ilu'l-Bīrūnī. *Containing four tracts*. 1367 H./1948 A. D.

RH = Majmū'at-Rasā'il li'l-'allāma al-fāsiḥif Abū 'Alī al-Ḥasan ibn al-Ḥaytham, 1358 H. plus the appendix: Risāla fī kharā'ij muthallāth min jihat al-'amād. 1366 H./1947 A. D.

RI = Rasā'ilu ibn-i-Sinān, by Ibrāhīm b. Sinān b. Thābit b. Qurra al-Ḥarrānī, containing six tracts. 1367H./1948 A. D.

RM = Rasā'ilu 'l-Muqarrifa fī'l-Hai'at li'l-mutaqaddimā wa mu'asiray il-Bīrūnī. *Containing eleven important treatises on astronomy and other subjects contributed by the famous predecessors and contemporaries of Al-Bīrūnī* (9th, 10th, 11th century A. D.). 1367 H./1948 A. D.

RN = Rasā'il Abi Naṣr ilā 'l-Bīrūnī, by Abū Naṣr Mansūr b. 'Alī b. 'Iṣḡ, for Al-Bīrūnī. *Containing fifteen tracts*. 1367 H./1948 A. D.

RT = Rasā'ilu ibn Qurra, by Thabit ibn Qurra al-Ḥarrānī. *Containing translations of two geometrical tracts of Archimedes*. 1366 H./1947 A. D.

Tnb = Muḥammad b. al-Ḥasan al-Karkhī, *Inḥā' al-miyāh al-ḥaqīqya*. 1359 H./1940 A. D.

The following abbreviations will also be used.

GAS = F. Sezgin, *Geschichte des Arabischen Schrifttums* (Leiden, Brill). Band 5, *Mathematik*, 1974; Band 6, *Astronomie*, 1978; Band 7, *Astrologie, Meteorologie und Verwandtes*, 1979.

DSB = *Dictionary of Scientific Biography*, 15 vols (New York: Scribner's Sons, 1972-78).

EI2 = *Encyclopaedia of Islam*, second edition. (Leiden-London: Brill, 1960 - ...).

Cat. (or catalogue), see footnote 1.

Dimirdash m n = line n of page m of *Ishtihāj al-anwār fī'l-dā'ira bi-kharā'ij al-ḥaḥf al-munḥana' al-wāqif' fīhā*. Taṣḥif Abī'l-Rayhān Muḥammad ibn Ahmad al-Bīrūnī. *Talḥiq al-usūḍ al-Ahmad Sa'īd al-Dimirdāsh. Murāja'at al-usūḍ al-'Abdalḥamīd Luḡī* (in Arabic). (Cairo, no date). This is the edition by Dimirdāsh of Al-Bīrūnī's *Extraction of Chords*.

Kennedy = E. S. Kennedy, "A survey of Islamic astronomical tables." *Transactions of the American Philosophical Society*, New Series, vol. 46, part 2. (Philadelphia, 1956).

Sansó = J. Sansó Moya, *Estudios sobre Abū Naṣr Mansūr b. 'Alī b. 'Iṣḡ* (in Spanish). Barcelona 1969.

JHAS = *Journal for the History of Arabic Science*.

IMI = *Istoriko-matematičeskie Issledovanija* (in Russian).

A

A 1 Cat 4 (p 63) f 40a-42b rest 1a-3b. *Maqāla li-Tharāhim ibn Sinān ibn Thābit ibn Qurra fī rasmi al-quḍā' al-ḥadātha*. Treatise by Ibrāhīm ibn Sinān ... on drawing the three conic sections. GAS 5, 293 no. 1. Printed: RI 4. Russian translation in "Ibrāhīm ibn Sinān ibn Thābit ibn Qurra, *Kniga o postroenii tresh homičeskh sečenii'*", IMI 16 (1965).

A 2 Cat 5 (p 63) f 42b-45a rest 3b-6a. *Risāla li-Ibrāhīm ibn Sinān ilā Abi Yūsuf al-Ḥasan ibn Isrā'il fī'l-aṣṭurlāb*. Letter from Ibrāhīm ibn Sinān to Abū Yūsuf ... on the astrolabe. GAS 6, 194 no. 2. Printed: RI 1.

A 3 Cat 6 (p 64) f 45a-47b rest 6a-8b. *Risāla fī'l-ab'ād wa'l-giṭm 'an Kūshyār ibn Labbān al-Jili*. Letter on the distances and sizes (of the celestial bodies) by Kūshyār ibn Labbān ... (This is part of *Al-Zij al-jami'* by the same author, see GAS 6, 248, no. 1) Printed: RM 11. See Kennedy, 125, 156-157.

A 4 Cat 7 (p 65) f 47b-50b rest 8b-11b. *Risālat Abi'l-Wafā' Muḥammad ibn Muḥammad al-Baḡḡānī ilā Abi 'Alī Ahmad ibn 'Alī ibn al-Sukr fī iqāmāt al-burhān 'alā 'l-dā'ir min al-falak min gaus al-nahār wa'rifa' nisf al-nahār wa'rifa' al-waqt*. Letter from Abū'l-Wafā' ... to Abū 'Alī ... on establishing the proof of the (rule for finding the) arc of revolution from the day arc, the noon altitude and the altitude at the time. GAS 6, 224, no. 3. Printed: RM 5. See Nadi Nadir, "Abū'l-Wafā' on the Solar Altitude", *The Mathematics Teacher* 51 (1960), 460-3. Dr. Richard Loreh and Dr. Haskell Isaacs have prepared an English translation, to be published in due course.

A 5 Cat 8 (p 66) f 50b-66b rest 11b-27b. *Risālat Abi Naṣr Mansūr ibn 'Alī ibn 'Iṣḡ Maqāla Amīr al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Bīrūnī fī barāhīn a'māl jadual al-laḡwīm fī zij Habash al-Hāsib*. Letter by Abū Naṣr ... to ... Al-Bīrūnī on the proofs of the procedures of the table of rectification in the Zij of Habash ... GAS 6, 342 no. 1. Printed RN 4. See Sansó 30; R. Irani, *The "Jadual al-Taḡwīm" of Habash al-Hāsib*', thesis, American University of Beirut, 1956; Kennedy, 153; the article Habash al-Hāsib by W. Hartner in EI², III, 8-9.

A 6 Cat 9 (p 67) f 66b-75b rest 27b-36b. *Risālat Abi Naṣr Mansūr ibn 'Alī ibn 'Iṣḡ Maqāla Amīr al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Bīrūnī fī taṣḥīḥ mā waḡa'a li-Abi Ja'far al-Khāzin min al-saḥw fī zij al-ṣafā'iḥ*. Letter by Abū Naṣr ... to ... Al-Bīrūnī on the correction of what Abū Ja'far al-Khāzin overlooked in the Zij of Plates. GAS 6, 243 no. 3. Printed RN 3. See Sansó 30; M. T. Debarnot, "Introduction du Triangle Polaire par Abū Naṣr b. 'Iṣḡ," JHAS 2 (1978), 126-136. On the Zij of Plates see D. King,

"New Light on the Zij al-Şafā'ih of Abū Ja'far al-Khāzin", *Centaurus*, 23 (1980) 105-117.

A 7 *Cat* 10 (p.67) f 75b-78a rest 36b-39a. *Maqālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin fi islah shakl min kitāb Mānālāwīs fi'l-kurūyāt 'adala fihī musallihū hāhāl-keitāb 'an maslakihī*.¹² Treatise by Abū Nasr ... on the correction of a proposition in the Spherics of Menelaos, in which the correctors of this book have digressed from his method. *GAS* 5, 339 no. 2. Printed: RNY 12. Spanish translation in *Samsó* 134-150, commentary in *Samsó* 60-70.

A 8 *Cat* 11 (p.68) f 78a-79b rest 39a-40b. *Maqālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin fi'l-burhān 'alā haqqat al-mas'ala allati waqā'at bayna Abi Hamid al-Şaghānī wa-bayna munajjimī al-Rayy fihā munāzā'a*. Treatise by Abū Nasr ... on the demonstration of the truth in the question on which there was a controversy between .. Al-Şaghānī and the astronomers of Rayy. *GAS* 6, 244 no.10. Printed: RN 13. Spanish translation in *Samsó* 115-120, commentary in *Samsó* 58-59.

A 9 *Cat* 12 (p.69) f 79b-83a, rest 40b-44b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi majāzāt dawā'ir al-sunā' fi'l-asturlāb*. Letter from Abū Nasr ... to ... Al-Birūnī on the crossings of the azimuthal circles on the astrolabe (i.e. their points of intersection with for example the equator). *GAS* 6, 244 no. 6. Printed RN 14. Spanish translation in *Samsó* 89-104, commentary in *Samsó* 49-53.

A 10 *Cat* 13 (p.69) f 83b-86b, rest 44b-47b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi 'Abdallāh Muḥammad ibn Ahmad al-Ma'mūnī fi san'at al-asturlāb bi'l-tariq al-sinā'ī*. Letter from Abū Nasr ... to ... Al-Ma'mūnī on the construction of the astrolabe in the practical way. *GAS* 6, 244 no.5. Printed: RNY 15. Spanish translation in *Samsó* 75-88, commentary in *Samsó* 46-49.

A 11 *Cat* 14 (p.70) f 86b-93b rest 47b-54b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī al-musammā jadwal al-daḡā'iq*. Letter from Abū Nasr ... to ... Al-Birūnī, called the Table of Minutes. *GAS* 6, 244 no. 7. Printed RN 5. See C. Jensen, "Abū Nasr Mansūr's approach to spherical astronomy as developed

12. The word used in the manuscript is *maslakihī* (instead of *maslakihī*), which makes little sense in the context. RN and *Samsó* read *shaklihī*. The word *maslak* also occurs at the beginning of the treatise (f. 75b:14,15 = RN 12, 3:10,12), thus confirming my reading.

in his treatise "The Table of Minutes", *Centaurus*, 16 (1972), 1-19; *Samsó* 31.

A 12 *Cat* 15 (p.70) f 93b-96b rest 54b-57b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi'l-burhān 'alā 'amal Muḥammad ibn al-Sabbāh fi'mitlān al-šams*. Letter from Abū Nasr ... to ... Al-Birūnī on the proof of the procedure of Muḥammad ibn al-Sabbāh in observing the sun. *GAS* 6, 244 no. 4. Printed: RN 2. Spanish translation in *Samsó* 121-133, commentary in *Samsó* 59-66. See also J. Samsó in *DSB IX*, 84.

A 13 *Cat* 16 (p.71) f 96b-98b rest 57b-59b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi'l-dawā'ir allati taḡaddu al-sā'at al-zamāniyya wa ba'q ma yattaslu bi-'amal al-asturlāb*. Letter from Abū Nasr ... to ... Al-Birūnī on the circles demarcating the unequal hours and on something related to working with the astrolabe. *GAS* 6, 224, no.8. Printed: RN 1. Spanish translation in *Samsó* 105-114, commentary in *Samsó* 53-58.

A 14 *Cat* 17 (p.72) f 99a-100a rest 60a-61a. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi'l-burhān 'alā 'amal Ḥabash fi ma'lāt al-samī fi ziyihī*. Letter from Abū Nasr ... to ... Al-Birūnī on the proof of the procedures of Ḥabash for the ascension of the azimuth in his zij. *GAS* 6, 243 no. 2 Printed: RN 11. See *Samsó* 32.

A 15 *Cat* 18 (p.72) f 100b-103a rest 61b-64a. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi ma'rifat al-qisī al-falakīyya ba'qtha min ba'q min ḡayr tariq ma'rifaitha bi'l-shakl al-ḡalīā' wa'l-nisbat al-mu'allafa*. Letter from Abū Nasr ... to ... Al-Birūnī on the determination (*lit.* extraction) of the arcs on the sphere from each other without the transversal figure and the compound ratio. *GAS* 5, 339 no. 3. Printed: RNY 8. German translation in P. Luncey, "Zur Entstehung der Kugeldreiecksrechnung", *Deutsche Mathematik*, 5 (1940), 405-446 See also *Samsó* 32.

A 16 *Cat* 19 (p.73) f 103a-106b rest 64a-67b. *Risālat Abi Nasr Mansūr ibn 'Alī ibn 'Irāq Maulā Amir al-Mu'minin ilā Abi'l-Rayhān Muḥammad ibn Ahmad al-Birūnī fi'l-jawāb 'an mas'āl handasyi'a sā'alahū 'anḡā*. Letter from Abū Nasr ... to ... Al-Birūnī on the answer to geometrical questions he (Al-Birūnī) asked him (Abū Nasr). *GAS* 5, 339 no. 4. Printed: RN 10. See *Samsó* 33.

A 17 *Cat* 20 (p 73) f 106b-109b rest 67b-70b. *Maqāla Abi Nasr Mansūr ibn ʿAlī ibn ʿIrāq fi kashf ʿanwāri al-Bāṭiniyya bimā manwāhā ʿalā ʿammathim fi ruʾyā al-abhilla*. Treatise by Abū Nasr ... on the disclosure of the error of the Bāṭiniyya (school of thought) with which they have misled their people in the observation of the new moon. *GAS* 6, 245 no.12. Printed: *RN* 6. See *Samsó* 36, on the Bāṭiniyya school see *EP*², I, 1131-1133.

A 18 *Cat* 21 (p 74) f 109b-110b rest 70b-71b. *Risālat Abi Nasr Mansūr ibn ʿAlī ibn ʿIrāq Maulā Amir al-Muʾminin ilā Abīl-Rayḥān Muḥammad ibn Ahmad al-Bīrūnī fi hall al-shubḥa ʿaradāt lahu fi'l-maqāla al-thāliṯha ʿashar min kitāb al-Uṣūl*. Letter from Abū Nasr ... to Al-Bīrūnī on the solution of an uncertainty which came to his (Al-Bīrūnī's) mind, in the 13th book of the *Elements* (of Euclid). *GAS* 5, 339 no. 1. Printed *RN* 7. See *Samsó* 33.

A 19 *Cat* 22 (p 74) f 110a-114a rest 71b-75a. *Faṣl min kitāb li-Abi Nasr Mansūr ibn ʿAlī ibn ʿIrāq Maulā Amir al-Muʾminin ilā Abīl-Rayḥān fi ḥurriyyat al-samāʾ*. Chapter from a book of Abū Nasr ... to Abū'l-Rayḥān (al-Bīrūnī) on the spherical shape of the heaven. *GAS* 6, 245 no. 11. Printed: *RN* 9. See *Samsó* 34.

A 20 *Cat* 23 (p 75) f 114b-115a rest 75b-76a. *Maqāla fi ʾstikhraj sāt mā bayna ʾtlu ʿ al-fajr wa'l-shams kull yawm min ayyām al-sana bi-madnat Qāʾin li-Abīl-Ḥasan ibn ʿAbdallāh ibn Bānshādh al-Qāʾini*. Treatise on the calculation of the hours between the beginning of dawn and sunrise on every day of the year for the city of Qāʾin by ... Al-Qāʾini. *GAS* 5, 337. Printed: *RM* 4. English translation with commentary in Marie L. Davidian, E.S. Kennedy, "Al-Qāʾini on the Duration of Dawn and Twilight," *Journal of Near Eastern Studies*, 20 (1961), 145-153.

A 21 *Cat* 24 (p 76) f 115b-117a rest 76b-78a. *Maqāla fi ʾstikhraj taʾriḥ al-Yahūd wa-ʿyādhim taʾlif Abi Jaʿfar Muḥammad ibn Mūsā al-Khwārizmī*. Treatise on the calculation of the calendar of the Jews and their feasts, composed by ... Al-Khwārizmī. *GAS* 6, 143 no. 4. Printed: *RM* 1. See E. S. Kennedy "Al-Khwārizmī on the Jewish Calendar", *Scripta Mathematica*, 27 (1964), 55-59.

A 22 *Cat* 25 (p 76) f 117a-118a rest 78a-79a. *Maqāla fi ʾstikhraj taʾriḥ al-Yahūd li-Abīl-Ḥasan ʿAlī ibn ʿAbdallāh ibn Muḥammad ibn Bānshādh al-Qāʾini*. Treatise on the calculation of the calendar of the Jews by ... Al-Qāʾini. *GAS* 6, 243 no. 3. Printed: *RM* 3.

A 23 *Cat* 26 (p 77) and *Cat* 1 (p 60) f 118a-124b, 323, 1a, rest 79a-87a (f. 1a has been catalogued wrongly as a separate treatise called "Ar-risālatu fi usūl al-rasād"). *Kitāb Ibrāhīm ibn Sinān ibn Thābit ibn Qurra fi Ḥarakāt al-Shams*. Book by Ibrāhīm ibn Sinān ... on the movements of the sun. *GAS* 6, 194 no. 1

Printed: ff. 118a-124b = *RI* 3, "Kitāb fi Ḥarakāt al-Shams" (beginning-34:8 gaws *AE*), f. 323 = *RB* 1, "Istikhraj al-Awtār" (209:10 mihl - 214:10 al-murabbʿ + figs. 115, 116). F. 1a, has not been printed. This is the reason why Saidan stated that the treatise is incomplete.¹³

A 24 *Cat* 2 (p 61) f 1b, 131a-132b rest 87a-89b. *Risālat Ibrāhīm ibn Sinān ibn Thābit fi wasf al-maʿāni allati ʾshakhrajahā fi'l-handasa wa ʾlhn al-nujūm*. Letter by Ibrāhīm ibn Sinān on the description of the notions he derived (i.e. the works he composed) in geometry and astronomy.¹⁴ *GAS* 5, 294 no. 4. Printed: f. 1b = *RI* 6, "Al-Handasa wa-ʾlhn al-Nujūm" (beginning - 5:7 al-ḥaqīq al-wāqīʿ); ff. 131a-132 = *RI* 3, "Kitāb fi Ḥarakāt al-Shams," 63:4 li-kull - end. Edition of the Arabic text in G. Saliba, "Risālat Ibrāhīm ibn Sinān ibn Thābit ibn Qurra fi'l-maʿāni allati ʾshakhrajahā fi'l-handasa wa'l-nujūm" (in Arabic), *Studia Arabica et Islamica, Festschrift for Ḥisn ʿAbbās*, ed. Wādād al-Qāḍī (American University of Beirut, 1981), pp. 195-203.

A 25 *Cat* 27 (p 78) f 132b-134b rest 89b-91b. *Kitāb Ibrāhīm ibn Sinān ibn Thābit fi misālat qaʿ al-makhrūj al-mukāfī*. Book by Ibrāhīm ibn Sinān ... on the area of the parabola. *GAS* 5, 293, no. 1. Printed: *RI* 5. German translation in H. Suter, "Die Abhandlung über die Ausmessung der Parabel von Ibrāhīm ibn Sinān ibn Thābit", *Vierteiljahresschrift der Naturforschenden Gesellschaft in Zürich*, 63 (1918), 214-222. See also B. A. Rosenfeld, M. M. Rožanskaja, "Geometricheskie predstavozanija i peremennye veličiny u Ibrāhima ibn Sināna" (in Russian), *Istorija i metodologija estestvennyh nauk*, 9 (1970).

A 26 *Cat* 28 (p 78) f 134b-141a rest 91b-98a. *Kitāb Arshimīdis fi'l-dawāʾir al-mutanāssa*. Book of Archimedes on tangent circles. *GAS* 5, 134 no.6. Printed: *RT* 2. Edition of the Arabic text and German translation in: *Archimedes Opera Mathematica vol. IV, Über einander berührende Kreise. Aus dem Arabischen von Yvonne Dold-Samplonius, Heinrich Hermelink, Mathias Schramm*. (Stuttgart: Teubner, 1972). Russian translation by B. Rosenfeld in I. N. Veselovskij (ed.), *Arhimed, Sočinenija* (Moscow, 1962). Spanish translation in J. Vernet, A. Catalá, "Arquímedes árabe: El tratado de los círculos tangentes," *Andalus*, 33 (1968), 53-93. See also Y. Dold-Samplonius, "Archimedes: Einander berührende Kreise", *Südhojfs Archiv*, 57 (1973), 15-40.

A 27 *Cat* 29 (p 79) f 141a-144b rest 98a-101b. *Kitāb Arshimīdis fi Uṣūl al-*

13. *Op. cit.* (see note 5), p. 174.

14. In *GAS* 6, 194 under no. 3 it has been stated, though wrongly, that this work deals with the geometry necessary for astronomical calculations. In *GAS* 5, 294 note 1 Sezgin mentions "autobiographical and bibliographical Angaben aus einer nicht identifizierbaren Schrift Sinān's"; these references are found in this work of Ibrāhīm ibn Sinān on ff. 131a-132b.

Handsa. Book of Archimedes on the Elements of Geometry. *GAS* V, 135-7. Printed: *RT* 1. This treatise is a version of the "Book of Assumptions by Aqāim". A facsimile-edition (of an Aya Sofya manuscript) with English translation and commentary (also on the present manuscript) is in Y. Samplonius, *Book of Assumptions of Aqāim*, thesis, Amsterdam 1977. See also Y. Dold-Samplonius, "Some remarks on the 'Book of Assumptions by Aqāim'", *JHAS*, 2 (1978), 255-263.

A 28 *Cat* 30 (p 80) f 144b-145b rest 101b-102b. *Faql fi takhṭi al-s'āt al-zamā-niyya fi kull qubba wa fi qubba yusta' malū lahā li'l-Faql ibn Ḥātim al-Nayrizi*. Chapter on drawing the lines demarcating the unequal hours in every cupola or in a cupola which is used for them, by... Al-Nayrizi (on sundials). *GAS* 6 192 no. 3. Printed: *RM* 2.

A 29 *Cat* 31 (p. 80) f 145b-169a, rest 102b-162a. *Risālat Abī 'Abdallāh al-Ḥasan ibn Muḥammad ibn Ḥamla al-ma'rūf bi'bn al-Baghdādī fī'l-maqādir al-musharika wa'l-mutabāyina*. Treatise by Abū 'Abdallāh ... known as Ibn al-Baghdādī, on commensurable and incommensurable magnitudes. *GAS* 5, 392. Printed: *RM* 9. Russian translation in: G. P. Matvevskaia, "Materialy k istorii učenija o čisle na srednevekovom Bliznem i Srednem Vostoke", in: *Iz istorii točnyh nauk na srednevekovom Bliznem i Srednem Vostoke* (Tashkent, 1972).

A 30 *Cat* 32 (p. 81-83) f 169a-188b rest 126a-145b. *Kitāb inbāi al-miyāh al-khafiyya tasnif Abī Bakr Muḥammad ibn al-Ḥasan al-Ḥasib al-Karajī*. Book on finding hidden waters, composed by ... al-Karajī. *GAS* 5, 328, 9. Printed: *Inb*. French translation by A. Mazahéri in: Al-Karajī, *La civilisation des eaux cachées* (Nice, 1973). The Persian translation of this work has been edited by Husayn Khadījam: *Istikhraj-i ābhā-yi pinhāni* (Teheran, Iranian Culture Foundation, 1966), 127 pp. See "Muḥammad ibn al-Husayn Karajī, *Kitāb-i istikhraj-i ābhā-yi pinhāni tarjuma-yi Husayn Khadījam*" (in Persian), *Solkhan-i 'Umi*, 4 (1344 (A.H. Solar)), 408-411. See also Mehdi Nadji, "Kardjis Erschliessung verborgener Gewässer", *Technikgeschichte*, 39 (1972), 11-24, and F. Brün, *Surveying and surveying instruments, being chapter 26, 27, 28, 29 and 30 of the book On Finding Hidden Waters by Abu Bakr Muḥammad al-Karajī*, *Biruni newsletter* no. 31. (American University of Beirut 1970).

A 31 *Cat* 33 (p 84) f 189a-191a rest 146a-148a. *Qawl Ibn al-Ḥaytham fi khaṭā'is al-muḥallāh min jihat al-'amūd*. Treatise by Ibn al-Ḥaytham on the properties of the triangle with respect to the perpendicular. *GAS* 5, 366 no. 4. Printed: *RH*, appendix, See H. Hermelink, "Zur Geschichte des Satzes von der Lotsumme im Dreieck", *Südhofts Archiv*, 48 (1964), 240-247. English translation

by F. A. Shamsi in: Hakim Muḥammed Said (ed.), *Ibn al-Ḥaytham, Proceedings of the Celebrations of 1000th Anniversary* (Karachi: Hamdard Academy 1969), pp. 228-246.

A 32 *Cat* 34 (p 84) f 191a-193b rest 148a-150b. *Risāla fi mis'ālat al-mujassam al-mukāfi li'l-shaykh Abī Sahl Wajyan ibn Rustan al-Qāhī*. Letter on the volume of the parabolic solid by the master ... Al-Qāhī. *GAS* 5, 318, no. 5. Printed: *RM* 6. German translation in: H. Suter, "Die Abhandlungen Thābit b. Kurras und Abū Sahl al-Kūhīs über die Ausmessung der paraboloiden." *Sitzungsberichte der Physikalisch-Medizinischen Societät zu Erlangen*, 48-49 (1916-7), 186-227. The preface in the MS on f 191a corresponds to the translation on p. 213-215.

A small collection of propositions without marginal number is appended to the preceding treatise. : A - *Cat* 35 (p 85) f 193b rest 150b. *Min kalām Abī Sahl al-Qāhī ayyān finā zāda min al-ashkāl fi amr al-maqāla al-thāniya min kitāb al-Uṣūl li-Ughdis lammā yuḥāyju ilayhi fī'l-maqāla al-thāniya wa'l-thalṭha min kitāb al-Makhrizā. From what the same ... Al-Qāhī said on the propositions he added to the second book of the Elements of Euclid because they are necessary in the second and third book of the Conics (of Apollonius) *GAS* 5, 319 no. 15. Not printed.*

A 33 *Cat* 36 (p 85) f 194a-195, 125-131, 196-217, 220-239b, rest 151a-200b. *Ifrād al-maqāl fi amr al-zilāl tasnif al-shaykh Abī'l-Rayḥān Muḥammad ibn 'Aḥmad al-Bīrūnī*. The exhaustive treatise on shadows, composed by the master ... Al-Bīrūnī. *GAS* 5, 380 no. 1. Printed: ff. 194a-195 = *RB* 2, "Ifrād al-Maqāl" (beginning - 5:10 *aḥaduhā*); ff. 125-131 = *RI* 3, "Kitāb fi Ḥarakāt al-Shams" (34:8 *min al-āḥḥar* - 63-4 *ūrijā*); ff. 196-217, 220-239b = *RB* 2, "Ifrād al-Maqāl" (5:10 *bi-annahā* - end). English translation with commentary in: E. S. Kennedy, *The Exhaustive Treatise on Shadows by Abu al-Rayḥān Muḥammad ibn 'Aḥmad al-Bīrūnī* (Aleppo: IHAS, 1976), 2 vols. See also H. Hermelink, "Bestimmung der Himmelsrichtungen aus einer einzigen Schattenbeobachtung nach Al-Bīrūnī", *Südhofts Archiv*, 44 (1960), 329-332; E. S. Kennedy, "Bīrūnī's graphical determination of the local meridian," *Scripta Mathematica*, 24 (1959), 251-255; E. S. Kennedy, Al-Bīrūnī on the Muslim Times of Prayer, in: P. J. Chelkowski (ed.) *The Scholar and the Saint, Studies in Commemoration of Abū'l-Rayḥān al-Bīrūnī and Jalāl al-Dīn al-Rāmi* (New York: New York University Press, 1975), p. 83-94; B. A. Rosenfeld, L. G. Utscha, "Some mathematical discoveries in al-Bīrūnī's Shadows", *JHAS*, 4 (1980), 332-336.

A 34 *Cat* 37 (p 88) f 239b-244b rest 200b-205b. *Maqālat Abī'l-Rayḥān Muḥam-*

mad ibn Ahmad al-Birūni rahimahu'llāh fi rashikāt al-Hind. Treatise by ... Al-Birūni, may God have mercy upon him, on the Indian rule of three. *GAS* 5, 380 no. 2. Printed: *RB* 4. Russian translation by B. A. Rosenfeld in *Iz istoričeskoj nauki i tehniki v stranah Vostočka* vol. 3, Moscow 1963. See also Abū'l-Qāsim Qurbānī, *Birūni-nāma* (in Persian), (Teheran, A.H. solar 1353), pp. 206-219. On the word *rashikāt* see E. Boilot, "l'Ouvrre d'al-Birūni, Essai Bibliographique," *Mélanges. Institut Dominicain d'Etudes Orientales du Caire*, 2 (1955), 161-256, esp. 188.

A 35 *Cat* 38 (p 89) f 245a-266b rest 206a-227b. *Tamhid al-mustaqarr li-talqiq ma'nā'l-manarr li-Abī'l-Rayhān Muḥammad ibn Ahmad al-Birūni*. Smoothing the basis for an investigation of the meaning of transits by Al-Birūni (this is the translation of Professor Kennedy). *GAS* 6, 267, no. 3. Printed: *RB* 3. English translation and commentary in: *Al-Birūni on transits. A study of an Arabic treatise entitled Tamhid al-Mustaqarr li-talqiq ma'nā'l-manarr by Al-Birūni Translated by Muh. Saffouri and Adnan Ifram. With a commentary by E. S. Kennedy*. American University of Beirut, 1959. See also G. J. Toomer, "Notes on Al-Birūni on transits," *Orientalia*. 34 (1965), 45-72.

A 36 *Cat* 39 (p 90) f 267a-276b rest 228a-237b. *Kitāb fi kayfiyyat tashih al-kurra 'alā saḥi al-aṣṭurlāb istiḥrāj Ahmad ibn Muḥammad ibn al-Ḥusayn al-Ṣaghānī*. Book on how to project the (celestial) sphere on the plane of the astrolabe... by ... Al-Ṣaghānī. *GAS* 5, 311, no. 4. Printed: *RM* 7.

A 37 *Cat* 40 (p 90) f 276b-279b rest 237b-240b. *Risālat Ahmad ibn Muḥammad ibn 'Aḍḍajālī al-Sijzī fi'l-shakl al-qatīā'*. Letter by ... Al-Sijzī on the transversal figure. *GAS* 5, 332 no. 15. Printed: *RM* 10, pp. 1-22. See J. L. Berggren, "Al-Sijzī on the Transversal Figure", *JHAS*, 5 (1981), 23-36.

A 38, not mentioned in the catalogue, f 279b-280a rest 240b-241a. *Al-shakl al-mutassā'*. The nine-sided figure (i.e. the regular nonagon). Anonymous, not mentioned in *GAS*. Printed: *RM* 10, p. 22-24. English translation and commentary by J. L. Berggren, "An Anonymous Treatise on the Regular Nonagon", *JHAS*, 5 (1981), 37-41.

A 39 *Cat* 41 (p 91) f 280a-282a rest 241a-243a. *Risāla li-Naṣr ibn 'Abdallāh fi anna 'l-aṣḥkāl kulluhā min al-dā'ira*. Letter by Naṣr ibn 'Abdallāh on (the fact) that all figures are derived from the circle. *GAS* 5, 314 no. 1. Printed: *RM* 8.

The author of this treatise was probably not Naṣr ibn 'Abdallāh but Al-Sijzī, for the following reasons. The author of the treatise mentions "our

book on Making Easy the Roads to the Geometrical Propositions (*Kitābunā fi tashih subul ilā 'l-aṣḥkāl al-handasiyya*, f 280a:24 = *RM* 8, 3:5) and "our book on the Properties of the Egg-shaped and Lentil-shaped Figures" (*Kitābunā fi kḥawāṣṣ al-shakl al-bayḍi wa'l-'adasi*, f 280b:18 = *RM* 8, 5:5) Al-Sijzī is known to have written a work on "making easy the roads for deriving geometrical figures" (*GAS* 7, 410 no. 38). Al-Sijzī refers to a work of his on the egg-shaped and lentil-shaped figure (the solids of revolution of an ellipse around its major and minor axis respectively) in his *Introduction to the Science of Geometry (al-Madhhal ilā 'ilm al-handasa*, *GAS* 5, 333 no. 20, MS. Dublin, Chester Beatty 3652, 16a:17). As far as is known, no other Arabic geometer has written about these figures. We know that Al-Sijzī also wrote on the "fact" that all figures are derived from the circle (*GAS* 7, 410, f) Thus Al-Sijzī is probably the author of the treatise A 39.

A 40 *Cat* 42 (p 92) f 282b-298, 326a, rest 243a-260a. *Kitāb Abī'l-Rayhān Muḥammad ibn Ahmad al-Birūni fi 'istiḥrāj al-anār fi'l-dā'ira bi-kḥawāṣṣ al-khaldī al-munḥanā al-wāqī' fihā*. Book of ... Al-Birūni on the extraction of chords in the circle by means of the properties of the inscribed broken line. *GAS* 5, 381 no. 3. Printed in *RB* 1, "Istiḥrāj al-Awātār": f 282b-298 = beginning - 108:8 *al-musāwiyā li-zāwiyā* + figs. 1-72; f 326a = 224:2 ... 0 — end + fig. 118, right side.

Edition of the Arabic text in ff. 282b-298 in *Dimirdāsh* 32 - 172:2; f. 326a: 21-29 is edited in *Dimirdāsh* 286:24-287:7. The text in f. 326a:1-20 has not been edited by Dimirdāsh (it is, however, on the facsimile of f. 326a on p. 31 of his edition). German translation with commentary, both based on a Leiden ms. in H. Suter, "Das Buch der Auffindung der Sehnen im Kreise von Abū'l-Rayhān Muḥammad al-Birūni," *Bibliotheca Mathematica*, 3. Folge, 11 (1910), 11-78. A facsimile edition of this Leiden MS (?) was published by Muḥammad Āthār Millī (?) (Teheran (?) 2535 Cyrus (?)), *Silsilat Intishārāt* 124. The Leiden and Bangalore manuscripts of the *Extraction of Chords* are slightly different. Russian translation by S. A. Krasnova, and I. A. Karpova with commentary by B. A. Rosenfeld and S. A. Krasnova in: *Iz istoričeskoj nauki i tehniki b stranah Vostočka*, vol. 3 (Moscow, 1963). See also Muḥammad Saūd, "A part of al-Birūni's Istiḥrāj al-Awātār fi 'l-Dā'irah" in *Hakim Muḥammed Saūd* (ed.), *Al-Birūni Commemoration Volume* (Karachi: Hamdard Academy, 1973), pp. 691-705. See also A. S. Saidan, "The Trigonometry of Al-Birūni" in the same volume, pp. 681-690.

On f. 326b there is a geometrical figure which apparently does not belong to any of the treatises and fragments in MS Bangalore 2468. The figure has been printed in *RB* 1, "Istiḥrāj al-Awātār", fig. 118, left side.

B

B is not mentioned in the catalogue. It consists of ff. 325, 322, 299-305, 309-316. B is a fragment of Al-Birūnī's *Maqāla fi'l-taḥlīl wa'l-taqīc' li'l-ta'dīl*: Treatise on the solution and the division of the (solar) equation, see section 4 of this paper. The treatise is listed in GAS 6, 273 no. 11 as a lost work. Printed in RB 1, "Istikhraj al-Awtār":

- f 325 = 219:16 *al-muḥawā* - 224:1 *'anhu*;
 f 322 = 206:12 *fa-anna* - 209:10 *min* + figs. 110-114;
 f 299 - 305 = 108:15 *al-mutasāwiyyatayn* - 138:1 *ṣāra*;
 f 309 - 316 = 153:9 *lanā* - 180:15 *ilā* + figs. 85-99.

The Arabic text in ff. 299-305 and ff. 309-316 has been edited in *Dimirdāsh* 172:2 - 245, as part of the *Extraction of Chords* of Al-Birūnī. Dimirdāsh realized that the text on f. 305b continues on f. 309a.

F. 316b:1-23 (= RB 1, 179:1-180:15) and ff. 322, 325 was not edited by Dimirdāsh.

Part of the treatise is discussed in E. S. Kennedy, Ahmad Murruwwa, "Birūnī on the solar equation," *Journal of Near Eastern Studies*, 17 (1958), 112-121.

C

C consists of two parts C₁ and C₂.

C₁: f 324, 321, 318, 320, 319, 306-308, 2-20b is part of Ibrāhīm ibn Sinān's *Al-Mas'ūl al-Mukhāra*, the exquisite problems. See section 5 of this paper. The work is mentioned in GAS 5, 294¹⁵ under no. 6. The first 8 extant leaves have been printed in RB 1, "Istikhraj al-Awtār":¹⁶

15. Sezgin mentions references made by Ibrāhīm ibn Sinān to Abū'l-ʿAlā' ibn Karnīb (GAS 5, 300), Abū'l-ʿAbbās ibn Yahyā (GAS 5, 300-301), Abū Yahyā (GAS 5, 303) and ʿAlī ibn al-Ḥasan ibn Maʿān (GAS 5, 304). These references are not in the "Letter ... on the description of the notions he derived in geometry and astronomy" (A 24) but in the "Exquisite Problems" (C₂). Sezgin says (GAS 5, 381 under no. 6) that a fragment of a book by Al-Birūnī on tangent circles has been preserved in the printed text of Al-Birūnī's *Extraction of Chords*, RB 1:218-129. However, this part of the printed text is part of the "Exquisite Problems" of Ibrāhīm ibn Sinān, who refers to his own book on tangent circles (which is mentioned in GAS 5, 294 no. 6). So GAS 5, 381 no. 6 has to be omitted.

Anbouba also remarked that Al-Birūnī probably did not write a book on tangent circles, and that Ibrāhīm ibn Sinān is the author of part of the text edited by Dimirdāsh as Al-Birūnī's *Extraction of Chords*. See A. Anbouba, *Tasbiḥ ad-dīra* (in Arabic), JHAS, 1 (1977), 352-384, esp. 382 note 6.
 16. At this point Sardan's references are not altogether correct (op. cit., see note 5).

f. 324 = 214:10 *wa-annahā* (MS: *wa lā annahā*) - 219:16 *maʿlūmā* + fig. 117.

f. 321 = 201:9 *hadhālika* - 206:11 *maʿlūm* + figs. 109, 111.

f. 318 = 184:12 *fa-TC* - 190:5 *murabbāʿahu* + fig. 102.

f. 320 = 196:1 *maʿlūman* - 201:9 *fādī* + figs. 107, 108.

f. 319 = 190:5 *fa-murabbāʿe BZ* - 196:1 *ilā DG* + figs. 103, 104.

(figs. 103, 104 are the same as figs. 105, 106 respectively)

f. 306a:1-4 = 108:9 *maʿlūma* - 108:14 *ilā HZ*.

f. 306a:4 - 308 = 138:1 *ilā khairi maʿlūm* - 153:19 *nuqta* + figs. 74-79.

ff. 2-20b has been printed in RI 6, "Al-Handasa wa 'Ilm al-Nujūm" (5:7 *idh* - end). ff. 324, 321, 318, 320, 319, 306-308b:24 have been edited in the correct order in *Dimirdāsh* 246:9-286:23 as part of the *Extraction of Chords* of Al-Birūnī.

C₂ Cat 3 (p 62) f 21a-39b. *Maqāla li-Ibrāhīm ibn Sinān fi tariq al-taḥlīl wa'l-tarkīb wa-sā'ir al-ʿamal fi'l-musā'il al-handasiyya*. Treatise by Ibrāhīm ibn Sinān on the method of analysis and synthesis and the other procedures in geometrical problems. GAS 5, 294, no. 2. Printed: RI 2. The last leaf of the treatise is missing. The complete text, which is in MS Paris, Bibliothèque Nationale, Fonds Arabe 2457, 1b-18b and MS Cairo, Dār al-Kutub Muṣṭafā Fāḍil Riyāda 40m, 130b-153b, is about 15 lines longer than the text in MS Bankipore 2468.

D

F. 317 is a fragment of Al-Birūnī's *Maqāla fi anna lanāzin tajazzī al-maqādir ilā lā nihāya qarība min al-khalīqayn alladhayn yaqrubān wa-lā yataqayyān fi'l-istibʿād*. Treatise on (the fact) that the necessities of the infinite subdivisibility of magnitudes are related to the matter of the two lines which approach each other but do not meet in the distance, see section 6 of this paper. The treatise is listed in GAS 5, 383 no. 13 as a lost work. Printed in RB 1, "Istikhraj al-Awtār" (180:15 *farāda* - 184:12 B + figs. 100-101). Russian translation with commentary in P. G. Bulgakov, A. A. Ahmedov, "Beruni i al-Kindi o teorii parallel'nih", *Obščestvennie nauki b Uzbekistane* (1977), 30-36. Review by E. S. Kennedy in *Mathematical Reviews*, March 1981, no. 81c: 01008. F. 317 was not edited by Dimirdāsh.

4. B: *The Treatise on the solution and the Division¹⁷ of the Equation by Al-Birūnī.*

17. The word division probably refers to the different ways in which the equation has to be calculated according to the different positions of the sun.

4.1 *The contents of B.*

The subject of **B** is the calculation of the equation of the sun. For a discussion of the problem and terminology I refer to the article by Kennedy and Muruw-wa mentioned in section 3 under **B**.

B is part of a text which once consisted of four *maqāla* (plural of *maqāla* = big chapter). The extant part of the first *maqāla* contains a fragment of a preface, which will be translated below, and a full discussion of terminology and preliminary theorems.

The second and third *maqāla* are extant in **B**. They present 16 methods for calculating the equation of the sun and give geometrical proofs for the correct methods. The 16 methods are discussed in full in the article by Kennedy and Muruw-wa mentioned above. These authors say that the text they discuss is part of the *Rasā'il al-Birūnī* (letters of Al-Birūnī), but they do not give a further identification.

Only part of the fourth *maqāla* is extant in **B**. Kennedy and Muruw-wa do not discuss this *maqāla*, but I use their notation for the second and third. It deals with the following problem. Given two of the following four parameters: the mean anomaly $\bar{\lambda}$, the true anomaly λ , the equation e and its maximal value e_{max} , required to determine the other two parameters. This leads to six combinations (*qirānāb*), which are listed in the extant part of the fourth *maqāla*: to solve the problem if 1. $\bar{\lambda}$, e_{max} , or 2. $\bar{\lambda}$, e , or 3. $\bar{\lambda}$, λ , or 4. e_{max} , e , or 5. e_{max} , λ , or 6. e , λ , are given. Then the text runs:

‘It is necessary that we finish the book by mentioning

them (the 6 combinations) in detail’ (*RB* 1, 179.13). Hence the fourth *maqāla* was the last one. The text is broken off in the middle of the second combination. In conclusion: **B** is almost the complete text of a treatise on the solar equation.

Dimirdāsh edited a small part of the first *maqāla* and the complete second and third *maqālas* as part of Al-Birūnī’s *Extraction of Chords*. He erroneously rendered the beginning of C_1 (the *Exquisite Problems* of Ibrāhīm ibn Sinān) as ‘fourth *maqāla*’. C_1 begins on p. 246 of his edition.

4.2 *Identification of B.*

In the list of the works he completed before the end of 427 H. Al-Birūnī says that he had composed:

because of a question of somebody who suspected (something) in the tables of the equation of the sun and who did not discover the method of Ḥabash for solving it, a treatise on the solution and division of the equation, in 70 leaves.¹⁸

18. The list has been edited in: *Chronologie* on: *Orientalischer Völkerver von Alberūnī. Herausgegeben von Dr. C. Eduard Sachau* (Leipzig 1878), pp. XXXVIII-XXXVIII. The quoted passage is on p. XXXXI lines 1-2. On Ḥabash see *GAAS* 6, 173-175.

This description agrees with the extant part of the preface in **B**. In order that the reader can check this statement himself, I give a tentative translation of f. 325a:1-15 = *RB* 1, 219:16 - 220:13 below. Dots indicate places where one or two words in the manuscript are illegible. All additions of mine are in parentheses. I have indicated in footnotes all places where I read the manuscript differently from the editors of *RB*.

...¹⁹ containing the values of the equation of the sun from the *zif* of Ḥabash. He found in ... a place where there was a big difference between the two lines in smallness (?) in the two margins... to it, and at this (point) the matter of those numbers became irregular. Therefore he asked me about this situation... as somebody trained in working with geometrical lines and accustomed to work on geometrical proofs ... at (that) time, with a¹⁹ number²⁰ of methods for calculation, to which thinking on them (the geometrical lines and proofs) had led me, some of them being easy, others being difficult. Thereupon none of them produced to the person who had asked²¹ (the question) anything which agreed²² with what he had asked about.

I was inclined to attribute this to the negligence of Ḥabash in calculating those tables, or to inattentiveness on the part of transcribers, till I returned to the collections of *zif*²³ mentioned above. Then I found in them a method of Ḥabash for solving the equation, dividing it, explaining it and making it clear. When I tried it, it produced for that place (in the table) a value equal to the value in the table. Thus I learnt that Ḥabash had used it, but nobody else.

Then I thought over its proof, and I enjoyed myself by thinking over the proofs of other (methods), till the ways to the knowledge of all of them had opened up, and the ways to the proof of them had been illuminated by tireless attention²⁴ to what made the perception of them obscure. Because of the multitude of them it was possible to devote a book to them, containing a very useful²⁵ specialism in astronomy, and for training those who dislike the dreariness of uncritical copying, nor the remaining (uncritical) followers. I have made it, and it is this book.

The following reference is also of interest for the identification of **B**. In the *Extraction of Chords* Al-Birūnī presents what he calls a

Solution of the equation < and division of it > for half of the deferent, from my book concerning this subject.

(*RB* 1, 72:1 — 74:14, the words in angle brackets are not in the Bankipore manuscript, but they are in the Leiden manuscript translated by Suter²⁶).

19. f 325a:1-4 have been printed incorrectly in *RB* 1, 219.16-220:1. I read the manuscript as follows:

المحتوى تعاديل الشمس من زيف حيش فوجد في ... مؤنسا عظم فيه تقاضل ما بين السطرين في صفة (?) في حاشيته ... إليه وزال به أمر تلك الأعداد عن النظام فسألني عن كيفية الحال ... كن يتدبرها بحارسه الخطوط المساحة ومعرفة البراهين الهندسية ... في الوقت بما حصرني

20. The MS is not very legible. I conjecturally read *sa*.

21. *RB* 1, 72:14-15 as in 325a:6, not *السائل* as in *RB* 1, 220:2.

22. *RB* 1 omits *مواظبا* which is in 325a:6 after *شيئا*.

23. *التعقبات الرجعية* as in 325a:8.

24. *بالذروب* as in 325a:12.

25. *عظم البناء* as in 325a:13.

26. *Op. cit.* (see section 3, A 40) p. 45-46 no. 11.

This is a proof of a method for calculating the equation of the sun, which is the same as the proof in the second section of the third *maqāla* in B. This section is entitled:

On the proof of a calculation which came to my mind by means of the properties of the broken line in the circle²⁷

From the material presented above we can draw the conclusion that B contains a version of Al-Bīrūnī's *Treatise on the solution and the division of the equation*. This treatise must have been written in or before 418 H./1027 A. D. because it is mentioned in the version of the *Extraction of Chords* which Al-Bīrūnī finished in Rajab 418 H./July-Aug. 1027 A.D.²⁸

5. Identification of C_1 as the Exquisite Problems of Ibrāhīm ibn Sinān.

Ibrāhīm ibn Sinān says in his *Letter on the Description of the Notions he Derived in Geometry and Astronomy* (A 24) that he wrote a treatise called the *Exquisite Problems*²⁹ (*Al-masā'il al-mukhtāra*). This treatise contained solutions of

²⁹ Geometrical problems, namely difficult problems on circles, lines, triangles, tangent circles and other things³⁰

Ibrāhīm ibn Sinān says that he

followed the method of analysis only, without giving syntheses, except in the case of three problems, where syntheses were necessary.³⁰

C_1 is a geometrical text from which only part of the preface is missing. In C_1 about 40 geometrical problems are treated. The exact number is to some extent arbitrary, depending on whether certain auxiliary problems are counted separately. The author of C_1 must be Ibrāhīm ibn Sinān ibn Thābit, because it contains a reference to "my grandfather Abū 'l-Ḥasan Thābit".³¹

In C_1 an analysis is given of all problems, but only three syntheses occur.³² So it is likely that C_1 is part of the *Exquisite Problems*.

²⁷ RB 1, 136:4-5 = f. 305b:7-8. The proof is in 305b:7-22, continued on f. 309a:1-7 = RB 1, 136:3-138:1, continued on RB 1, 153:19-154:10, edited in *Dimirdāsh* 213-216. The corresponding method of calculation is explained in the second section of the second *maqāla*, which is in RB 1, 117:13-118:10, edited in *Dimirdāsh* 184-185. It is discussed by Kennedy and Murawwa *op. cit.* p. 115-116 as method 2. Note that RB 1, 117:13-118:10 is part of C_1 , and that the remark of Kennedy and Murawwa, p. 116 left column, "In this connection Bīrūnī gives the proof... Apollonius is mentioned several times in the passage" refers in fact to C_1 , not to B.

²⁸ The date is in MS Bankipore 2468, f. 326a, printed in RB 1, 226:11-12; = *Dimirdāsh* 287:6-7.

²⁹ f. 133a:23-24 = RI 3, 69:5 = edition by Saliba (see A 24), 200:230.

³⁰ f. 132a:12-15 = RI 3, 68:8-11 = ed. Saliba 200:20-22.

³¹ f. 308b:20 = RB 1, 153:1 = *Dimirdāsh* 286:19. This was already pointed out by Ambouba in "Tasbiḥ ad-Dīra", *JHAS*, 1 (1977), 382 note 6.

³² The syntheses are on f. 308a:28-308b:30 = RB 1, 150:4-152:12 = *Dimirdāsh* 285:7-286:18; f. 3a:7-29 = RI 6, 11:1-12:8; f. 10a:10-10b:18 = RI 6, 46:12-50:16.

That this is really so is proved by reference in a work of Al-Sijzī, namely the "Book by Ahmad ibn Muḥammad ibn 'Abdaljalīl (al-Sijzī) on the exquisite problems which were currently being discussed between him and the geometers of Shirāz and Khorāsān, and his (own) additions" (*Kitāb Ahmad ibn Muḥammad ibn 'Abdaljalīl fi'l-masā'il al-mukhtāra allatī jarat baynahu wa-bayna muḥandisi Shirāz wa-Khorāsān wa-ta'līqātuhu*, see *GAS* 5, 333 no. 23). A version of this work survives in two manuscripts:

I = MS Istanbul, Reṣit 1191, ff. 31b-62a, undated.

X = MS Dublin, Chester Beatty Library 3652, ff. 35a-52b, dated 612 H./1215 A.D.³³

In I and X Al-Sijzī presents his synthesis of what he calls an important proposition from the book of Ibrāhīm ibn Sinān on the Exquisite Problems. The corresponding analysis is in C_1 on f. 4b, which proves that C_1 is in fact the *Exquisite Problems*.

Below I give English translations of the synthesis of Al-Sijzī and the analysis of Ibrāhīm ibn Sinān. My reason for doing this is first to present in detail the sources for identifying C_1 , and secondly to draw the attention of the reader to a hitherto neglected work which seems to be of some importance for the history of geometry.

In the translated passages Ibrāhīm ibn Sinān and Al-Sijzī deal with the following problem (fig. 1):

Given: three straight lines AB, EZ, EH and three collinear points G, D, K. Required: two straight lines GL, DL intersecting in a point L such that the following relations are satisfied:

1. L is on AB.

2. GL intersects EZ in a point T, and DL intersects EH in a point I such that T, I and K are collinear.

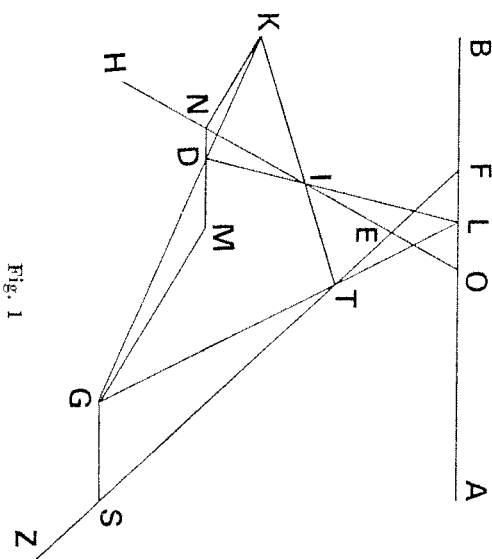


Fig. 1

I have added some words to the texts in the manuscripts; these words will be translated in pointed brackets. Arabic letters in geometrical figures

³³ See A. J. Arberry, *A Handlist of the Arabic Manuscripts in the Chester Beatty Library*, vol. 3 (Dublin, 1958), p. 59 no. 7.

6. **D**: a fragment of Al-Birūnī's *Treatise on (the fact) that the necessities of the infinite subdivisibility of magnitudes are related to the matter of the two lines which approach each other but do not meet in the distance*.

6.1 Description of **D**.

The beginning of **D** is a proof that any segment of a straight line is infinitely subdivisible; the proof uses the existence of parallels and the fact that one can find infinitely many points on a straight line on one side of a given point. This proof is attributed in **D** to Al-Kindi (died after 256H./870 A.D.).⁵⁵

Next **D** discusses an "objection" by a person whose name is not mentioned. This objection probably refers to part of the text which is now lost. It is based on the opinion of this person that the existence of parallel (that is: non-intersecting and being in the same plane) straight lines which approach each other "on one side" would not be surprising.

The author of **D** states that in his opinion two non-parallel straight lines do meet.⁵⁶ But then he gives some examples of in which two non-parallel straight lines (i. e. line segments) approach each other without ever meeting. The idea is basically that one or both of the segments may be extended an indefinite number of times in such a way that the point of intersection is approached but not reached.

The historical interest of **D** is in the reference to the person who believed that there may in fact be parallel straight lines which approach each other in one direction. This assumption contradicts the parallel postulate of Euclid (if the other axioms of Euclid are assumed to hold). However, asymptotically approaching parallels exist in hyperbolic geometry, which was created by Bolyai, Lobachevski and Gauss early in the 19th century.⁵⁶ The idea of two converging parallels is mentioned by earlier geometers, for example Proclus (5th century A.D.)⁵⁶ and Saccheri (1733).⁵⁷ However, these geometers at-

53. See GAS 5, 255-259 for the mathematical work of Al-Kindi.

54. The meaning of this statement is not clear, because by definition two non-parallel straight lines have a point of intersection. Perhaps the author meant that he believed the parallel postulate to be true; or he may have taken *mutawāzin* in the sense of "equidistant".

55. For a survey of the history of the parallel postulate and non-Euclidean geometry and references see M. Kline, *Mathematical thought from ancient to modern times* (New York: Oxford University Press, 1972), Ch. 36 (pp. 861-88), R. Bonola, *Non-Euclidean Geometry, a critical and historical study of its development* (translated from the Italian), (New York: Dover, Reprint 1955); see also notes 56 and 57. For attempts by medieval Islamic geometers to prove the parallel postulate see A. P. Juschkewitch, *Geschichte der Mathematik im Mittelalter* (translated from the Russian), (Leipzig, 1964), pp. 277-288.

56. See Proclus, *Commentary on the First Book of Euclid's Elements*, Translated by Glenn R. Morrow (Princeton, 1970), pp. 151, 285 (= Proclus, ed. Friedlein, Leipzig 1873, pp. 192, 364-5).

57. See G. Saccheri, *Eucledis ab omni naseo vindicatus, sive conatus geometricus quo stabiliumur*

tempted to disprove the existence of such lines because they believed the parallel postulate to be true. It is therefore interesting that **D** refers to a medieval geometer who doubted some of the consequences of the parallel postulate.

6.2 Conjectural identification of **D**.

Following Saidan, Bulgakov and Ahmedov⁶ I shall attempt to identify **D**. Al-Birūnī says in the list of his own works mentioned above that he wrote a

Treatise on (the fact) that the necessities of the infinite subdivisibility of magnitudes are related to the matter of the two lines which approach each other but do not meet in the distance, in ten leaves.⁵⁸

The contents of **D** agree with this title, especially the reference to the two converging parallel lines. The infinite subdivisibility of magnitudes is also used in **D**.

It seems plausible that **D** is part of a work of Al-Birūnī, because **D** is found between other works of Al-Birūnī (**B**, **A 40**) and because **D** is well-written in a concise Arabic, which is different from the monotonous style of many other geometers.

So **D** is probably a fragment of the above-mentioned work on parallels of Al-Birūnī. This fragment seems to contain between one-half and one-fifth of the work.⁵⁹

→ *prima ipsa universae geometriae principia* (Milano, 1733), book 1, propositions 30-33. Translated into German in: F. Engel, P. Stackel, *Die Theorie der Parallellinien von Euklid bis auf Gauss* (Leipzig, 1895), pp. 41-136 (esp. pp. 104-109). Translated into English in G.-B. Halsted, *Girolamo Saccheri's Eucledis vindicatus* (Chicago-London: Open Court Publications, 1920).

58. Sachau, *Chronologie* (see note 18), XXXIV:4. Following the MS (Leiden, Or 133, 45:12-13), Sachau reads *tajazzu' al-magādir lā ilā nihāya*. The correct reading is *tajazzu' al-magādir ilā lā nihāya*.

59. This is apparent from a comparison between the length of some other treatises of Al-Birūnī in MS Bankipore 2468 and their length according to his own indications in the list edited by Sachau (note 18). See the following table:

Treatise	No. of leaves in	No. of leaves	see Sachau page
	Bankipore 2468	acc. to Al-Birūnī	
A 33	50	200	XXXXXI
A 34	5½	15	XXXXXII
A 35	22	60	XXXXXII
A 40	17½	80	XXXXXIV
B	18 extant	70	XXXXXI
D	1 extant	10	XXXXXIV