This paper contains a critical edition of the Arabic text with English translation and notes of "Book XV" of the Revision of the Elements. It is still unclear whether this was an Arabic translation of a Greek text or a translation of a lost Arabic source (al-Magribi). It is an adaptation of a lost Arabic source (al-Maghraibi) which by al-Maghrabi is related to, but contains more than Book XIV of Euclid's Elements.

"Book XIV" of Euclid's Elements was addressed as "Book X" of the Arabic text of the Elements, which was added as "Book XIV" to the Greek text of the Elements. When was added as "Book XIV" to the Greek text of the Elements, the differences from the English translation and volume of the text are not, however, the primary problem. The French version of the English translation was published in one single volume, differing in solution of the general problem of the translation and notes. "Book XIV" of the Arabic text with English translation and notes of "Book XV" of the Revision of the Elements is the text examined in the Arabic manuscripts in Tunis, Omdurman, and Madrid. Book XIV of the Arabic text with English translation and notes of "Book XV" of the Revision of the Elements is the text examined in the Arabic manuscripts in Tunis, Omdurman, and Madrid.
Al-Maghribi describes the subject of XVI and MX in his Preface.

The mathematical contents of Book XVI of the Revision are as follows:

1. MXI: Proportional to Slides, Proportioned, SURFACE AND VOLUME.
   a. DMX: Of the ratio of the cubes (such as MXI, GEF) to the ratio of the faces of the solids.
   b. DMX: Of the ratio of the squares (such as MXI, GEF) to the ratio of the sides of the solids.

2. MXI: The ratio of the areas (such as MXI, GEF) to the ratio of the surfaces of the solids.
   a. DMX: Of the ratio of the plane figures (such as MXI, GEF) to the ratio of the plane surfaces of the solids.
   b. DMX: Of the ratio of the plane figures (such as MXI, GEF) to the ratio of the plane surfaces of the solids.

3. MXI: Of the ratio of the lines (such as MXI, GEF) to the ratio of the sides of the solids.
   a. DMX: Of the ratio of the lines (such as MXI, GEF) to the ratio of the sides of the solids.
   b. DMX: Of the ratio of the lines (such as MXI, GEF) to the ratio of the sides of the solids.

4. MXI: Of the ratio of the angles (such as MXI, GEF) to the ratio of the angles of the solids.
   a. DMX: Of the ratio of the angles (such as MXI, GEF) to the ratio of the angles of the solids.
   b. DMX: Of the ratio of the angles (such as MXI, GEF) to the ratio of the angles of the solids.

5. MXI: Of the ratio of the points (such as MXI, GEF) to the ratio of the points of the solids.
   a. DMX: Of the ratio of the points (such as MXI, GEF) to the ratio of the points of the solids.
   b. DMX: Of the ratio of the points (such as MXI, GEF) to the ratio of the points of the solids.

6. MXI: Of the ratio of the solids (such as MXI, GEF) to the ratio of the solids.
   a. DMX: Of the ratio of the solids (such as MXI, GEF) to the ratio of the solids.
   b. DMX: Of the ratio of the solids (such as MXI, GEF) to the ratio of the solids.

The purpose of Book XVI is to explain the geometric constructions corresponding to the book.

The main difference between Book XVI and his original version of the Elements is that the former introduces new propositions, while the latter simply restates and extends the theorems from the original.

There are interesting additions in the Revision such as a proof of the parallel postulate.

References:
- Suter, Art. No. 376 (p. 155).
- Krause, Sphélik, pp. 74-75.
- Tekeli in DSB IX, 555-557.
- The preceding information is taken from a thematic observational notebook.
Dealing with polyhedra, such as the tetrahedron, octahedron, and icosahedron, most of the proofs are very elaborate, and the manipulations of proportions are given in great detail, in the way of Books V and VI of the Elements. The ratios are determined correctly, except for the ratio between the volume of the octahedron and the icosahedron in Book XV.29. The proof of this proposition contains an error that must have escaped al-Maghribi. By making minimal changes in the text of Book XV.29 one obtains a correct theorem, and it will be shown in the notes to the translation that the proof in the text can be extended to form a correct proof in the style of ancient and medieval geometry. Because the proofs are very elaborate and the manipulations of proportions are very detailed, such as the one in Book XV, 4.7–22, most of the
The ratio between the bases and the ratio between the heights. Let $s_20$ and $s_8$ be the rectangles in the ratio $a+x$, $a+x'$ and $a+x''$ respectively.

**Theorem:** The bases are divided in the same ratio as the squares.

**Theorem:** The bases are divided in the same ratio as the squares.

**Construction:** Consider two rectangles with bases in the ratio $a+x$, $a+x'$ and $a+x''$ respectively.

**Construction:** Consider two rectangles with bases in the ratio $a+x$, $a+x'$ and $a+x''$ respectively.

**Construction:** Consider two rectangles with bases in the ratio $a+x$, $a+x'$ and $a+x''$ respectively.

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**Construction:** Consider two rectangles with bases in the ratio $a+x$, $a+x'$ and $a+x''$ respectively.
The historical background to the text

Book XV of al-Maghribi includes almost the whole mathematical content of 'Book XIV' of the Elements, written by Hypsicles. The theorems in MXV could not have been directly copied from (the Arabic translation of) El.XIV, because the points in the geometrical figures are labelled in a different way and because there are many differences in the details of the mathematical proofs. However, there are striking similarities in the theorems themselves. Thus the area of a pentagon is shown to be equal to the product of the five-sixths of the diagonal of the pentagon in El.XIV §7 and in MXV:11. There are many other ways of expressing the area of the pentagon, and therefore we can assume that the resemblance between Hypsicles and al-Maghribi is not accidental. The proposition El.XIV §6 (first proof of prop.6 in Heath, vol.3, p.516) is missing in MXV, but al-Maghribi gives in MXV:9-10 the preliminaries to this proposition which are not used anywhere in MXV. Similarly, the proof of El.XIV §10 (prop.8 in Heath, vol.3, p.518) does not occur in MXV, but al-Maghribi gives in MXV:22 a preliminary to this proof; again, this preliminary is not used. Thus it seems that the theorems on the icosahedron and the dodecahedron in Book XV of the Revision are based on a source detected in al-Maghribi's text, which was probably inspired by more Greek sources than Hypsicles' Book XIV of the Elements, in particular theorems of the icosahedron and the dodecahedron in Book XV of al-Maghribi, written in the early years of the 9th century.

The Hebrew translation can be divided into the following parts:

I. An introduction, containing references to the original (or perhaps a rendering of Latinus) and to a modern Arabicized form of Hypsicles that was used in the Hebrew translation.

II. A review of the theorems and the copernation, corresponding to Section 1 of the Introduction of the Elements.

III. A review of the theorems and the copernation, corresponding to Section 2 of the Introduction of the Elements.

IV. A review of the theorems and the copernation, corresponding to Section 3 of the Introduction of the Elements.

V. A review of the theorems and the copernation, corresponding to Section 4 of the Introduction of the Elements.

VI. Concluding theorems.

The translation of the Hebrew text is based on a modern Arabicized form of Hypsicles' Book XIV of the Elements, written in the early years of the 9th century. The translation includes references to the original (or perhaps a rendering of Latinus) and to a modern Arabicized form of Hypsicles that was used in the Hebrew translation. The translation is based on a modern Arabicized form of Hypsicles' Book XIV of the Elements, written in the early years of the 9th century.
Al-Kindi's mathematical works are listed by Sezgin in GAS V, 256.

XV. Al-Kindi (9th century A.D.) is known to have written a treatise on the correction of Books XIV and XV of the Elements of Euclid. 

I do not want to suggest that this was A, which is not concerned with Book XV, but the titles of Al-Kindi's works show that there are also in the other manuscripts and they may have been in the

Von Hoensbroech
It is important to restore the original text of Book XCVII of the Elements to gain a better understanding of the propositions in this section of the Elements. In particular, when the propositions are presented in the manuscripts, there may be differences in the text. To restore the original text, I have attempted to modernize the orthography, add hamzas, and make trivial corrections involving slashes and diacritical marks. I have also mentioned the scribal errors and variants in the manuscripts. In the Arabic text, the aid of the alphabetic numbers with slashes does not pose any philological problems.

Manuscript D is of little use for the reconstruction of the original text of Book XCVII, as it never has a plausible variant in cases where there are difficulties in the text. In Book XCVII, al-Maghribi often refers to previous propositions in his revision of the Elements, using alphabetical numbers with slashes. These alphabetical numbers can very easily be confused with slashes. In the manuscripts, these variations can be confusing.

In Book XCVII, the text is on the whole very simple, and the edition of al-Maghribi. The text in these manuscripts has been given for the recognition of the original of practical philosophers. In the Arabic text, the aid of the alphabetic numbers with slashes does not pose any philological problems.

I have added full stops and commas in the Arabic text for better legibility. Square brackets in the Arabic text contain the page numbers in the manuscripts. In the Arabic text and the apparatus, the four manuscripts have been abbreviated as follows: D = DMagdab, T = T��r, F = F Medina, H = H Medina. The same procedure as in the previous books is followed in this book.
Revision of the Elements by Michael de Pennington
Revision of the Elements by Mohiy al-Din al-Maghribi
Regression of the Elements by Mutual Interaction

Jan P. Hoogenraad
In the translation, I have added brackets to indicate changes I have made to the Arabic text. These changes are typical in translation, and I have done so to make the text more comprehensible to modern readers. For example, I have added punctuation, including parentheses and commas, to improve clarity.

In some cases, I have also added explanatory notes to help readers understand the text. These notes are indicated by a superscript number, and they are located at the end of the page.

The figures are printed in the Arabic translation, and some of the three-dimensional figures may look odd to modern eyes because they have been drawn in the way they appear in the manuscripts. We can suppose that more medieval students of Book XV of the Elements were familiar with these figures.

As long as the Hebrew text has not been published, it is not possible to give an adequate historical commentary on individual propositions in the translation. A few are however known to have been published, and I have noted these in the translation. For example, proposition 46 of Book IV of the Elements is a theorem that is not found in the Arabic text, and I have added it to the translation for the benefit of modern readers.

The references in the translation are to the English translation of Euclid's Elements by Heath, as well as to the Arabic text of the Elements, which is not available in a modern edition. The modern reader is assumed to have a detailed knowledge of Euclid's Elements, and the translation is intended to assist in understanding the text.

I have attempted to make the translation as faithful as possible to the original text, but I have also made some changes to improve clarity. These changes are indicated by a superscript number, and they are located at the end of the page.

In conclusion, the translation is intended to be a helpful resource for modern students of mathematics and the history of mathematics.
Translation of Book XV of the Revision of the Elements.

1. The square of the altitude of every equilateral triangle is three-fourths of the square of its side. Thus let triangle ABG be equilateral, and (let) its altitude (be) AD. I say that the square of AD is three-fourths of the square of AB. 

Proof: Angles ABG and AGB are equal, that is, and equal to the square of the altitude of the triangle ABG. Thus the square of AD is equal to the square of the altitude of the triangle ABG. Thus the square of AD is three-fourths of the square of AB.

2. The side of the octahedron is equal to the altitude of the triangle of the pyramid inscribed in the same sphere. 

Proof: The square of the diameter of the sphere is one and a half times the square of the side of the octahedron, that is, AB, because the excenters are drawn from the center of the circle to the excenters of the circles drawn on the sides of the pyramid, and the center of the sphere is equal to the excenter of one of the sides of the pyramid. Thus the square of AB is one-fourth of the square of DB. Let the square DB be twice the square of AB. Hence, the square of AB is three-fourths of the square of DB. Three-fourths of the square of DB is equal to twice the square of AB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. Hence, the square of AB is equal to the square of DB. 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5. The square of the side of the octahedron is one and a half times the square of the side of the cube.

6. To every circle, the sum of the squares of the sides of the regular pentagon and the chord of the angle of the pentagon is five times the square of the radius of the circle circumscribing the pentagon.

Example: Let there be a circle with diameter AB. (Let the side of the regular pentagon be GD, and the chord of the angle of the pentagon be DE.) Then the square of the side of the circle is twice the square of the radius of this circle, the side of the circle is equal to the square of GD, and (since) the square of the chord, the sum of the squares of GD and DE is equal to the square of AB. I say that the square of GD is equal to the square of the side of the regular pentagon (GD), and the square of DE is equal to the square of the chord of the regular pentagon (DE).
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Then the ratio of MN to CO is equal to the ratio of QN to SO, XIV:106. Thus the ratio of the square of MN to the square of CO is equal to the ratio of the square of QN to the square of SO, VI:22. But it has been shown that the square of MN is five times the square of OC, XV:4. Thus the square of QN is five times the square of SO. Thus the (sum of the) squares of MN, NQ is fifteen times the square of LK, XIII:15 (= E1.XIII:10), that is to say three times the square of LK, XIII:118. Thus the (sum of the) squares of MN, NQ is fifteen times the square of LK.

Again, the ratio of the square of MN to the square of BE, that is the side of the cube, is equal to the ratio of the square of QN to the square of FE. But the square of MN is three times the square of BE, XV:3 (= E1.XIII:9). Thus the square of QN is three times the square of FE. Thus the (sum of the) squares of MN, NQ is three times the (sum of the) squares of BE, EF. But the (sum of the) squares of BE, EF is five times the square of DZ, XV:4. Thus the (sum of the) squares of MN, NQ is fifteen times the square of DZ. Thus the squares of LK and DZ are equal, so lines LK and DZ are equal. Thus the circles ABGDE and HTK are equal. That is what we wanted to prove.

It has become clear from this that the perpendicular falling from the centre of the sphere on the face of the dodecahedron is equal to the perpendicular falling from the centre of the sphere on the face of the icosahedron, because the perpendiculars falling from the centre of the sphere on equal circles drawn on its surface are equal. That is what we wanted.

5.6. The reference should be to MXII:10 (see the appendix for the text and translation).
5.7. This is an interesting reference. We locate the diameter MN of the sphere and the perpendicular falling from the centre of the sphere to a face of the dodecahedron.

5.10. Proved in a lemma at the end of MXIII, cf. note 3.2.
1. The proof is in the same geometrical style as that by Pythagoras in Euclid IX § 1.

2. The text is in Latin, with some Greek script.

3. The diagram illustrates a geometric figure, likely a cube and a related polyhedron, with labeled points and lines.

4. The text discusses the ratio of the side of the cube to the side of the icosahedron and provides a proof involving squares and ratios.

5. The text refers to Propositions and Theorems from previous works, including those by Euclid.

6. The proof involves algebraic manipulation and geometric constructions.

7. The text concludes with a statement about the equality of certain segments.

8. The reference to MV:14 (= XI:16) and VI:15 is made, likely referring to previous works or propositions.

9. The text contains mathematical symbols and notation, typical of Euclidean geometry.

10. The text is in a form that requires careful reading and understanding of mathematical language.
The ratio of the triangle of the octahedron to the square of the side of the cube is equal to the ratio of half the altitude of the triangle to two-thirds of its side. 

Proof: We make the triangle of the octahedron AGH, with center Z, and let the perpendicular drawn from Z to the base AB be center D, and let the perpendicular drawn from the center D to AB be the side of the cube. That is what we wanted to prove.

The product of the perpendicular drawn from the center of the dodecahedron to the side of the dodecahedron is equal to one-third of the surface of the dodecahedron.

Example: Let the triangle of the dodecahedron be triangle ABC, with the side of the dodecahedron divided into six congruent triangle ABC. Then the product of the perpendicular drawn from the center of the dodecahedron to the side of the triangle is equal to one-third of the surface of the dodecahedron.

Proof: We make the triangle of the dodecahedron ABC, with center D, and let the perpendicular drawn from D to the base AB be center E. That is what we wanted to prove.

The ratio of the triangle of the octahedron to the square of the side of the cube is equal to the ratio of half the altitude of the triangle to two-thirds of its side. That is what we wanted to prove.

The product of the perpendicular drawn from the center of the dodecahedron to the side of the dodecahedron is equal to one-third of the surface of the dodecahedron.

Example: Let the triangle of the dodecahedron be triangle ABC, with the side of the dodecahedron divided into six congruent triangle ABC. Then the product of the perpendicular drawn from the center of the dodecahedron to the side of the triangle is equal to one-third of the surface of the dodecahedron.

Proof: We make the triangle of the dodecahedron ABC, with center D, and let the perpendicular drawn from D to the base AB be center E. That is what we wanted to prove.
The ratio of the square of the side of the cube to the surface of the octahedron is equal to the ratio of the square of the side of the equilateral triangle to the altitude. Thus the product of the altitude of the octahedron is equal to the altitude of the equilateral triangle. This is the same as the product of the cube and the octahedron. 

The Revision of the Elements by Muhā¯lI’I’sān

1. The area of every pentagon circumscribed by a circle is equal to the product of three-fourths of the diameter of the circle and five-sixths of the chord of the angle of the pentagon. Thus let pentagon $ABGDE$ be inscribed in a circle with diameter $AZH$ and centre $Z$, and let $BE$ be the chord of the angle of the pentagon $ABGDE$. Then let $AZ$ be perpendicular to $ABGDE$. Then the product of $AK$ and $BT$ is equal to the product of the altitude of the pentagon $ABGDE$. That is what we wanted to prove.
The theorem is valid for a pentagon and decagon inscribed in any circle. In the proof the circle is assumed to have a "circle of the icosahedron," that is an circle which circumscribes a regular pentagon formed by five sides of an icosahedron.

13.1. The references should be to IX.15 (= XI.10) or MXIII:15 (= XI.10).

13.3. The references should be to IX.14 (= XI.14).

The theorem is valid for a pentagon and decagon inscribed in any circle. In the proof the circle is assumed to have a "circle of the icosahedron," that is an circle which circumscribes a regular pentagon formed by five sides of an icosahedron.

The theorem is valid for a pentagon and decagon inscribed in any circle. In the proof the circle is assumed to have a "circle of the icosahedron," that is an circle which circumscribes a regular pentagon formed by five sides of an icosahedron.

13.2. I do not know which lemma is meant here. The lemma must have been a
The ratio of the surface of the icosahedron to the surface of the octahedron is equal to the ratio of five times the square of $B$ to the square of $A$.

Proof: We make the diagram and then find the square of $A$ and $B$ and divide $BE$ in the two parts of proportionate to $BE$ and $DG$ and divide $BE$ in the two parts of proportionate to $BE$ and $DG$.

The ratio of the excess of the surface of the icosahedron over the surface of the octahedron to the ratio of the excess of the surface of the icosahedron over the surface of the octahedron is equal to the ratio of $B$ to the square of $A$.

Proof: We make the diagram and then find the square of $A$ and $B$ and divide $BE$ in the two parts of proportionate to $BE$ and $DG$.
I. The surface of the octahedron is equal to the sum of the surface of the product of the plane triangles of the equilateral triangles and the square of the side of the decagon.

II. The ratio of the surface of the product of the plane triangles of the equilateral triangles to the square of the side of the decagon is equal to the ratio of the surface of the product of the plane triangles of the equilateral triangles to the square of the side of the dodecahedron.

Proof: We make \( BE \) and \( HI \) equal to the product of \( AB \) and \( DF \). Then the product of \( AB \) and \( DF \) is equal to the product of \( AB \) and \( HI \). This makes \( BE \) and \( HI \) equal to the product of \( AB \) and \( DF \).

The ratio of the surface of the octahedron to the surface of the dodecahedron is equal to the square of the ratio of the surface of the dodecahedron to the surface of the octahedron.

Thus, the ratio of the surface of the dodecahedron to the surface of the octahedron is equal to the ratio of the square of the product of the plane triangles of the equilateral triangles to the square of the side of the decagon.

This becomes clear from this fact, compounded, the ratio of the surface of the dodecahedron to the surface of the tetrahedron is equal to the square of the ratio of the surface of the tetrahedron to the surface of the dodecahedron.
16.1. The Utrecht and Oxford manuscripts have one of the triangles instead of “the surface. The text in the Mhrishah manuscript is correct (the whole passage is missing in the Ayasofya manuscipt.)

16.2. The abbreviation “prop.” serves as a translation of the abbreviations in the manuscripts (for “chakl,” meaning proposition).

17.1. The text uses the same word (qaida) for “base” (of a parallelepiped) and “face” (of a regular polyhedron).

17.2. The text sometimes uses the terms “pyramidalsolid,” “octahedralsolid,” etc., mainly in cases where we would use the terms “volume of a pyramid,” “volume of an octahedron,” etc.

18.1. The text should be read at the end of the previous passage of the commentary.

18.2. The text sometimes uses the terms “pyramidalsolid,” “octahedralsolid,” etc., mainly in cases where we would use the terms “volume of a pyramid,” “volume of an octahedron,” etc.

Proof: The parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

It follows from this that the parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

The ratio of the tetrahedral solid to the (circumscribing) sphere is equal to the octahedral solid to the circumscribing sphere.

Proof: The octahedron with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

The ratio of the tetrahedral solid to the (circumscribing) sphere is equal to the octahedral solid to the circumscribing sphere.

Proof: The parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

It follows from this that the parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

The ratio of the tetrahedral solid to the (circumscribing) sphere is equal to the octahedral solid to the circumscribing sphere.

Proof: The parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

It follows from this that the parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

The ratio of the tetrahedral solid to the (circumscribing) sphere is equal to the octahedral solid to the circumscribing sphere.

Proof: The parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.

It follows from this that the parallelepipedal solid with base equal to one-ninth of the diameter of the circumscribing sphere is equal to the octahedron.
21. (For) every equilateral and equalangular solid circumscribed by a sphere, if the solids and the center of the sphere are joined by straight lines, the number of faces of the solid, the number of which is equal to the number of faces of the equilateral solid, is equal to the ratio of the area of the base of one of the solids to the area of the base of the other.

22. (For) every two pyramids whose axes are equal and perpendicular to the bases.

22.1. In propositions 21-23 the pyramidal figures and pyramids are not necessarily regular. Even though the text uses the terms "pyramid" and "pyramidal," which usually mean a solid whose base is a polygon and whose vertices are the center of the sphere, this theorem does not imply that the pyramids are necessarily regular. The text uses the terms "pyramid" and "pyramidal" to indicate a solid that is divided into equal and similar pyramids. The axes of these pyramids are equal and perpendicular to the bases. The number of these pyramids is equal to the number of faces of the equilateral solid. The intersection of these figures is equal to the ratio of the area of the base of one of the solids to the area of the base of the equilateral solid.

Example: Let there be a cube with a side of 2 and a sphere of radius 1. Join the vertices of the cube with the center of the sphere to form six pyramids. The area of the base of each pyramid is equal to the area of a face of the cube. Therefore, the number of pyramids is equal to the number of faces of the cube. The intersection of these figures is equal to the ratio of the area of the base of one of the solids to the area of the base of the cube.
211. The Revision of the Elements by Muhyi al-Din al-Misri, 1212

212. The table shows the regular pentagon of the right angle of the circle, but not of the triangle.

213. The ratio of the solid of the cubo to the octahedron is equal to the side of the equilateral triangle to its altitude.

214. The parallelepiped solid of the base of which is equal to the area of the equilateral triangle.

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300. The parallelepiped solid of the base of which is equal to the area of the equilateral triangle.
The ratio of the perpendicular of the icosahedron to the diameter of the circumscribing sphere is equal to the ratio of the side of the decagon plus (half of) the side of the hexagon to twice the altitude of the triangle of the side of its pentagon.

Thus let the pentagon of the icosahedron be ABODE, let its centre be Z and let the perpendicular drawn from the centre be ZH. Let L be the perpendicular of the icosahedron and K the diameter of the circumscribing sphere. We construct on ZH the equilateral triangle DLG. We join LH; it is clearly perpendicular to ZH.

Is the ratio of ZH, which is equal to one-third of L, to twice HL, which is equal to the height of the icosahedron, equal to the ratio of the surface of the icosahedron to the surface of the pentagon ABGDE?

Thus the ratio of ZH to twice HL is equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K.

Thus let the perpendicular of the icosahedron be ABGDE, let its centre be Z and let the perpendicular drawn from the centre be ZH. Let L be the perpendicular of the icosahedron and K the diameter of the circumscribing sphere. We construct on ZH the equilateral triangle DLG. We join LH; it is clearly perpendicular to ZH.

Is the ratio of ZH, which is equal to one-third of L, to twice HL, which is equal to the height of the icosahedron, equal to the ratio of the surface of the icosahedron to the surface of the pentagon ABGDE?

Thus the ratio of ZH to twice HL is equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K.

Thus let the perpendicular of the icosahedron be ABGDE, let its centre be Z and let the perpendicular drawn from the centre be ZH. Let L be the perpendicular of the icosahedron and K the diameter of the circumscribing sphere. We construct on ZH the equilateral triangle DLG. We join LH; it is clearly perpendicular to ZH.

Is the ratio of ZH, which is equal to one-third of L, to twice HL, which is equal to the height of the icosahedron, equal to the ratio of the surface of the icosahedron to the surface of the pentagon ABGDE?

Thus the ratio of ZH to twice HL is equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K.

Thus let the perpendicular of the icosahedron be ABGDE, let its centre be Z and let the perpendicular drawn from the centre be ZH. Let L be the perpendicular of the icosahedron and K the diameter of the circumscribing sphere. We construct on ZH the equilateral triangle DLG. We join LH; it is clearly perpendicular to ZH.

Is the ratio of ZH, which is equal to one-third of L, to twice HL, which is equal to the height of the icosahedron, equal to the ratio of the surface of the icosahedron to the surface of the pentagon ABGDE?

Thus the ratio of ZH to twice HL is equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of one-third of L to one-third of K, that is, equal to the ratio of the surface of the pentagon ABGDE to the surface of the icosahedron, that is, equal to the ratio of one-third of L to one-third of K.
Proof: The perpendicular of the octahedron is equal to the perpendicular of the cube, XV:3. But the square of the radius of the sphere is three times the square of the perpendicular of the cube, XIV:3'. Therefore it is three times the square of the perpendicular of the octahedron. But the square of the altitude of the equilateral triangle is three times the square of half of its side, XV:1 and 11:4. Therefore the ratio of the square of the radius of the sphere to the square of the perpendicular of the octahedron is equal to the ratio of the square of the altitude of the (equilateral) triangle to the square of half of its side. Therefore the ratio of the radius of the sphere to the perpendicular of the octahedron is equal to the ratio of the altitude of the triangle to half of its side, VI:22. That is what we wanted.

27. The ratio of the perpendicular of the icosahedron solid to the perpendicular of the octahedron is equal to the ratio of the sum of the sides of the hexagon and the decagon inscribed in one circle to the side of the pentagon of the (i.e. this) circle.

Proof: Since the ratio of the perpendicular of the icosahedron solid to the diameter of the sphere is equal to the ratio of half of the side of the decagon and half of the side of the hexagon taken together to twice the altitude of the triangle of the icosahedron, XV:25, and since the ratio of the diameter of the sphere to the perpendicular of the octahedron is equal to the ratio of twice the altitude of the triangle of the icosahedron to half of its side, XV:26, and it follows, by the sum of the three, that

26. Euclid proves in El. XI. 15 that the square of the diameter of the sphere is three times the square of the side of the cube. Al-Maghribi concludes in a corollary to MXIV:3 (= El. XI. 15) that the square of the radius is three times the square of the perpendicular from the center to any face of the cube.

27.1. It is tacitly assumed here that the above-mentioned decagon and hexagon are inscribed in a circle through five angular points of the icosahedron.

27.2. The passage "and it follows...triangle" is mathematically superfluous, because it repeats what has already been said. However, it is not the exact repetition of the preceding passage, so it cannot be ascribed to error.

27.3. "By the sum of the three" is my translation of the Arabic bi-majmuduhim. What is necessary here is not addition but multiplication of the ratios. The Euclidean terms for the "composition" of ratios (derived from suntithra), can have an additive and a multiplicative meaning (see Heath's translation of the Elements, vol. 2, p. 135). We could therefore explain the nonsensical "sum" in our text by assuming that proposition 27 is of Greek origin, that a technical term like "suntithra" in the Greek was mistranslated into Arabic in the additive meaning, and that the passage following was an attempt to explain the rule. However, it is not the composition of the ratios.

28. The ratio of every two parallelepipeds is compounded of the ratio of their bases and the ratio of their breadths.

Proof: We extend the CH and we cut the Kl. We extend the ZH and we cut the HK. We extend the KH and we cut the KZ. The length of the KL is equal to the KH.

22. The ratio of every two parallelepipeds rectangular solids is equal to the sum of the sides of the icosahedron solid to the perpendicular of the octahedron solid.

23. The revolution of the equilateral triangle is equal to the sum of the sides of the icosahedron solid to the perpendicular of the octahedron solid.
The ratio of the volume of the tetrahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon. Therefore, we can make the base of the parallelepiped solid of the icosahedron equal to the base of the octahedron, and the height of the parallelepiped solid of the octahedron equal to the height of the parallelepiped solid of the icosahedron.

**Proof:** We make the parallelepiped solid of the icosahedron equal to the height of the parallelepiped solid of the octahedron.

Then the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

29.4. Theorem 29.3 actually states that the ratio of the side of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the pentagon to the side of the hexagon.

29.5. Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

29.6. Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

29.7. Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Let DE be the side of the decagon of the (certain) circle, and let solid C be the solid of the octahedron. Then the ratio of the side of the decagon of the (certain) circle to the side of the pentagon is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Then the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

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Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.

Therefore, the ratio of the volume of the icosahedron to the volume of the octahedron is equal to the ratio of the side of the hexagon to the side of the pentagon.
The proposition in question is the following: Let the ratio of the surfaces of the two solids be equal to the ratio of their volumes. Then, the ratio of their surfaces is equal to the ratio of their volumes.

Proof: Let the ratio of the surfaces of the two solids be $\frac{a}{b}$, and let the ratio of their volumes be $\frac{c}{d}$. Then, we have $a:b = c:d$.

By the properties of similar solids, we know that the ratio of their volumes is equal to the cube of the ratio of their corresponding linear dimensions. Therefore, $\left(\frac{a}{b}\right)^3 = \frac{c}{d}$.

Solving for $\frac{a}{b}$, we get $\frac{a}{b} = \left(\frac{c}{d}\right)^{\frac{1}{3}}$.

This proves the proposition, as the ratio of the surfaces of the two solids is equal to the ratio of their volumes.

...
we want to find lines proportional to the sides of the five figures. Thus we draw the circle circumscribing the pentagon of the dodecahedron and the triangle of the icosahedron. Let the side of its pentagon be $AB$, (let) the side of its triangle be $GD$, and (let) the chord of two-fifths of it be $AE$. Then it (AE) is the side of the cube, X14:9.

Let the square of $ZH$ be equal to twice the square of $AE$. We bisect it ($ZH$) at $N$, and we draw with centre $N$ and radius $NH$ a circle, let $TK$ be the side of its triangle, and $LM$ be the side of its square. Then the square of $ZH$ is four times the square of $NH$. But the square of $NI$ is one-third of the square of $TK$, (by prop.) X13:11'. Therefore the square of $ZH$ is one and a third times the square of $TK$.

But the square of the diameter of the sphere is one and a half times the square of the side of the pyramid, X14:1 (E1.X13:12), and it is twice the square of the side of the octahedron, X14:5 (E1.X13:14). Therefore the ratio of the square of the side of the pyramid to the square of the side of the octahedron is to the ratio of the square of $TK$ to the square of $LM$ is equal to the ratio of the side of the octahedron to the side of the cube. Therefore the ratio of the square of $Z$, that is the square of the side of the pyramid, to the square of $TK$, that is the square of the side of the octahedron, is equal to the ratio of the square of $TX$ to the square of $LM$. That is, the square of the side of the pyramid is to the square of the side of the octahedron as the square of $TX$ is to the square of $LM$. That is the side of the cube to the side of the icosahedron.

Therefore the ratio of $ZR$ to $UK$ is equal to the ratio of the side of the pyramid to the side of the octahedron and the ratio of $GK$ to $AL$ is equal to the ratio of the side of the icosahedron to the side of the dodecahedron. Since $AB$ is the side of the dodecahedron and $GD$ is the side of the icosahedron, the ratio of $GD$ to $AB$ is greater than the composition, and that the side of the icosahedron is greater than the side of the octahedron, and that the side of the dodecahedron is greater than the side of the cube. It has become clear from this that the side of the pentagon is greater than the side of the circle.

It is noteworthy that the side of the dodecahedron is greater than the side of the composition, and that the side of the icosahedron is greater than the side of the octahedron. The reason is that the side of the dodecahedron is greater than the side of the triangle, while the side of the pentagon is greater than the side of the circle. If we take the side of the dodecahedron as the radius of the circle, then the side of the pentagon is greater than the side of the triangle, and the side of the icosahedron is greater than the side of the octahedron. Therefore the ratio of $ZR$ to $UK$ is equal to the ratio of the side of the pyramid to the side of the octahedron.

Therefore the side of the tetrahedron is greater than the side of the octahedron and the side of the icosahedron is greater than the side of the dodecahedron and the side of the pyramids is greater than the side of the octahedron and the side of the icosahedron is greater than the side of the pyramids. Therefore the ratio of the side of the pyramid to the side of the octahedron is to the ratio of the side of the tetrahedron to the side of the icosahedron as the ratio of the side of the pyramid to the side of the octahedron.
Equal to L.

32. The ratio of the perpendicular of the pyramid to the perpendicular of the cube is equal to the ratio of one-third of the altitude of the equilateral triangle to halfof its side. Thus let the perpendicular of the pyramid be A, (let) the radius of the sphere circumscribing it (be) B, and (let) the perpendicular of the cube (be) G. Let D be the altitude of the equilateral triangle, (let) halfof its side (be) E, and (let) Z (be) three times E.

Then A is one-sixth of the diameter of the sphere, XIV:1 (= E1/6).

Thus we draw a circle such that the side of the parallel AB and AD equal to A.

33. We want the ratio of the squares of the perpendiculaers to the bases.

\[ \frac{A^2}{G^2} = \frac{D^2}{Z^2} \]

32.1 By MXIV:3, corollary (see note 26.1)

We want to find the ratio of the perpendiculars to the bases.

To find the ratio of the perpendiculaers to the bases.

\[ \frac{A^2}{G^2} = \frac{D^2}{Z^2} \]

Thus we draw a circle such that the side of the parallel AB and AD equal to A.

32.2 The term perpendiculaers indicates a reasoning of the following form: if A = B, then E = Z. Therefore the squares in the following.

Pierce to halfof its side, that is, one-third of Z. That is what we wanted to prove.
34. The proportion of the perpendiculars, namely, that of the height to the side of the pyramid is equal to the proportion of the side of the pyramid to the side of the base.

35. We join BD. Then the square of AB is three times the square of BD, because of the difference.

36. We make AE equal to AB and AD.

37. We construct the quadrangle ABCD. We draw the altitude of the pyramid, and the altitude of the octahedron.

38. Therefore the ratio of the square of the perpendiculars of the pyramid to the square of the side of the pyramid is equal to the ratio of the square of the side of the pyramid to the square of the side of the base.

39. Then we divide BD in extreme and mean ratio at E, let the greater part BE be equal to BD. Then the square of AB is three times the square of BD.

40. We make AF equal to AB and AD.

41. We construct the quadrangle ABCD. We draw the altitude of the pyramid, and the altitude of the octahedron.

42. Therefore the ratio of the square of the perpendiculars of the pyramid to the square of the side of the pyramid is equal to the ratio of the square of the side of the pyramid to the square of the side of the base.

43. Then we divide BD in extreme and mean ratio at E, let the greater part BE be equal to BD. Then the square of AB is three times the square of BD.

44. We make AF equal to AB and AD.

45. We construct the quadrangle ABCD. We draw the altitude of the pyramid, and the altitude of the octahedron.

46. Therefore the ratio of the square of the perpendiculars of the pyramid to the square of the side of the pyramid is equal to the ratio of the square of the side of the pyramid to the square of the side of the base.

47. Then we divide BD in extreme and mean ratio at E, let the greater part BE be equal to BD. Then the square of AB is three times the square of BD.
The ratio of the face of one of the five solids to that of the other is equal to the ratio of the surface of the face of the former to that of the后者.

The ratio of the face of the octahedron to that of the cube is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the octahedron to that of the pyramid is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the pyramid to that of the cube is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the tetrahedron to that of the pyramid is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the pyramid to that of the cube is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the tetrahedron to that of the pyramid is equal to the ratio of the surface of the face of the former to that of the latter.

The ratio of the face of the pyramid to that of the cube is equal to the ratio of the surface of the face of the former to that of the latter.
37. We want to find the lines proportional to the five solids.

38. We have $KAD:MN:SO = c:d:e:f:3$, and therefore the following correct solution of the problem of prop. 35 (cf. note 35): $K:E:F:G:H:M:W:3:3$. The ratio of the face of the octahedron to the face of the pyramid is equal to the ratio of half of MN to two-thirds of SO. Therefore, the ratio of the face of the cube to the face of the pyramid is equal to the ratio of the face of the cube to the face of the pyramid. Thus, we have found the five lines proportional to the five solids. The inversion of the cube is equal to the inversion of the cube.

The inversion of the cube is equal to the inversion of the cube.

39. The inversion of the cube is equal to the inversion of the cube.

40. The inversion of the cube is equal to the inversion of the cube.
Then the number of propositions in the book, except the lemmas and the different cases, is 516.

Appendix. Propositions XI.10, 14 of the Revision of the Elements.

10. (For) every pair of different lines, each of which is divided in

14. If the side of the heptagon is divided in extreme and mean ratio,

Book XI. Prop. 14

10 7 6 5 4 3 2 1

A B C D E F G

is what we wanted.

The ratio of AF to GO, and equal to the ratio of EF to ZD, V:18, Thea.

The Revision of the Elements by D. E. Smith, 1906.


Cambridge 1925, Reprint New York (Cover) 1996.


(1925) 1932."Appendix. Propositions XI.10, 14 of the Revision of the Elements."}

10. (For) everypairofdifferentlines,eachofwhichisdividedin

extremeandmeanratio,theratioofoneline to thesecond(line)isequaltoseratiorofitis greaterpart to the greaterpart of thesecond(line),andequaltetheratiorofits lesserpart (of the first line) to the lesserpart (of the second line).’.
Acknowledgements.

The Revision of the Elements by Al-Adim al-Maghribi

Institutions for their help.

The Bodleian Library at Oxford made available to me a unique manuscript of al-Maghribi's work on the Elements, which was preserved in the library. The permission to consult the manuscript was granted by the library director. A handwritten copy of the manuscript was made available in June 1996, which I used to support my research. The manuscript was discovered and is now preserved in the library.

The Oxford University Press provided me with a translation of the Elements by Dr. M. H. Kahn. The translation was provided by Dr. J. T. L. Koshy.

The only medieval Arabic mathematical manuscript in the Utrecht University Library is the anonymous and incomplete MS. Or. 448. The manuscript was edited here, and I wish to thank the staff of the library for their generous help. Prof. D. Y. Young of the American University of Cairo drew my attention to the anonymous MS. Or. 448, and thanks to his reference, the authorship of the manuscript was confirmed. The manuscript contains fragments of the Elements, which were restored by Dr. M. H. Kahn. The manuscript was provided by the Bodleian Library at Oxford.

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