Current Studies in the History of Mathematics and Astronomy in Islamic Civilization (2nd - 9th centuries of the Hegira)

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1 Introduction

The history of Mathematics and Astronomy in Islamic Civilization is currently being researched in many countries in the world. The subject is very extensive and books and papers are published in many languages, so it is not easy to get an overview. I will devote the main part of my lecture to a personal impression of some new directions of research which have been important in the last ten years, and which may somewhat change the overall perspective on the development of science in the medieval Islamic world. I will discuss some concrete examples, and it is therefore inevitable that a very large part of current research will be left out in this lecture because time is short. I ask your indulgence for such omissions, which do not imply any value judgement. I can only discuss a few examples here and I have had to omit a lot of extremely valuable recent research [1].

Until recently, the emphasis in research in the history of Islamic mathematics and astronomy was to a large extent on the history of arithmetic, and on theoretical texts on algebra, pure geometry, pure trigonometry, and astronomy, mainly theory of the motion of the sun, moon and planets. Of course, the research in these fields still continues, and important discoveries have been made in the last years. A good example is the discovery by Professor Menso Folkerts of Munich in Germany of a complete Latin manuscript of the *Arithmetic* of Muḥammad ibn Mūsā Al-Khwārizmī [2]. The *Arithmetic* of Al-Khwārizmī is of major importance in the history of mathematics as a whole. The historical role of this text is such that we still call any method of computation an *algorism*. Thus far, this fundamental text by al-Khwārizmī was only known in incomplete Latin manuscripts, and the Arabic version has never been found.

In recent years, some important Arabic texts have also been published for the first time. An example is the *Memoir on Cosmology* (al-Tadhkira fī ᶜIlm al-Hay'at) by Naṣīr al-Dīn al-Ṭūsī, published (with an English translation and an excellent commantary) by Jamil Ragep in 1993 [3]. A lot of work has also been done on mathematics and astronomy in Islamic Spain [4].

The emphasis in recent research has also widened to include the history of practical mathematics and practical astronomy, the interaction of theory and practice, and science in context. I will now describe some characteristic examples of this type, which will indicate the future potential of this kind of research.

2 Astronomical tables: estimation of parameter values

The Islamic mathematicians and astronomers have left us more than one hundred different astronomical handbooks which usually contain more than 100 numerical tables, to be used in astronomical computations. The computer can be used for extracting information about the ways in which these tables were computed and the underlying parameter values. These methods have been applied to various medieval texts by two young researchers, Benno van Dalen from Frankfurt (Germany) and Glen van Brummelen from Edmonton (Canada). I will discuss some of the work of Benno van Dalen on the *Jāmiʿ Zīj* of the Iranian astronomer Kūshyār ibn Labbān al-Jīlī (ca. 390 H.)

mean solar motion	equation of time	mean solar motion	equation of time	mean solar motion	equation of time
0	9,48	120	12,0	240	27,56
6	11,52	126	11,56	246	25,48
12	13,52	132	12,12	252	23,16
18	15,52	138	12,52	258	20,32
24	17,40	144	13,52	264	17,24
30	19,08	150	15,08	270	14,20
36	20,16	156	16,36	276	11,12
42	21,12	162	18,24	282	8,20
48	21,48	168	20,12	288	5,48
54	21,56	174	22,12	294	3,40
60	21,44	180	24,08	300	1,56
66	21,08	186	26,02	306	0,44
72	20,20	192	27,44	312	0,08
78	19,16	198	29,16	318	0,00
84	17,56	204	30,24	324	0,20
90	16,44	210	31,12	330	1,08
96	15,24	216	31,32	336	2,24
102	14,08	222	31,24	342	3,52
108	13,16	228	30,48	348	5,40
114	12,28	234	29,40	354	7,40

Kūshyār's table for the equation of time

Mr. van Dalen's work concerns Kūshyār's table for the equation of time, so I will begin with a non-technical introduction of this concept. During the day, the time can be measured from the position of the sun in the sky, and in this way one obtains the true solar time. The period between two successive culminations of the sun is approximately

24 hours, but the exact interval is subject to small seasonal variations. In other words, it is not possible to set a watch in such a way that the sun is exactly in the South at 12 o'clock on all days of the year. The variation is caused by the fact that the sun moves on the ecliptic (not the celestial equator), with a non-constant velocity.

The cumulative effect of this variation could be computed by the medieval Islamic astronomers. This quantity is called the "equation of time." One has to subtract the equation of time from the the true solar time to obtain the mean solar time, which is used in astronomical computations.

Kūshyār's table for the equation of time follows here. One enters the table with the mean solar longitude (which can be computed when one knows the date in the solar calendar), subject to a small displacement which does not concern us here. The table shows the equation of time in minutes and seconds. For example, in the month Esfand (February 22 - March 21), the mean solar motion is between 330 and 360 degrees so the equation of time varies between 1 minute and 8 minutes.

Kūshyār computed the values of the equation of time E according to the following function:

$$E(\tilde{\lambda}) = \frac{1}{15}(\tilde{\lambda} + \Delta - \alpha(\tilde{\lambda} + \Delta - \bar{q}(\tilde{\lambda} + \Delta)) + c),$$

with

$$\alpha(x) = \arctan(\cos\epsilon \cdot \tan x),\ \ \bar{q}(x) = \arctan(\frac{e\sin(x - \lambda_A)}{60 + e\cos(x - \lambda_A)}).$$

Here $\tilde{\lambda}$ is the argument in the table (the "displaced mean solar longitude"), ϵ is the obliquity of the ecliptic, e the solar eccentricity, λ_A the solar apogee, c the so-called epoch constant, and Δ the so-called displacement. The details of the definitions are not important; what is important is that we have a table in a medieval manuscript which was computed according to a known mathematical function using parameter values ϵ, e, λ_A, c and Δ which were known to the medieval astronomer but which are usually unknown to us.

In this case, Kūshyār tells us some of the parameter values that he used, but in most cases such information is not given in the texts. Even in this case, one would like to check if these parameter values were really used in the table, for this would be a proof of the authenticity of the table. Therefore one needs statistical estimators to recover the parameter values from the numbers in the table. Statistics is necessary because the computation involves rounding and other random errors.

Mr. van Dalen developed these estimators and he showed that the parameter values lie in the following 95 % confidence intervals:

ϵ	$(23°33'4'', 23°35'0'')$
e	$(2°4'27'', 2°4'55'')$
λ_A	$(84°1'48'', 84°14'26'')$
c	$(4°3'44'', 4°4'2'')$
Δ	$(1°57'37'', 2°2'27'')$

In this case it turns out that the table in the Zīj al-Jāmiᶜ is authentic, for Kūshyār tells us that he used $\Delta = 2$, $c = 4°4'$, $\lambda_A = 84$. Elsewhere Kūshyār used values $\epsilon = 23°35'$ and $e = 2°4'45''$ so one can assume that he also used them in this table. All parameter values have now been determined, hence one can recompute the table and try to find even more information on Kūshyār's methods of computation.

In this example, the computer is used as a tool for extracting information on parameter values, and for gaining insight in the *practical* mathematical methods that were used by Kūshyār. There are still lots of source material waiting to be analyzed by means of the computer. By means of such analyses, the available information on *practical* computations and astronomical parameters will increase enormously.

3 Analysis of the mathematical structure of tables

Kūshyār's table in the previous section contained numbers which were values of a mathematical function of known type. Some astronomical handbooks contain tables with numbers which are in modern terms the values of functions of unknown type. The task of the historian is to find the exact function that was used.

An example of this type is the lunar visibility table in the astronomical handbook of al-Khwārizmī (ca. 200 H.) The Arabic text of this work is lost, but the work was revised by the Spanish astronomer al-Majrīṭī (died 462 A.H. lunar), and a medieval Latin translation of this revision is extant. The first half of the lunar visiblity table in this work is as follows [7]:

Sign Face	Aries	Taurus	Gemini	Cancer	Leo	Virgo
10	9°26′	9°19′	9°33′	11°29′	15°58′	20°20′
20	9°25′	9°18′	9°57′	12°48′	17°31′	21°04′
30	9°21′	9°21′	10°37′	14°15′	19°11′	21°17′

The table is used as follows. Suppose one wants to predict whether the lunar crescent may be visible at a given evening, one or two days after conjunction of the sun and moon. One enters the table with the longitude of the sun. For example, if the date is 10 days after Nawrūz, the longitude of the sun is (approximately) 10 degrees. One then finds in the table the number 9°26′. This means that if at the moment of sunset, the difference between lunar and solar longitude is more than 9°26′ (and if the lunar latitude is zero), the moon may be seen; if the difference is less than that number, the moon will not be seen.

The text does not tell us how this table was computed. In the medieval Islamic world, there were many theories for the determination of conditions for lunar visibility, so one does not know exactly what mathematical function was computed here. Once a theory is chosen, the computation also depends on the geopgraphical latitude of the locality. On the basis of computer calculations, Kennedy and Janjanian suggested in 1965 that the table was computed according to a criterion for lunar visibility that was developed in India before the advent of Islam. According to this criterion, the moon is visible if moonset is more than 48 minutes after sunset. Kennedy and Krikorian then recomputed the numbers for various geographical latitudes and found the best agreement with the text for a locality with geographical latitude 42 degrees and 30 minutes. However, the agreement was not close, and the geographical latitude is historically implausible, because there was no scientific centre so far North in the Western Islamic world at the time of either al-Khwārizmī or Al-Majrīṭī.

After the personal computer became widespread, it was easy to do many calculations on the entries in the table, and so a small improvement was possible. On the basis of a very general assumption which is incorrect but nevertheless satisfied by many historical lunar visibility theories (namely that the visibility can be predicted on the basis of the geographical latitude, the solar depression at moonset and the solar azimuth at moonset), it was possible to develop a method for reconstructing the entire lunar visibility theory from the numbers in the table. It turned out that the first half of the table (which is displayed here) was computed very accurately according to the Indian criterion, for a geographical latitude of 41°35′. In the fifth century of the Hejira there was one scientific centre in the Western Islamic world at such a high latitude, namely Saragossa in the very North of Spain. Hence the table for lunar visibility was not computed by al-Majrīṭī, who worked in Cordoba.

For this kind of investigation one needs a personal computer and also a list of geographical latitudes of localities in the medieval Islamic world. Of course, a list of modern latitudes is useful, but it is even more useful to have a sorting of the geographical latitudes which are mentioned in the medieval works themselves. Professor E.S. Kennedy and Mrs. M.-H. Kennedy have published such a listing in 1987, in which the latitude of 41°35′ for Saragossa is mentioned [8]. This book shows that the the computer is also very useful in historical research for sorting large amounts of data and making them easily accessible.

The preceding two examples have shown how the computer can be used to extract new kinds of information from Islamic astronomical tables using sophisticated mathematical methods. The computer thus gives a new dimension to the pioneering work on Islamic astronomical tables which has been done by Kennedy and King [9].

4 Optics

We now turn out attention to applications of geometry. Geometry can be applied in the science of optics, especially in the theory of mirrors and refraction. As an example of recent studies I will discuss an important discovery in a text on burning mirrors *Kitāb al-ḥaraqāt* by Abū Saᶜd al-ᶜAlā' ibn Sahl. This mathematician lived in the fourth century of the Hejira and he wrote his treatise for king Ṣamṣām al-Dawla, of the Buwayh dynasty, who reigned between the years 372 and 376 of the Hejira in Baghdād. We do not know Al-ᶜAlā's origin, but he was in contact with several Iranian mathematicians such as Abū Sahl al-Kūhī and Aḥmad ibn Muḥammad ibn ᶜAbdaljalīl al-Sijzī around the year 360 of the Hejira, when they were in Shirāz. The treatise by Al-ᶜAlā' on burning mirrors has recently been published by Rashed [10].

The most important part of the treatise is about hyperbolic lenses. Al-^{c}Alā' proves that certain hyperbolic lenses refract all solar rays to one of the foci of the hyperbola, and in his proof he assumes implicitly the law of refraction which is called "the law of Snell" (after the Dutch scientist Willibrord Snell (1591-1626)). I will explain Al-^{c}Alā's proof in some detail in the following geometrical diagram.

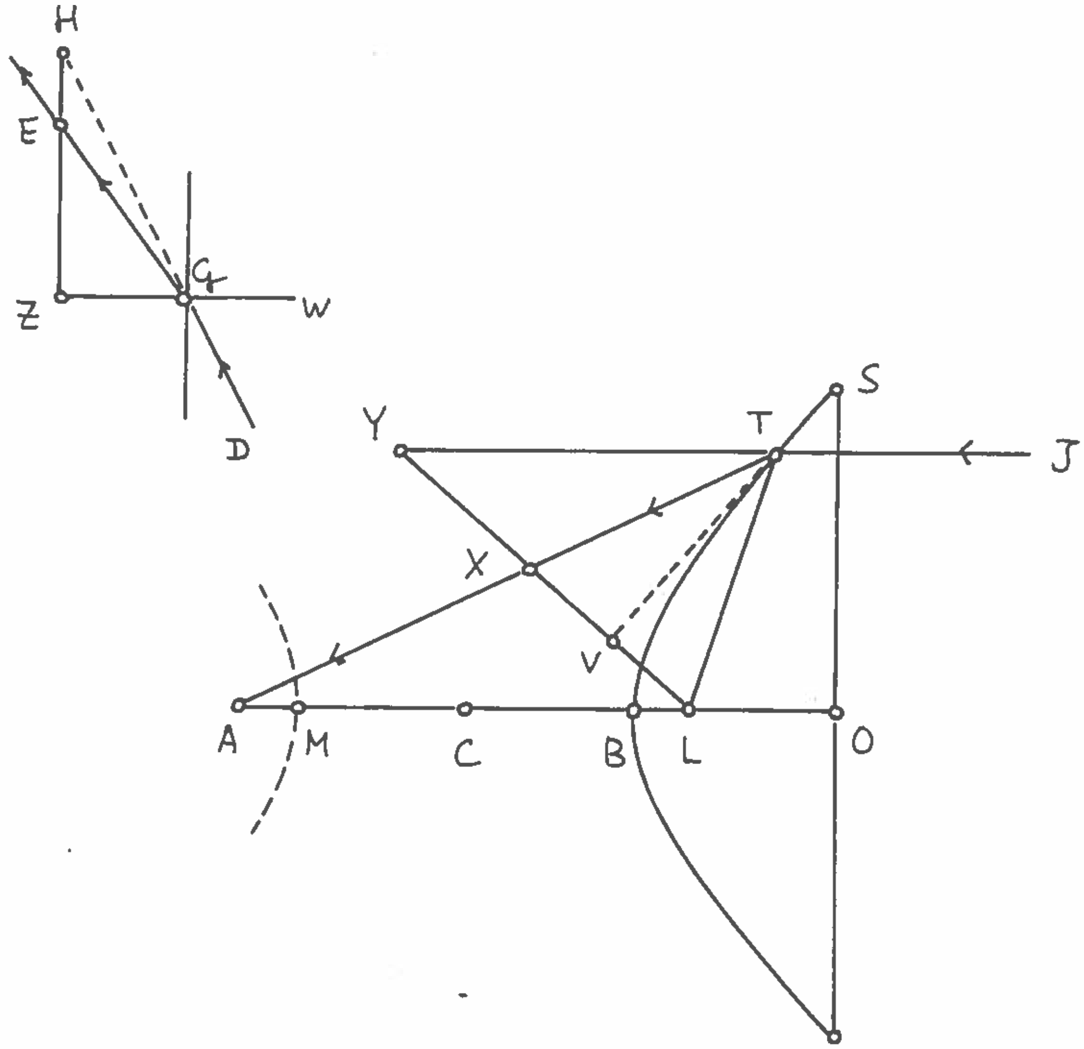

Figure 1

In Figure 1, Al-^{c}Alā' considers a ray of light DG which emerges from a crystal at point G and is refracted at the boundary WGZ of the surface. From any point E on the refracted ray GE he draws a perpendicular EZ to WGZ (which is considered a straight line) and he extends ZE to meet DG extended at point H.

He then draws a hyperbola with transverse axis MB and foci A and L, such that $AL : BM = GH : GE$. If we call the centre of the hyperbola C and put $|CB| = |CM| = a, |CL| = |CA| = f$, the eccentricity of the hyperbola is $f : a = GH : GE = \frac{GZ}{GE} : \frac{GZ}{GH}$, that is the ratio between the sines of the angle of refraction and the angle of incidence (measured with respect to the normal at G). Al-ᶜAlā' considers the branch of the hyperbola with axis BL. He cuts off a segment by a perpendicular SO and he rotates the segment around the axis. Thus he obtains a hyperbolic lense. He assumes that this lense is made of crystal. Let a solar ray JT parallel to the axis meets the lense at point T (which we can assume to be in the plane of the paper.) Draw TL, TA, draw the tangent TV, drop line LV perpendicular to TV, and extend LV to meet TA at X and JT extended at Y.

According to two well-known theorems about the hyperbola (proved by Apollonius of Perga in his *Conics* (Kitāb al-Makhrūṭāt), which was translated into Arabic and known to Al-ᶜAlā'), we have $AT - LT = MB = 2a$ and $\angle LTV = \angle VTX$. Hence $LT = TX$, so $2a = MB = AT - TX = AX$. Thus $TY : TX = AL : AX = 2f : 2a = f : a = GH : GE$. If $\angle JTV = \angle DGZ$, figures $YTXV$ and $HGEZ$ are similar, and hence the ray JT is refracted to point X, so it will reach point A.

However, Al-ᶜAlā' wants to prove that all light rays are refracted to point A, also light rays for which $\angle JTV \neq \angle DGZ$. In his proof he assumes that any light ray JT is refracted at point T as a ray TX in such a way that $TY : TX = GH : GE = f : a$. This means that the ratio $TY : TX$ is constant and does not depend on the angle JTV. We can also say that the ratio $GH : GE$ is constant and does not depend on $\angle DGZ$. Thus the ratio between the sine of the angle of refraction and the sine of the angle of incidence is assumed to be constant. This is the law of Snell for refraction. Thus if a hyperbolic lense is such that the eccentricity is equal to the index of refraction (from crystal to air), it will refract all rays of the sun parallel to the axis to one of the foci, and hence it can cause burning at that point.

Al-ᶜAlā' was not the first scientist in history who studied refraction. The Greek scientist Ptolemy (A.D. 150) had also investigated refraction, and his *Optics* contained a simple table, listing the angle of refraction for various angles of incidence. The *Optics* was translated into Arabic and known to Al-ᶜAlā'. However, Ptolemy did not know that the ratio of the sines of these angles is constant. Thus the law of refraction is Al-ᶜAlā's discovery.

Al-ᶜAlā' does not state the law of refraction explicitly and he does not give any numerical value of the refraction index. Thus one wonders how he discovered the law. Did he perform experiments, and did he discover the law in this way? Or did he have a theoretical motivation for the law? Or did he begin with the question which type of lense reflects the solar rays to one point, make the conjecture that this must be a hyperbolic lense, and then derive the law of refraction in that way? This is not an unreasonable assumption, because the only curves which were studied at that time by the mathematicians were the circle and conic sections; a circular, elliptic, and parabolic lense do not have the required property. Perhaps the question can be resolved by further study of the text of Al-ᶜAlā' or by the discovery of new documents. The question also arises why the law was not used by scientists after Al-ᶜAlā', such as Ibn al-Haytham and Kamāl al-Dīn al-Fārisī. In any case, the law of refraction is an example of a discovery which has been attributed to 17th-century

European scholars, although it had already been made in the medieval Islamic world.

Sometimes people ask if there are more examples of this kind, and whether many results of 17th-century European science, and especially mathematics, were already discovered in the medieval Islamic world. This question is difficult to answer. On one hand, some examples can be cited similar to the law of refraction, and there are many medieval Islamic works which have not been studied, and which may contain even more examples. On the other hand, one should not *want* to expect too many similarities with European science.

In recent years, some historians have stated that, for example, Sharaf al-Dīn al-Ṭūsī knew how to find the derivative of a function, or that some medieval Islamic mathematicians knew the infinitesimal (integral) calculus, or that they studied curves by means of equations just like Descartes, or that Al-Sijzī knew four-dimensional geometry, and so on. However, in this way the works of the medieval Islamic scientists are distorted. Most Islamic mathematicians carefully avoided infinitesimals, in the same way as their ancient Greek predecessors. When one reads the original works of Sharaf al-Dīn al-Ṭūsī and al-Sijzī, one does not find the derivative of a function or four-dimensional geometry. Perhaps some modern historians make the unconscious assumption that modern European science is in some sense the "best" science, so they want to see as much European science as possible in the medieval Islamic works. I think that this assumption should be avoided, because the value of medieval Islamic science is not dependent on possible similarities with 17th-century European science.

5 Geometry and Architecture

Geometry was applied not only in sciences but also in the arts and in architecture. It is widely believed today that the arts and architecture were one of the motivations for the study of geometry in medieval Islamic civilization. However, if one asks for specific examples, very few such examples can be given. Few sources have been studied thus far, and perhaps the historians have been looking in the wrong places. Maybe one can find Iranian craftsmen who know about the old methods and who can explain many connections to us.

A clear connection between geometry and architecture are the descriptions by Jamshīd Ghiyāth al-Dīn Al-Kāshī (died in 832 Hejira) in his *Key to Arithmetic* (Miftāḥ al-Ḥisāb) of *muqarnas* and of surface areas and volumes of cupolas of mosques. These descriptions have been analysed by Yvonne Dold, and she has also prepared a video showing a computer-animated construction of a cupola according to Al-Kāshī's instructions, as well as the relations between Al-Kāshī's work and several monuments in Samarkand.[11]

One less obvious connection between a theoretical text on geometry and a problem in the decorative arts has recently been described by A. Özdural.[12] In a short treatise on Algebra, which was published in Tehran in 1960 [13], ʿUmar Khayyām discusses the construction of a right-angled triangle ABG such that the hypotenuse is equal to one of the sides plus the altitude; in Figure 2, $AG = AB + BD$. ʿUmar Khayyām puts $AD = 10$, $BD = x$, and he shows that the construction of the triangle leads to a cubic equation

$x^3 + 200x = 20x^2 + 2000$ (The reader may try to derive this equation from the triangle.) He then solves this equation by means of a hyperbola and a circle.

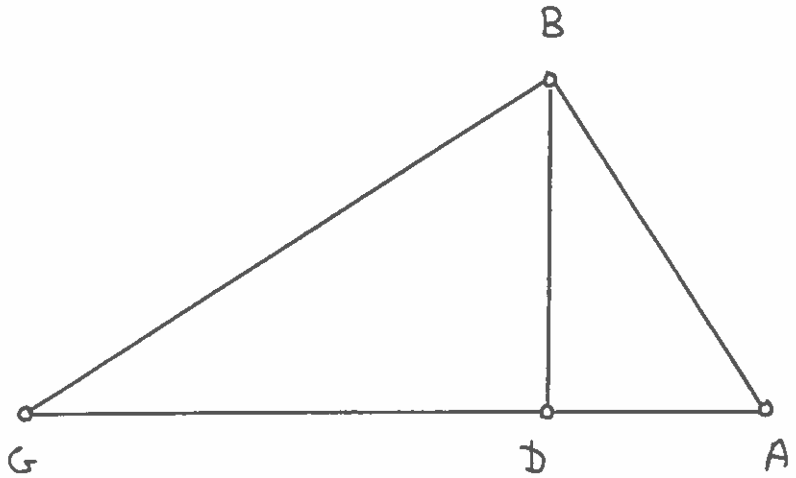

Figure 2

Solution by ᶜUmar al-Khayyām of $x^3 + bx = ax^2 + c$ by means of a circle and hyperbola (in modern notation, see Figure 3):

Draw a straight line ADB such that $|AD| = \frac{c}{b}$ and $|AB| = a$. Construct AG perpendicular to AB such that $|AG| = \sqrt{b}$. Draw GE parallel to AD.

Draw a hyperbola through D with asymptotes AG, GE.

Draw a circle with diameter BD.

Let the circle and hyperbola intersect at point K. Drop perpendicular KL onto BD. Then $x = AL$ is the required solution.

Proof: Put $KL = y$.

Since K is on the circle, we have $KL^2 = BL.LD$, or $y^2 = (a - x)(x - \frac{c}{b})$.

Since K is on the hyperbola, we have $KL : LD = TL : LA$, or $y : (x - \frac{c}{b}) = \sqrt{b} : x$.

Hence $x^2 : b = (x - \frac{c}{b})^2 : y^2 = (x - \frac{c}{b}) : (a - x)$.

Therefore $ax^2 - x^3 = bx - c$.

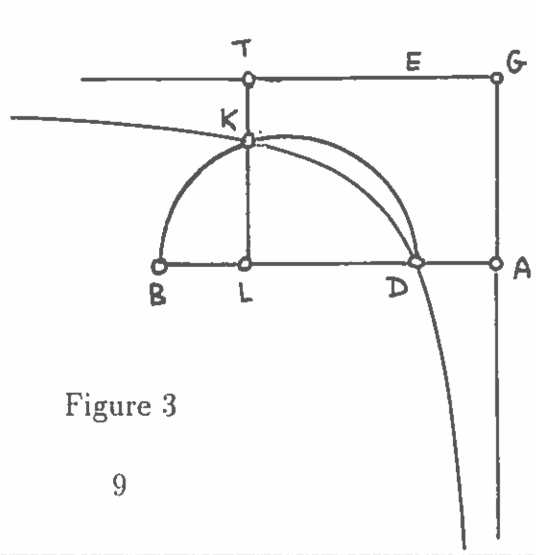

Figure 3

Özdural remarked that the same triangle is found in a Persian manuscript which was probably written in Isfahan in the 16th century.[14] This manuscript contains many practical constructions and geometric patterns, and the triangle also occurs in one such pattern (Figure 4). The text says that the pattern was also constructed by Ibn al-Haytham by means of conic sections.

Here we see an interaction between artisans on one hand, and mathematicians such as Al-Khayyām and Ibn al-Haytham on the other hand. Before Özdural's discovery, nobody had any idea of the context of this triangle. The next question is, of course, whether there is a connection between this triangle and any surviving architectural monuments.

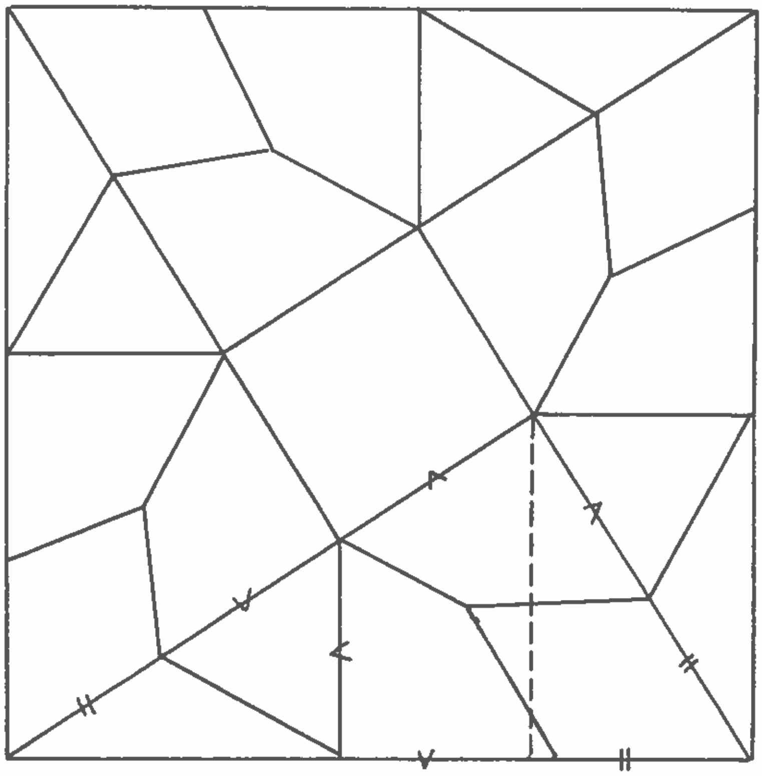

Figure 4

There may be many more unknown connections between geometrical problems in manuscripts and applications in architectural monuments or the decorative arts. I want to give you some questions to think about, from an unpublished letter by Aḥmad ibn Muḥammad ibn ᶜAbdaljalīl al-Sijzī (fourth century H.), which is entitled *Answer to questions which were asked to him by one of the geometers from Shirāz*.[15] See Figure 5.

Problem 3: We want to divide parallelogram AD into four parts $AGEHT, THEZM, MZDN, AMNB$ which are all equal in area, and such that $|HT| = |TM|$.

Problem 4: We want to divide parallelogram $ABDG$ into four parts $ABZE, BZTD, AGHE, HEZT$ such that $AEHG = BZTD$, $AEZB : ABGD$ is a given ratio and $ZEHT : AEHG$ is another given ratio.

Problem 6: We want to divide parallelogram AD in such a way that $AGCME = EMCKH = HKFT = ZTFQL = DQLZB$ and such that their sum is equal to $AHTB$ or has a given ratio to $AHTB$.

Problem 8: We want to divide parallelogram $ABDG$ in such a way that $AEHM = DZTN = EGZTH = BMN$ and such that the division is by lines parallel to the diagonals of the parallelogram.

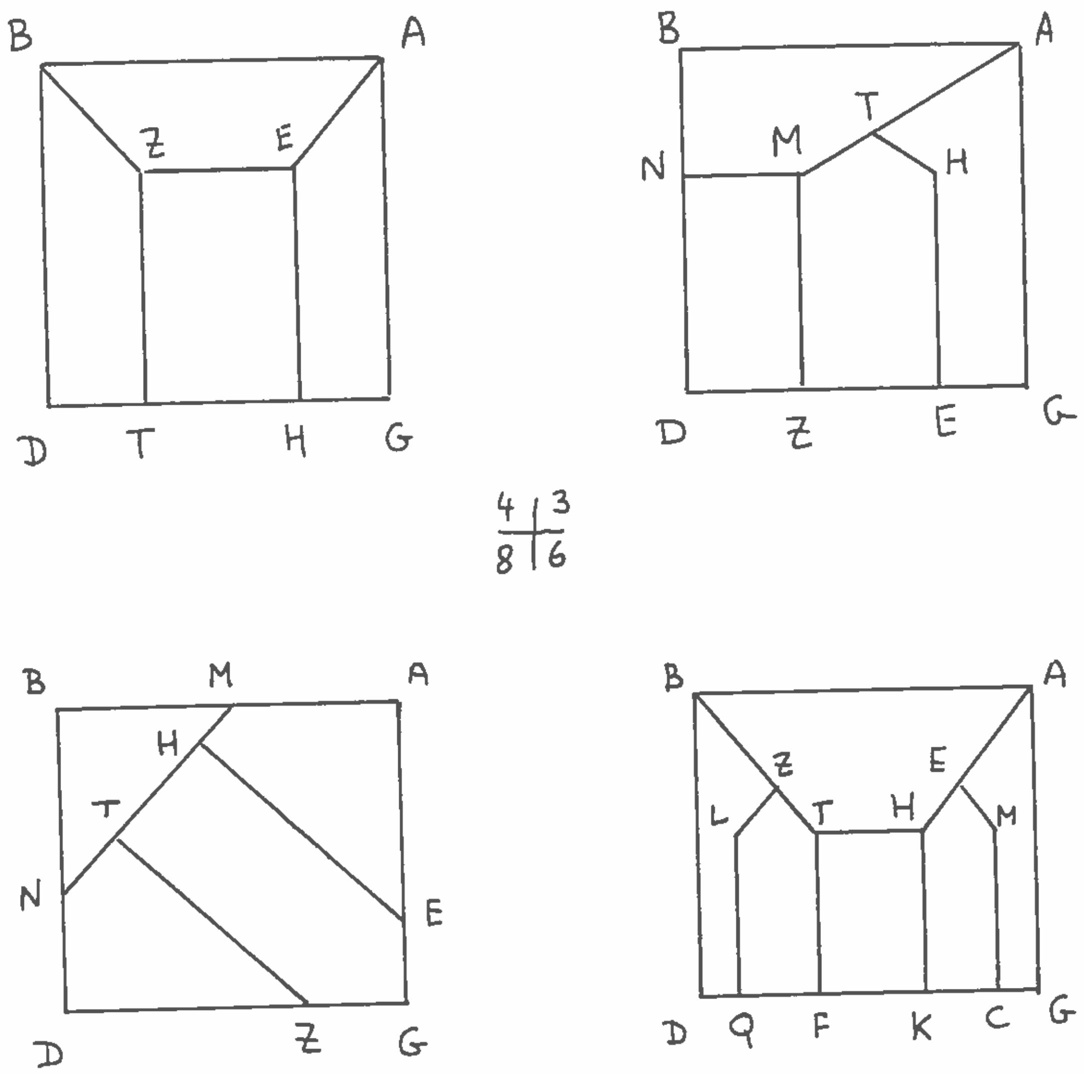

Figure 5

These problems are all solved by al-Sijzī. I am not concerned with the solutions here. (You may try to find solutions yourself!) My questions are:

1. Are these problems related to any decorative patterns or any problems in architecture in medieval Islamic civilization?

2. If so, can these patterns and architectural monuments still be seen in Iran or elsewhere?

3. If the answer to question no. 1 is negative, why were the geometers in Shirāz interested in these problems?

The connection between geometry and arts has a great potential for mathematics education. Suppose one has some connections between geometrical constructions in manuscripts and mosaics or architectural monuments which one can visit. One can treat the constructions in school and then make an excursion to see the patterns or monuments in reality. Or, if this is not feasible, one can also ask the children to draw the patterns and make works of traditional art. Many children today hate mathematics, at least in the Western world, but they may change their mind if they see a real connection between mathematics, history, and beautiful art.

6 Instruments

Thus far, most researchers in the history of mathematics and astronomy in medieval Islamic civilization have used written texts as their source material. However, other possible sources also exist. In the last section we have seen that architectural monuments and decorative patterns may be one type of alternative sources, and perhaps there is still an oral tradition in this field. Surviving astronomical instruments such as astrolabes, and timekeeping instruments such as sundials, are also important sources for the history of science.

Until recently, these instruments have been neglected, but fortunately this situation is now changing. About ten years ago, Professor David King, from the University of Frankfurt in Germany, started a very large project to investigate many types of Islamic and European astronomical instruments which were made before the year 1550 of the Christian era. He and his collaborators are working on a many-volume catalogue which is to contain precise descriptions and photographs of these instruments. In addition, Professor King and his collaborators are also using modern techniques such as computer graphics for the analysis of medieval instruments.

Professor King works with a "hands-on" approach, which he describes as follows:

> "The basic philosophy behind the preparation of the catalogue is that it is essential to hold the instrument in one's hands, to take it apart, to examine each part carefully - in short, to play with it for a while - in order to begin to understand it properly. Photographs are inadequate for this purpose." ([16], p. 4)

The investigations have already produced masses of new information. Many anonymous instruments have approximately been dated. Often, instruments of a particular period provide new information on the astronomical knowledge of that period, not found in manuscript texts.

One of the many instruments studied by Professor King is the astrolabe which Ḥāmid ibn Khiḍr al-Khujandī costructed in Baghdad in the year 374 of the Hejira. Photos of parts of the astrolabe have been published by King in [17], the complete astrolabe is unpublished. This is the oldest Islamic astrolabe which is known to exist; it is now in a private collection in Kuwayt. The workmanship of the astrolabe is evidence of the development of the instrument in the fourth century of the Hegira.

King remarked that the spider of the astrolabe contains an ornament in the form of a quatrefoil [= with four leaves]. This ornament was often used in late medieval European gothic art, and we now see that its origin may be in the Islamic Middle East. This example shows the interdisciplinary nature of the study of astrolabes. The first plate of the astrolabe contains among other things a stereographic projection of the celestial sphere with altitude circles for each three degrees of interval and azimuth circles for each five degrees of interval, for the latitude of Baghdad (33 degrees). The second plate of the astrolabe was used for finding the astrological houses and progressions for the latitude of Baghdad. From this plate one can deduce the theory of astrological aspects ("shu^{c}ā^{c}āt") and progressions ("tasyīrāt") which al-Khujandī used. In this theory, the celestial sphere is divided in sections by means of circles through the North and South points of the horizon which intersect the celestial equator at points with regular intervals.

The Islamic astronomical instruments are scattered in museums and private collections around the world, so one has to make long trips if one wants to study them by means of professor King's "hands-on" approach (which I really believe is the best approach.) Professor Sezgin of the Institute for History of Arabic-Islamic sciences in Frankfurt has initiated a project to construct replicas of the most important Islamic astronomical instruments, and to put these replicas on display at his Institute in Frankfurt. I believe that this is also an important project, not only because the hands-on approach is the best approach for the researcher, but also because in my own experience, the astrolabe has a great didactic potential for a large audience. It is easy to make an astrolabe with students or school children using simple materials. A primitive example, which I have drawn myself (on the basis of a Yemeni astrolabe) will be shown during the lecture. I think it would be very worthwhile if an institute for history of science could publish a more sophisticated example of a historic Islamic astrolabe from cardboard, for example a good copy of the astrolabe of Al-Khujandī of which pictures have been shown during the lecture.

The astrolabe can be used to explain some basic principles of astronomy, such as the rising and setting of stars, the motion of the sun, the varying length of the day and the cause of the seasons. In our country, most school children and even many university students are hopelessly ignorant of these elementary subjects. The astrolabe can also be used to tell the (true solar) time, and to find the northern direction. With some special plates, it can be used to find graphical solutions to spherical trigonometrical problems, for example finding the qibla for a location with given geographical latitude. For students on a higher mathematical level, one can explain the mathematical theory of stereographic projection, and develop the theory to construct the astrolabe, have them draw the astrolabe themselves, and then ask them to perform some astronomical observations with the astrolabes which they have constructed.

I am taking some time for this digression because I believe that the history of science should be more than an interesting subject of academic study for the specialist. In my view, the history of science should be brought to the attention of many people, because only this will bring the subject to life. The astrolabe is one of the possible means to do this, because of its rich history, its user-friendliness and its beauty. The astrolabe can be of interest to many people today, just as it was meaningful to many people in the past. I would like to quote as evidence a few sentences from Mawlānā Jalāl al-Dīn Rūmī with references to the astrolabe.

In the Mathnawī Mawlānā says:

> ᶜishq usṭurlab-e asrār-e Khodāst (love is the astrolabe of the secrets of God)

In the end of Chapter 1 of his work *fīhi mā fīhi*, Rūmī says:

> Man is the astrolabe of God, but one needs an astronomer who knows the astrolabe. If a greengrocer or vendor would possess an astrolabe, of what use would it be to him? And what would he know, even with this astrolabe, of the motions of the (celestial) spheres, their circles (epicycles?), the signs of the zodiac, their influences, the transits of the stars (planets?) and so on? But in the hands of the astronomer the astrolabe is very useful, for *who knows himself knows his Lord.*
>
> Just as the astrolabe is the mirror of the heavens, so is the human being - *We have honoured the children of Adam* (Sūra 17:70) an astrolabe of God. If God gives to someone knowledge of Him and familiarity with Him, this (man) sees each instant, through the astrolabe of his own being, the manifestations of God and his unlimited beauty, and this beauty is never absent from this mirror. [18]

To understand this passage one needs to know that the spider of the astrolabe is the projection of the celestial sphere, and that the motion of the spider around the plate represents the (apparent) motion of the stars around the earth.

7 Conclusion

The preceding examples all reflect an interest in the history of the applications of mathematics and astronomy. I believe that this interest is increasingly important in current studies, and that in the background there is always the wish to put mathematics and astronomy in their social and cultural context. Many more examples could be discussed, and of course my choice also reflects my own personal interest and my limitations. Thus I have not discussed examples of recent research on the transmission of scientific terms from other languages (such as Greek, Syriac, Pahlavi and Sanskrit) into Arabic.

It is certain that we have not yet arrived at a picture of the history of mathematics and astronomy in Islamic civilization which is in any sense complete. Researchers are uncovering more and more pieces of an enormously intricate mosaic, of which the general pattern is usually not very clear. One of the problems is the vastness of the field; another problem is caused by the fact that the source material and the researchers are scattered around the world, so there is no easy way to find all the relevant source material or all the information which has been published on any given subject. Original papers on the history of Islamic mathematics and astronomy appear in many languages (Arabic, Turkish, Persian, English, French, German, Russian, Spanish, Catalan, Italian, etc.) Fortunately there are bibliographical works on the history of Islamic mathematics and astronomy, one published by Fuat Sezgin in 1974 and 1978, and one by Rozenfeld and Matvievskaya in 1983 [19] (soon to appear in an English translation). These works are enormously useful, but they are already somewhat out of date as soon as they have been printed.

An international group of researchers with nucleus in Canada, Holland and Iran is now making plans to put a database (comparable to the works of Sezgin and Rosenfeld-Matvievskaya) on the internet and to continuously update it with recent publications. This database should be accessible free of charge for any individual anywhere in the world with access to the internet. The project is enormous, and there are several initial difficulties which have not yet been resolved. Yet we hope that this database will eventually be realized. Then one of the difficulties of research will be resolved to some extent. Another difficulty in this research is caused by the fact that many researchers work on their own or in very small groups, with limited time for their research, and that some of the few centers for the history of Arabic-Islamic science are working in difficult circumstances. However, I believe that good conditions for this kind of work are now being created in Iran. I hope that the new Iranian institutions will train many researchers in the history of Islamic mathematics and astronomy, and I hope that their efforts will be richly rewarded.

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