

New findings of Greek geometrical fragments in the Arabic tradition

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Book VII of the Mathematical Collection of Pappus of Alexandria (ca. A.D. 250) is basically a commentary to a group of Greek works in advanced geometry which were written in the fourth and third century B. C. by Euclid, Apollonius and others. Only few of these works have come down to us in Greek (the *Data* of Euclid, Books I-IV of the *Conics* of Apollonius) or in Arabic translation (the *Data* of Euclid, Books I-VII of the *Conics* of Apollonius, *Cutting-Off of a Ratio* by Apollonius). From the 17th century on, European geometers and historians of mathematics have attempted to reconstruct the contents of the remaining (lost) works from the information given by Pappus. From the *Fihrist* of Ibn al-Nadīm we know that some of these (lost) works had also been translated into Arabic, but no manuscripts of such translations have turned up until the present. However, it appears that treatises of Ibrāhīm ibn Sinān (ca. A.D. 940) and al-Sijzī (ca. A.D. 980) contain traces of these lost works. Here we make some general remarks on the identification; for details see [1] and [2].

The treatises of Ibrāhīm ibn Sinān and al-Sijzī contain a number of geometrical propositions which are explicitly attributed by them to Apollonius, and which do not occur in the extant works of Apollonius. The information given by Pappus makes it possible to determine from which lost works these propositions were taken (*Neuseis*, *Plane Loci*, *Tangencies*). Some auxiliary theorems in Pappus' commentary to these lost works correspond very precisely with these Arabic propositions, and in these cases, the Arabic propositions explain the motivation of Pappus' auxiliary theorems. Thus it is even possible to locate

exactly the part of the lost Apollonian works from which the propositions in the texts of Ibrāhīm and al-Sijzī were derived. One can now compare the newly discovered Arabic propositions with the European reconstructions from the 17th century and later. The fact that the Arabic solutions are often simpler than the reconstructions, confirms that the Arabic solutions are due to Apollonius, one of the great geometers of antiquity.

The treatises of Ibrāhīm ibn Sinān and al-Sijzī also contain some very obscure and mysterious problems involving extremely complicated geometrical figures. Ibrāhīm ibn Sinān and al-Sijzī give no information on the origin of these problems and figures, but they are unrelated to any other mathematical problems in Arabic texts known to me, and thus one wonders why Ibrāhīm and al-Sijzī would study them at all. From Pappus' information it is possible to conclude that these problems are related to propositions in Euclid's lost work on *Porisms*, which is one of the most exciting mathematical works of antiquity (see [2] for all details). The Greek word *πόρισμα* has two meanings, namely 1. corollary, and 2. a very complicated kind of geometrical problem (such as Euclid discussed in his *Porisms*). The *Fihrist* of Ibn al-Nadīm contains a reference to a work attributed to Euclid on *Fawā'id* (= corollaries), which could be a translation of *πόρισμα*, suggesting that (part of) Euclid's *Porisms* was transmitted into Arabic. According to the classification of geometrical problems in Ibrāhīm ibn Sinān's treatise *On Analysis and Synthesis*, a Euclidean *porisma* would be an ill-posed problem, which required a "correction". The above-mentioned obscure problems in the text of Ibrāhīm and al-Sijzī can be derived from certain Euclidean *porisms* (mentioned by Pappus) by means of such a "correction". Thus these problems are in all probability influenced by the *Porisms*.

References

- [1] J.P. Hogendijk, 'Arabic translations of lost works of Apollonius', *Archive for History of Exact Sciences* 35 (1986), pp. 187-253.
- [2] J.P. Hogendijk, 'On Euclid's lost Porisms and its Arabic traces', *Bollettino di Storia delle Scienze Matematiche* 7 (1987), pp. 93-115.