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Two Editions of Ibn al-Haytham's *Completion of the Conics*

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The lost Book VIII of the *Conics* of Apollonius of Perga (ca. 200 B.C.) was reconstructed by the Islamic mathematician Ibn al-Haytham (ca. A.D. 965–1041) in his *Completion of the Conics*. The Arabic text of this reconstruction with English translation and commentary was published as J. P. Hogendijk, *Ibn al-Haytham's Completion of the Conics* (New York: Springer-Verlag, 1985). In a new Arabic edition with French translation and commentary (R. Rashed, *Les mathématiques infinitésimales du IXe au XIe siécle. Vol. 3.*, London: Al-Furqan Foundation, 2000), it was claimed that my edition is faulty. In this paper the similarities and differences between the two editions, translations, and commentaries are discussed, with due consideration for readers who do not know Arabic. The facts will speak for themselves.

ملخص: لقد حاول الرياضي المسلم ابن الهيثم (حوالي ٩٦٥ - ١٠٤١ م)، في كتابه «تمام كتاب المخروطات»، أن يُحرر من جديد المقالة الثامنة المفقودة لكتاب «المخروطات» لأبلونيوس البرغائي (حوالي ٢٠٠ قبل الميلاد). ونُشر النص العربي لهذا التحرير، مع ترجمة انجليزية وشرح، في كتابي [هوخندايك ١٩٨٥].

ثم نُشِر، بعد هذا، تحقيق جديد لنفس التحرير، مع ترجمة فرنسية وشرح في [راشد ٢٠٠٠ - أ]، ذُكِرَ فيه أنْ [هوخندايك ١٩٨٥] كان مخطئاً. سنُناقِش، في هذه المقالة، ما هو متشابه وما هو مختلف بين التحقيقين والترجمتين والشرحين، ونترك الأشياء تتكلم بنفسها.

چکیده: ابن هیثم، ریاضیدان دورهٔ اسلامی (حدود ٣٥٤ - ٤٣٠ هجری قمری)، مقالهٔ هشتم کتاب المخروطات آپولونیوس پرگایی (حدود ٢٠٠ سال پیش از میلاد) را، که مفقود شده بود، در اثری به نام مقالة في تمام کتاب المخروطات بازسازی کرده است. متن عربی این بازسازی را همراه با ترجمه و شرح آن به انگلیسی در سال ١٩٨٥ اینجانب در کتاب خود [Hogendijk 1985] منتشر کردم. ویرایش تازه‌ای از این متن عربی با ترجمه و شرح آن به فرانسه را رُشدی راشد [Rashed 2000a] عرضه کرده، که در آن ادعا نموده است که تألیف مذکور اینجانب مغلوط است. در این مقاله شباهتها و تفاوتهای میان این دو ویرایش، ترجمه و شرح، مورد بحث قرار گرفته است به نحوی که برای خوانندگان نا آشنا به زبان عربی قابل استفاده باشد. واقعیات خود سخن خواهند گفت.

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1. INTRODUCTION

The *Conics* of Apollonius of Perga (ca. 200 B.C.) is one of the fundamental texts of ancient Greek geometry. In this work, Apollonius introduced the terms parabola, hyperbola, ellipse, and asymptote which are still used today. The *Conics* originally consisted of eight "Books," i.e., large chapters. Only Books I–IV are extant in an ancient Greek version [Heiberg 1891–1893]. Books V–VII have survived in an Arabic translation made in the ninth century A.D. [Toomer 1990]. Book VIII seems to be irretrievably lost, and we have virtually no information about its contents.

Around 1970, a medieval Arabic manuscript was discovered which contained a reconstruction of Book VIII by the medieval Islamic mathematician al-Ḥasan ibn al-Haytham (ca. 965–1041), who was known in the Latin west as Alhazen. This manuscript is preserved in Manisa (Turkey). Ibn al-Haytham's reconstruction is entitled *Completion of the Conics*, and it consists of a brief preface and 31 geometrical propositions.

A facsimile of the Manisa manuscript with Turkish and German translations of Ibn al-Haytham's preface appeared in [Terzioğlu 1974]. [Hogendijk 1985] contains a critical edition of the Arabic text of Ibn al-Haytham's *Completion of the Conics* with English translation and commentary. Another edition, with French translation and commentary, has recently appeared [Rashed 2000a, 1–272]; this edition is also based on the Manisa manuscript.

In his preface, Rashed calls my edition "faulty," and he prints a list of 125 corrections to my Arabic text [Rashed 2000a, 22–26]. Rashed says that he prefers not to discuss the errors in my translation and commentary; see the footnote for the complete French text of his judgment.[1] Rashed's Arabic edition contains a number of references to my earlier edition, but in his entire translation and commentary, Rashed refers to my book just once [Rashed 2000a, 93].

This paper has been written for a wide audience. We shall compare [Rashed 2000a, 1–272] with [Hogendijk 1985] in order to present an overview of the similarities and differences between the two editions, translations, and commentaries. The facts will speak for themselves. As a preliminary, we shall try to give the reader some idea of the detective work involved in editing Arabic geometrical texts. No familiarity with the Arabic language

[1] "J. P. Hogendijk a publié une thèse de doctorat dans laquelle il entreprend une édition critique, une traduction anglaise et un commentaire historique et mathématique volumineux. Cette publication a eu le grand avantage de faire connaître en Occident ce livre d'Ibn al-Haytham, ainsi que les résultats auxquels ce dernier a pu parvenir. Nous notions à l'instant que N. Terzioğlu, en raison même de la grande qualité de la copie, s'était contenté de la reproduire. J. P. Hogendijk en revanche a jugé bon d'en proposer une édition critique (signalée par la lettre *ḥ* dans notre apparat critique). Celle-ci, quoique fautive, est une édition quand-même. Un grand nombre de fautes sont d'ailleurs dues à une volonté, louable mais malheureuse, de rectifier un texte arabe au demeurant parfaitement correct. On se contentera ici de relever les erreurs introduites dans le texte arabe transmis, en laissant au lecteur le soin de corriger les erreurs systématiques liées à l'orthographie ancienne (al-mukāfī, iḥdayhimā), et de rectifier les mélectures des lettres utilisées dans les raisonnements et les figures géométriques. Pour les contresens qu'il faut déplorer dans la traduction anglaise aussi bien que dans les commentaires, j'aime mieux ne pas les évoquer ici" [Rashed 2000a, 22].

is presupposed. A few Arabic letters and passages will be displayed so that the reader can see what they look like. The paper has three appendixes with detailed information.

Henceforth, H and R will be used as abbreviations for the two editions [Hogendijk 1985] and [Rashed 2000a, 1–272]. Notations such as H223:7, H169n6, R173:1f will refer to line 7 of page 223 of my edition, note 6 to page 169 in my edition, and footnote to line 1 of page 173 of Rashed's edition.

2. THE DETECTIVE WORK OF EDITING ARABIC GEOMETRICAL TEXTS

The unique Arabic manuscript of Ibn al-Haytham's *Completion of the Conics* was written by a scribe who was not a mathematician and who misunderstood most of the text. Thus, there are many scribal errors in the manuscript, and labels of points in geometrical figures (A, B, ...) are often written ambiguously or incorrectly.

It may be useful to explain some terminology concerning a widely accepted methodology for editions of ancient and medieval manuscripts. In a critical edition of an ancient or medieval text, the modern editor tries to restore the text which the author (in this case Ibn al-Haytham) wrote. The "edited text" is what the modern editor believes to be the original text by the author (Ibn al-Haytham). An emendation is a correction of a scribal error in the manuscript(s), made by the modern editor in order to restore the original text by the author. If emendations are necessary, the incorrect manuscript readings are usually displayed in a so-called critical apparatus, at the bottom of each page.[2] In case a text is edited twice, the later editor will probably want to follow some of the emendations made by the earlier editor. According to generally accepted academic standards, these emendations should be credited to the earlier editor by the later editor. The latter can define a special symbol for this purpose and use it in the critical apparatus.

The detective work involved in this editing process will now be illustrated by an example from Ibn al-Haytham's *Completion of the Conics*, using the transcription in H, which is almost the same as the now standard transcription of Arabic letters proposed in [Hermelink and Kennedy 1962]. The passage occurs near the beginning of Proposition 6 (cf. H168–169, R170–173). Ibn al-Haytham considers a hyperbola or ellipse ABG, with axis AD, vertices A, D, and center E, and he wants to construct a line BS tangent at some point B and intersecting the axis at S in such a way that $BS : SD$ is equal to a given ratio $H : W$. The Manisa manuscript contains two figures, one for the ellipse and one for the hyperbola. The two figures are reproduced in Fig. 1 [from Terzioğlu 1974, f. 6b], with transcriptions of the names of the points. Corrections to the figures are indicated as follows: $[M]S$ means that the manuscript has M and S is my correction.

In his analysis in proposition 6, Ibn al-Haytham assumes that the tangent BS at B has been found in such a way that $BS : SD = H : W$, and he draws AG parallel to the tangent, to meet the conic at point G. Through the center E he draws EB to meet AG at T, then $AT = TG$ according to the *Conics* of Apollonius. Ibn al-Haytham drops perpendiculars BK, TF, GO to the axis AD. Then the manuscript has a corrupt passage (Fig. 2, [Terzioğlu 1974, f. 6a:9-12]) which can be tentatively translated as follows:

[2] If there are many incorrect words in the manuscript(s), some editors prefer to present only the most important corrections. Different editors can have different policies in this regard.

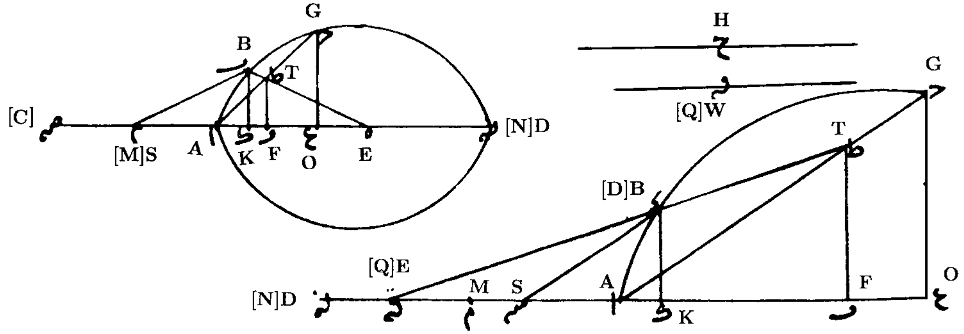

FIGURE 1

> Then the ratio of the product of ¿O and OA to the square of OG is equal to the ratio of the transverse diameter A¿ to its latus rectum. The same is the ratio of the product of ¿K and KA to the square of KF, as is proven in Proposition 21 of the first Book. So the ratio of the product of E¡ and ¡A to the square of ¡T is that same ratio, because these lines are also and those lines.

The symbol ¿ stands for an ambiguous letter which could possibly be interpreted as D د R ر or Z ز, and the symbol ¡ can be interpreted as B ب, or perhaps as h ث or t ت, or possibly even F ف. (The shape of the letters in the manuscript is somewhat different from the printed form.) In the quoted passage, all other labels of points in the geometrical figure (O, G, etc.) are more or less clear.

To understand what the passage means, we look at the figures, and we consult Proposition 21 of Book 1 of the *Conics* of Apollonius [Heath 1896, 19; Ver Eecke 1923, 43–44]. This proposition is to the effect that in the figures $DO \cdot OA/OG^2 = AD/p$ and $DK \cdot KA/KB^2 = AD/p$, where p is the segment which Apollonius called *latus rectum*. The precise meaning of this term is of no importance here. Thus we read ¿ as D, and we decide that KF كف is a scribal error for KB كب. The meaning of the last sentence escapes us, until we interpret *also and* ايضًا و as a corruption of *halves* انصاف, and translate "because these lines are halves of those lines." Now the passage makes sense if we interpret ¡ as F: because $EF = DO/2$, $FA = OA/2$, and $FT = OG/2$, we have $EF \cdot FA/FT^2 = DO \cdot OA/OG^2 = AD/p$. We conclude that "these lines" in the quoted passage are EF, FA, FT and that "those lines" are DO, OA, OG.

We now adopt in our edited text the (restored) readings KB and *halves*, and we indicate at the bottom of the page, using footnotes or line numbers, the manuscript readings KF (H169n6) and *also and* (H169n7). In the critical apparatus, we do not note our interpretations ¿ $= D$ and ¡ $= F$, because they are not inconsistent with the manuscript. We translate the text on the basis of the restored readings (H168:12–17).

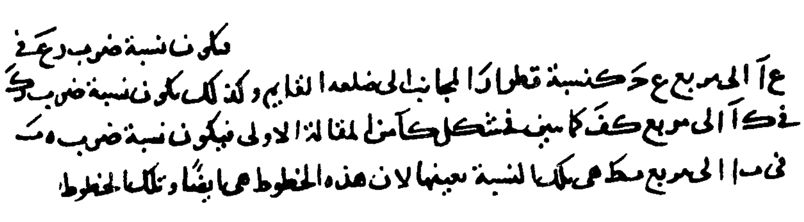

FIGURE 2

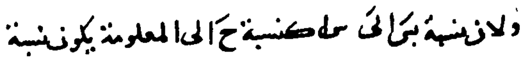

FIGURE 3

Rashed made precisely the same emendations KB (R173:1f) and *also and* (R173:3f), without mentioning anywhere that they were already found in my edition: compare his translation (R170:17–172:3).[3]

3. RASHED'S REFERENCES TO HOGENDIJK'S EDITION

For the sake of clarity, Sections 3–5 and Appendixes 1–3 of this paper will be written in the third person: thus "Hogendijk's edition" instead of "my edition."

In his critical apparatus to the Arabic text, Rashed lists a total of 581 emendations. We first illustrate his (lack of) references to Hogendijk by a "mixed" example from Ibn al-Haytham's Proposition 6. The Manisa manuscript reads as follows (f. 6b:3–4, see Fig. 3):

> Since the ratio of BS to SA is equal to the ratio of H to the known, the ratio of . . . .

Hogendijk emended SA to SD to make mathematical sense (H171n4) and restored a missing word W to the text. He translates: "Since the ratio of BS to SD is equal to the known ratio of H to $\langle W \rangle$, the ratio of . . . " (H170:10; the angular brackets indicate that the word is missing in the manuscript).

Rashed also emended SA to SD, without reference to Hogendijk (R173:11f). He added $\langle W \rangle$ just like Hogendijk, and with a reference *ḥ* to Hogendijk (R173:12f). He translates: "Puisque le rapport de BS à SD est égal au rapport connu de H à F,[3] le rapport de . . . " (R172:13–14).

This example illustrates Rashed's general policy. In 53 cases, Rashed restores one or more words to the text in the same way as Hogendijk. Rashed mentions these restorations in the critical apparatus with a reference to Hogendijk. In 464 other cases, Rashed emends a word or passage in the manuscript in exactly the same way as Hogendijk. Rashed does not give credit to Hogendijk for any of these emendations.

Slightly more than half of the 464 emendations are corrections of labels of points or lines (such as $KF \to KB$ and $SA \to SD$ above). The majority of the remaining emendations are simple: corrections to the mathematical argument ("and the ratio" → "so the ratio," etc.), insertion of passages in the margin of the manuscript into the text, removal of passages which the scribe repeated by mistake, corrections of slight grammatical errors, and so on. There are dozens of nontrivial emendations, including changes of words and passages that the scribe of the manuscript misunderstood. An example is the emendation of *also and* to *halves* mentioned above.[4]

[3] To obtain Rashed's system for transcribing Arabic letters from the systems of Hermelink-Kennedy and Hogendijk, a few changes should be made, including $F \to P$, $G \to C$, $T \to I$, $W \to F$. Thus FT in my translation appears as PI in Rashed's translation. Rashed does not use a special symbolism to indicate translations of words (such as "F") that were added to the Arabic manuscript in order to restore the original text.

[4] Some more examples: R149:21f = H139n3 "renewal" *tajdīd* → "diorismos" *taḥdīd*. Note that [Terzioğlu 1974, 11] in his translation of Ibn al-Haytham's preface read the word as Rekonstruktion, similar in meaning to renewal; R161:20f = H155n1 "after" *min ba*ᶜ*d* → "its square" *murabba*ᶜ*uhu*, R171:12f = H169n3 "it remained from" *fa-baqiya min* → "we assume" *fa-nafriḍu*, R181:9f = H181n5 "the two poles" *al-quṭbayn* → "the two points" *al-nuqṭatayn*, R259:15f = H283n8 "on" ᶜ*alā* → "so LZ" *fa-lz*.

Thus, approximately 89% of the emendations in Rashed's critical apparatus occur also in Hogendijk's edition; 9% are credited to Hogendijk, and 80% appear without acknowledgement.[5]

Half of the remaining 11% of Rashed's emendations are related to the fact that Hogendijk and Rashed read a few ambiguous labels of points in geometrical figures differently; see Appendix 3 for a list. In Proposition 5, for example, Hogendijk reads the label of a point as the Arabic letter *yā'* (I), whereas Rashed reads the same label as the Arabic letter *tā'* (in his transcription T). With one exception, to be discussed below, these differences do not concern the mathematical interpretation, for if Hogendijk reads I in Proposition 5, Rashed always reads T, and so on.

A similar example of a text that has been edited twice is the medieval Arabic translation of Diocles' *On Burning Mirrors*. The text was edited in [Toomer 1976] and again in [Rashed 2000b, 1–151]. In [Hogendijk 2002] it is shown that 75% of Rashed's emendations to the Arabic manuscript occur in Toomer's edition; only 8% are credited to Toomer, and 67% appear without acknowledgement.[6]

4. RASHED'S CORRECTIONS TO HOGENDIJK'S EDITION

On pp. 23–26 of his edition, Rashed lists 125 corrections to Hogendijk's edition, in Arabic only.[7] Three of these corrections are improvements which affect Hogendijk's translation: in H136:25–26 change "understanding" to "brilliance" (of stars in the night sky); in H162:20 change "we went on previously" to "we have explained before," and in H236:23–24 change "that line $HA \ldots$ and that $HM \ldots$" to "either that line $HA \ldots$ or that $HM \ldots$." A fourth correction made by Rashed is an improvement of Hogendijk's text and figure of Proposition 23. In the text (H256:13–23) Mt should be changed to Rt and Rt to Mt, and in the figure on H250 point t should be placed on AR extended such that $Rt = IA$. These readings

[5] This may be compared with [Rashed 2000a, 738–757], a critical edition with French translation of a text by al-Sijzī (10th century) on the construction of the regular heptagon and the trisection of the angle, which had previously been critically edited with English translation in [Hogendijk 1984, 292–316] on the basis of the three extant Arabic manuscripts. The following is stated in [Rashed 2000a, 655] about the edition [Hogendijk 1984]: "L'édition (cf. plus loin apparat critique) et la traduction anglaise demeurent tout à fait insuffisantes, en dépit d'un effort remarqué." In the critical apparatus, Rashed indicates the approximately 40 differences between his edition and the previous edition. Most of the differences are small and only four differences affect the meaning of the text (cf. the translations [Rashed 2000a, 738:23, 744:15–18, 746:5] with [Hogendijk 1984, 306 Sect. 12, 311 Sect. 44–45, Sect. 48].) In almost all the remaining 140 items in the apparatus, Rashed's reading is the same as in the previous edition, but this is nowhere mentioned. In the following instances, Rashed's reading is different from all three manuscripts but the same as in the previous edition: *ghauṣ* (R739:9), *li-l-ladhī* (R745:12), *khaṭā* (R747:10), and *al-ḍil*^{c} (R749:7). The close relationship between [Rashed 2000a, 738–757] and [Hogendijk 1984, 292–316] is further illustrated by the fact that the German translation of the same text of al-Sijzī in [Schoy 1926, 21–31] is very different.

[6] Similar cases have occurred in the past. In the edition [Rashed 1984] of the Arabic books of Diophantus' *Arithmetica*, Rashed made some emendations which appear in the previous editions [Sesiano 1975, 1982] but are not credited to Sesiano, even though they are not found in Rashed's first edition [Rashed 1975]; see [Toomer 1985, Jaouiche 1987]. In the critical edition [Rashed 1996, 781–833] of a treatise by Abū Ja^{c}far al-Khāzin (10th century) on isoperimetric problems, the reader is not informed of the fact that approximately 75% of the emendations are the same as in the earlier edition of al-Khāzin's text in [Lorch 1986, 158–214].

[7] Rashed does not mention these corrections in his critical apparatus. Compare the similar list of 209 corrections in [Rashed 2000b], in Arabic only, to G. J. Toomer's edition of Diocles' *On Burning Mirrors* [Toomer 1976]; see [Hogendijk 2002].

do not imply an essential change in the mathematics, but they are closer to the manuscript.[8] Because the fourth correction concerns labels in geometrical figures, Rashed did not mention it in his list. Fifteen corrections in Rashed's list are improvements of Hogendijk's Arabic text but do not affect the translation.[9]

Many of Rashed's remaining corrections are small. A few characteristic examples follow.

1. Ibn al-Haytham begins four of his Propositions (1, 14, 18, 20) with "If there is a known (conic section)" and one Proposition (16) with "If a (conic section) is known." Hogendijk sticks to the text in the Manisa manuscript.[10] Rashed prefers the text in the beginning of Proposition 16, and he therefore emends the beginning of Propositions 1, 14, 18, 20. In the critical apparatus to Proposition 1 (R149:23f), Rashed says in Arabic that the manuscript reading (i.e., Hogendijk's text) is also possible. However, in the list of corrections to Hogendijk, Rashed presents his emendation four times as a correction to Hogendijk. Rashed's own translations of the five passages are inconsistent.[11]

2. Hogendijk adds at the beginning of Proposition 1 an Arabic word and an Arabic passage to the Manisa manuscript, so that the restored text is similar to the beginnings of other propositions. Hogendijk's translation reads (H140:5,10, words restored by Hogendijk are in angular brackets): "⟨let⟩ the (conic) section first ⟨be⟩ a parabola ... ⟨So we assume this by way of analysis. Let it (the tangent) be BE.⟩" In his list of corrections to Hogendijk, Rashed says that the Arabic word and Arabic passage have to be deleted (corrections to H141:4, H141:8 on R23). But Rashed translates Hogendijk's edited text rather than his own: "Que la section soit d'abord une parabole ... ⟨Supposons cela par l'analyse, qu'elle soit BE⟩" (R150:1,4–5).

A similar example is at the end of Proposition 7, where Hogendijk restored ⟨in two points⟩ to the text (Arabic text H181:10, translation H180:13). In the list of corrections (R24), Rashed says that Hogendijk's addition should be deleted, but in the translation R180:10 Rashed adds the footnote: "sous-entendu: en deux points" ... The opposite occurs

[8] The four corrections correspond to the following changes in the Arabic text: H137:18 read *dirya* as *durriyya*, cf. R149:15f; H163:15 change *qaddamnā* to *qad bayyannāhu* (R167:21f); H237:17 change *huwa anna* to *huwa* ⟨*immā*⟩ *anna*; H257:10–16 change *rā'-tā'* to *mīm-tā'* and *mīm-tā'* to *rā'-tā'*. Hogendijk's emendations H257n4–n8, n10–n11 are unnecessary.

[9] H137:7 change *anna* to *inna*; H141:3 change *al-qiṭ*^{c} *nisba* to *al-qiṭ*^{c} *mithl nisba* as in the manuscript; H149:15 change *li-l-qiṭ*^{c} *AG* to *li-qiṭ*^{c} *AG*; H153:14,15, H175:7, H181:5, H229:9 change *taqa*^{cc}*ur* to the manuscript reading *muqa*^{cc}*ar*; this is uncommon but Rashed has also found the same unusual word in other works of Ibn al-Haytham, cf R161:11f; H181:6 emend *wa-innahu* to *fa-innahu*, as in R181:6f; H211:16 change ^{c}*an il-qiṭ*^{c}*a* to ^{c}*an il-qiṭ*^{c}; H213:18 change *yaqṭa*^{c}*u* to *taqṭa*^{c}*u*; H231:14 change *ma*^{c}*lūm* (2) to *ma*^{c}*lūma*; H247:14 change *al-tartīb* to *tartīb*; H279:8 emend *quṭr al-qiṭ*^{c} to *quṭran li-l-qiṭ*^{c} as in R257:3f (thus Rashed made the Arabic text agree with the translation "a diameter of the conic section" in H278:10); H291:11 change *al-maqāla* to *maqāla*. Rashed says that the Arabic word for parabola should be written as *mukāfi'*, not *mukāfī* as in Hogendijk's edition. There is no agreement on the orthography of this word in the literature that I have consulted.

[10] See the translations on H140:1, H216:1, H230:1, H238:1, H220:1.

[11] See R23–25, corrections to H141:1, H217:1, H231:1, and H239:1. Rashed emends (*idhā kāna qiṭ*^{c}*un* ...) *ma*^{c}*lūmun* to *ma*^{c}*lūman*. His translations are as follows: Proposition 1 "[s]oit une section conique donnée" (R148:35), Proposition 14 "[s]i on a une parabole connue" (R206:3), Proposition 16 "[s]i on a une hyperbole ou une ellipse connue" (R208:10), Proposition 18 "[s]i une section conique est connue" (R216:13), Proposition 20 "[s]i on a une section de cône donnée" (R224:6). These translations correspond to Rashed's Arabic text in Propositions 1, 18, to Hogendijk's text in Proposition 14, 20, and to neither in Proposition 16. Rashed's translations of *ma*^{c}*lūm* as "known" and "given" take no account of the subtle philosophical difference between the two terms.

in Proposition 19. Here the manuscript reads "If T is greater" and Hogendijk translates (H234:12) "If T is greater (than K)," where (than K) is Hogendijk's explanatory addition. Rashed restores "If T is greater ⟨than K⟩" and he presents his restoration as a correction to Hogendijk (R25, correction to H235:8, cf. R218:23).

3. Rashed sometimes emends the text in a way which is slightly different from but equivalent to Hogendijk's emendation. He then lists his emendation as a correction to Hogendijk. In the end of Proposition 5, for example, the Manisa manuscript mentions "the hyperbola which was not drawn through M." Hogendijk excises the Arabic word "not," emends the next word (H165n7), and translates "the hyperbola drawn through M" (H164:26). Rashed emends the Arabic word "not" (R169:14f) and translates (R168:16) "L'hyperbole tracée par le point M." Rashed presents his emendation as a correction to Hogendijk's emendation (R24, correction to H165:22).

4. The text contains 33 references to the *Conics*, often in the form "as is proven in Proposition x of Book y," and sometimes in slightly different forms that can easily be interpreted as scribal errors. Hogendijk made the corresponding small emendations but Rashed prefers to keep the manuscript text. In Hogendijk's translation, this involves at most changing "as" to "because of what," and "in" to "by." Rashed lists his changes as nine separate corrections to Hogendijk. Rashed is consistent neither in these emendations nor in their translation.[12]

In Appendix 2 the reader will find the few corrections by Rashed that have a significant effect on the translation and that have not been mentioned above. Most of Rashed's corrections, however, do not cause any change in the translation. The differences between labels of points in geometrical figures are listed in Appendix 3, together with a number of errors in Rashed's figures. These errors do not result from differences in mathematical interpretation of the text. In conclusion, Rashed's new edition and translation are only infinitesimally different from the earlier edition and translation by Hogendijk.

5. RASHED'S MATHEMATICAL AND HISTORICAL COMMENTARY

Rashed's commentaries to most mathematical problems in Ibn al-Haytham's text consist of two parts. The first part is a transcription of Ibn al-Haytham's mathematical argument in modernized notation. Hogendijk argues that the manuscript text contains interpolations by one or more later authors, who was or were responsible for some primitive mathematical mistakes in the text (H120–122). Rashed, on the other hand, does not discuss the possibility of interpolations in the text, and he does not react to Hogendijk's arguments. Apparently, Rashed accepts the contradictions arising from his opinion that the whole text is authentic. Compare the following sentence in his commentary on the figures for Propositions 12–13: "Ibn al-Haytham dit que l'arc capable considéré (i.e., the circular arc) coupe la section 'dans tous les cas'; ceci est inexact, comme Ibn al-Haytham le montre plus loin . . . " (R69).

The second part of Rashed's commentaries concerns the number of solutions of the problems. For most of the problems, Ibn al-Haytham presents a *diorismos* (plural: *diorismoi*),

[12] The corrections are in R23–26 to H141:14, H143:6, H145:8, H145:11, H181:8, H193:8, H217:14, H269:8, H273:11. Rashed emends "because of what" (*limā*) to "as" (*kamā*) in a reference to *Conics* VII:13 in R255:2f, following Hogendijk's emendation H277n1. Rashed usually translates *kamā* as "comme ce qui" and *limā* as "d'après ce qui" or "en raison de ce qui," but in R150:21 he translates *kamā* as "d'après ce qui."

i.e., a necessary and sufficient condition for the existence of a solution. Hogendijk identified various mathematical mistakes in Ibn al-Haytham's diorismoi in Propositions 5, 12–13, 22, 23, and 27, and he also determined correct diorismoi in the cases where Ibn al-Haytham was wrong.

In the commentary on Proposition 13, Rashed refers to Hogendijk's elementary solution of the problem, from which the diorismos can easily be found (R93). This is the only reference to Hogendijk in Rashed's entire mathematical and historical commentary. However, Rashed's mathematically correct diorismoi for Propositions 5, 22, 23, and 27 are also closely related to the diorismoi presented by Hogendijk. Because Rashed uses a different notation, the diorismoi will be rendered here in both notations, together with the substitutions that have to be made to relate them to one another. The precise definitions of the various notations can be found in the two editions.

Proposition 5: Hogendijk's diorismos: $\alpha \geq \alpha_0$ with $\alpha_0 > 2$ and $z = \alpha_0^2$ a root of the polynomial $z^3 + z^2(-8 - 11\eta) + z(16 + 12\eta - \eta^2) + \eta^2 = 0$ (H11, H327–328). Rashed's diorismos: $\frac{G}{H} > 2$ and $k^2 \frac{G^2}{H^2}(\frac{G^2}{H^2} - 4)^2 - k\frac{G^2}{H^2}(11\frac{G^2}{H^2} - 12) - \frac{G^2}{H^2} + 1 \geq 0$ (R45). Substituting $\alpha \to \frac{G}{H}$ $z \to \frac{G^2}{H^2}$, $\eta \to \frac{1}{k}$, we obtain Rashed's polynomial from Hogendijk's polynomial.

Proposition 22: Hogendijk's diorismos: $MA \leq \frac{\delta}{4}(\frac{r}{\rho} + \frac{\rho}{r})$ (H364, note 22.14). Rashed's diorismos: $kcm - \frac{G^2}{4} \leq \frac{k^2c^2}{4}$ (R113). Substituting $MA \to m, \delta \to G, r \to c, \rho \to \frac{G}{k}$ we obtain Rashed's diorismos from Hogendijk's diorismos.

Proposition 23: Rashed's diorismos is mathematically correct (R119): $\lambda \leq \lambda_0$ with λ_0 the least positive root of $\lambda_0^2 + \frac{\lambda_0}{d}(G^2 - 4m(m+d)) + G^2 = 0$. Substituting $m \to AM$, $m + d \to RM$, $G \to \delta, \lambda \to \frac{r\delta}{\rho}$ in $\lambda^2 + \frac{\lambda}{d}(G^2 - 4m(m+d)) + G^2 \geq 0$ we obtain $AM \cdot RM \leq \frac{1}{4}(\delta^2 + d\delta(\frac{r}{\rho} + \frac{\rho}{r}))$, which is Hogendijk's diorismos with d^2 corrected to δ^2 (H368). To Hogendijk's diorismos one should add the condition $r < \rho$, which is obviously necessary in order for the hyperbola and the ellipse to intersect.

Proposition 27: The problem has a nontrivial diorismos for the hyperbola $y^2/x(x + d_0) = r_0/d_0$ if $r_0 > 3d_0$. Ibn al-Haytham does not mention this case. Hogendijk's diorismos for this case: $EZ^2 \geq 8d_0(r_0 - d_0)$ (H373 note 27.10). Rashed's diorismos for this case: $EG^2 \geq 8AD \cdot (AI - AD)$ (R131), which we obtain from Hogendijk's diorismos by the substitutions $EZ \to EG, d_0 \to AD, r_0 \to AI$. Hogendijk and Rashed both mentioned that the correct diorismos can be deduced from Apollonius' *Conics* VII : 40 [Toomer 1990 I : 490–497]; see H372–374 and R129n28, R132.

Rashed's historical commentary is also closely related to Hogendijk's commentary, as is shown by the following two examples. The *Completion of the Conics* is not mentioned in the three medieval lists of Ibn al-Haytham's work that have survived. Rashed (R20) follows Hogendijk's conclusion (H62–63) that it is an authentic work by Ibn al-Haytham. Hogendijk analyzes the available information on the lost Book VIII of Apollonius' *Conics* (H41–51) and he concludes that Ibn al-Haytham had even less information on Book VIII than is available today (H66–67). Rashed presents a similar investigation with the same conclusion (R2–15).

Rashed's commentary is related to Hogendijk's commentary in multiple other ways. Because both commentaries are in Western languages, it would not be difficult for the interested reader to find further examples. See also the end of Appendix 3, concerning Ibn al-Haytham's work on the regular heptagon.

6. THE HISTORICAL SIGNIFICANCE OF IBN AL-HAYTHAM'S TEXT

In the previous sections we have seen that Rashed's Arabic edition, translation and mathematical commentary of Ibn al-Haytham's *Completion of the Conics* are very closely related to my own Arabic edition, translation and mathematical commentary of the same text. I will now briefly comment on Rashed's opinion of the historical significance of Ibn al-Haytham's *Completion of the Conics* as a whole. While our editions may be closely related, our interpretations on this score differ significantly. Here, I want to make it perfectly clear that Rashed has every right to hold his own opinion on the value of Ibn al-Haytham's work. However, I equally want to emphasize that his opinion may not be as supportable from the extant historical evidence as one might hope.

The *Completion of the Conics* is by far Ibn al-Haytham's most important work on conic sections, and it leads Rashed to the following view on the position of Ibn al-Haytham in the history of geometrical constructions by means of conic sections:[13] Before Ibn al-Haytham, the Greek and Islamic geometers studied isolated problems by means of intersecting conic sections. Rashed says that these problems appeared in a sporadic way. In Rashed's opinion, we see in Ibn al-Haytham's work that the study of problems by means of intersecting conic sections (or even pencils of conic sections) becomes a systematic method of investigation in geometry. Most of these problems are solid (i.e., algebraically equivalent to an irreducible cubic or quartic equation), but Rashed thinks that Ibn al-Haytham also consciously uses conic sections to study problems that are solvable by ruler and compass. Rashed believes that Ibn al-Haytham carefully studies the existence and the number of solutions by determining the number of intersections of the conic sections on the basis of their asymptotic properties and their tangency. For Rashed, the history of Islamic geometry is a story of progress beyond the work of Archimedes and Apollonius, and the culmination of Islamic geometry is the work of Ibn al-Haytham.[14]

I disagree with this opinion for the following reasons. Ibn al-Haytham does not relate the number of solutions of geometric problems to the tangency of conic sections used in their solution (H100–103). Proposition 5 of the *Completion of the Conics* comes closest, and can be summarized as follows. In that proposition, Ibn al-Haytham studies a geometric construction involving conic sections and a given ratio α, and he presents a diorismos of the form $\alpha \geq \alpha_1$. For $\alpha = \alpha_1$, Ibn al-Haytham shows that two conic sections used in the construction intersect at two (possibly closely located) points, and he concludes that the problem has two solutions. Ibn al-Haytham then presents an incorrect "proof" that the two conics do not intersect for $\alpha < \alpha_1$ (H326–327), and he concludes that the problem has no solution in this case. As a matter of fact, there is a ratio $\alpha_0 < \alpha_1$ such that the two conics are tangent if $\alpha = \alpha_0$ (so the problem has one solution) and intersect at two points if $\alpha > \alpha_0$. The diorismos $\alpha \geq \alpha_0$ is equivalent to the correct diorismos for Proposition 5 mentioned in Section 5 above. Thus Ibn al-Haytham cannot have been aware of the relationship between diorismoi and tangency of conic sections when he was writing the *Completion of the Conics*.

As far as is known, the only Islamic mathematician who knew this relationship and actually used it in his own work was Abū Sahl al-Kūhī, who flourished around 970 in Iran. Al-Kūhī's text is available in an English translation in [Berggren 1996]. A number of Hellenistic geometers, including Archimedes and Apollonius, were certainly able to relate

[13] See Rashed's Introduction, p. xvii, R18, R48, R143.
[14] See Rashed's Introduction, pp. v–vi, xiii–xiv, xix.

the number of solutions of problems to the tangency of conics used in their construction. The relation is used explicitly in a construction, probably by Archimedes, in the commentary of Eutocius on Archimedes' *On the Sphere and Cylinder* II : 4, and implicitly by Apollonius in *Conics* V (H103–104). Eutocius' commentary was translated into Arabic but apparently unknown to Ibn al-Haytham, who seems to have been unaware of some ancient Greek discoveries.

A number of other mathematical errors and unnecessary difficulties in Ibn al-Haytham's *Completion of the Conics* have been identified (H85–96), and many of the problems that were studied by Ibn al-Haytham could have been solved correctly or more easily by Apollonius (H361 note 21.13, H373 note 27.10, H379 note 31.5, H382–384). Rashed tries to explain most of the errors and difficulties in Ibn al-Haytham's *Completion of the Conics* as the results of conscious choices on the part of Ibn al-Haytham (cf. R47, R95, R132, R143). In other cases, Rashed does not present an explanation (R112, R117).

In my view, Ibn al-Haytham was a creative mathematician whose *Completion of the Conics*, while an interesting historical text, reaches the heights neither of most geometrical works by Archimedes nor of the *Conics* of Apollonius. This difference of interpretation suggests a cautionary tale. Medieval mathematical texts can often be clarified by an analysis of their contents in terms of more recent mathematics, but what kinds of historical conclusions can legitimately be drawn from such an analysis? Interpreting Islamic mathematics in terms of 17th- and 19th-century concepts leaves one open to the danger of attributing concepts to Islamic mathematicians that they did not possess and of describing developments in Islamic mathematics that did not actually happen. Such interpretations may lead to a naive glorification of medieval Islamic mathematics. A methodology that draws from the extensive modern literature on Greek mathematics and that thus places medieval Islamic mathematics in context helps avoid these pitfalls. Moreover, analyzing the achievements of medieval Islamic mathematicians in context allows for an examination of their failures and errors as well as their successes. While the successes are important, the errors and failures reveal the limits of the mathematical capabilities of the medieval Islamic mathematicians, and thereby aid us in reaching a fuller understanding of the rich complexity of the medieval Islamic mathematical tradition.

APPENDIX 1

In this Appendix the entire corpus of items in both the critical apparatus of Hogendijk and Rashed is presented, and the relationships are indicated. Two emendations are deemed equal if the edited texts are equal. For each page, Hogendijk presented his apparatus in the form of numbered footnotes, and Rashed in the form of one or more items indicated by the line number.

The following notations will be used in this Appendix:

H135n1	footnote 1 to page 135 of Hogendijk's edition
R147:2	the single item in Rashed's apparatus for line 2 of page 147 (This notation replaces R147:2f in the main text of this paper)
R147:6-i	the first item in Rashed's apparatus for line 6 of page 147
R151:8*	not in Rashed's critical apparatus, but in Rashed's edited text.
H135:5*	not in Hogendijk's critical apparatus, but in Hogendijk's edited text.
O	emendation by Hogendijk omitted by Rashed

N	new emendation made by Rashed, not made by Hogendijk
CrH	emendation which Rashed credits to Hogendijk
NegH	Rashed criticizes Hogendijk
[R209:10]	the item R209:10 is not an emendation

The notation R175:11 = +H175n4 means that Hogendijk changes more than Rashed, but Rashed's emendation is part of Hogendijk's.

Rashed's emendations which are the same as Hogendijk's emendations and which Rashed does not credit to Hogendijk are printed in italics.

COMPLETE COMPARISON OF RASHED'S CRITICAL APPARATUS WITH HOGENDIJK'S CRITICAL APPARATUS OF IBN AL-HAYTHAM'S COMPLETION OF THE CONICS

Preface: _R147:2 = H135n1, R147:6-i = H135:5_*, _R147:6-ii = H135n2, R147:8 = H135n3, R147:9-i = H135:8_*, _R147:9-ii = H135:8_*, R147:11-i = N (cf. H135n4), _R147:11-ii = H135n5, R147:12 = H135n6, R147:13 = H135:14_*, R147:14 = CrH135:14, _R147:15 = H135n7, R147:17-i = H135n8,_ R147:17-ii = N (cf. H135n9), O = H137n1, O = H137n2, _R149:12 = H137n3, R149:14 = H137n4_, R149:15-i = N, R149:15-ii = N (cf. H137n5), O = H137n6, O = H137n7, R149:16 = N, O = H139n1, O = H139n2, _R149:21 = H139n3._

Proposition 1: R149:23 = N, _R151:1 = H141n1, R151:2 = H141n1, R151:3 = H141n2, R151:4 = H141n3, R151:5 = H141n4, R151:6 = H141n5,_ O = H141n6, R151:8* = H141n7, _R151:10–11 = H151n8._

Proposition 2: _R151:14 = H143n1_, O = H143n2, _R153:3 = H143n3._

Proposition 3: _R153:6-i = H145n1, R153:6-ii = H145n2, R153:6-iii = H145n3, R153:7 = H145n4,_ O = H145n5, O = H145n6, _R155:5 = H145n7_, R155:6 = N, O = H145n8, _R155:7-i = H145n9, R155:7-ii = H145n10, R 155:8-i = H145n11, R155:8-ii = H147n1, R155:8-iii = H147:1_*, R155:11–12 = N (cf. H147:5), _R155:13-i = H147:6_*, _R155:13-ii = H147n2, R155:13-iii = H147n3, R155:13-iv = H147n4, R155:13-v = H147:7_*, _R157:3 = H147n5, R157:4 = H147n6, R157:5–6 = H147n7, R157:9 = H147n8._

Proposition 4: _R157:12 = H149:3_*, _R157:13 = H149:5_*, _R157:14-i = H149n1, R157:14-ii = H149:7_*, R157:15 = N (cf. H149n2, H149n3, H151n3, H153n1, H161n1, H161n3, H163n9, H163n12, H167n8), _R159:1 = H149n4,_ O = H149n5, _R159:6 = H151n1, R159:9-i = H151n2, R159:9-iii = H151:6_*, _R159:10 = +H151n3,_ O = H151n4, _R159:14–15 = H151n5, R159:14 = H151:11_*, _R159:15 = H151n6, R159:16 = H151:14_*, O = H153n1, _R161:6-i = H153n2, R161:6-ii = H153n3, R161:7 = H153n4, R161:8-i = H153n5, R161:8-ii = H153n6,_ R161:10 = N (cf. H153n7), R161:11 = N (cf. H153n8, H153n9), _R161:13 = H153n10._

Proposition 5: _R161:20-i = H155n1, R161:20-ii = H155n2, R163:3 = H155n3, R163:4 = H155n3,_ R163:5-i = N (cf. H155n4), _R163:5-ii = H155n5, R163:7 = H155n6, R163:8 = H155n7,_ O = H157n1, _R163:15–16 = H157n2, R163:17 = H157n3,_ R163:18* = H157n4, R163:19 = CrH157:13, R165:1 = CrH159:1, _R165:2-i = H159n1, R165:2-ii = H159:2_*, _R165:2-iii = H159n2, R165:3 = H159:4_*, R165:4* = H159n3, _R165:7 = H159:9_*, O = H159n4, _R165:12-i = H159n5, R165:12-ii = H159:16_*, _R165:13-i = H159n6, R165:13-ii = H159n7, R165:13-iii = H159n8, R165:18 = H159n9,_ O = H161n1, _R165:21-i = H161:3_*, _R165:21-ii = H161n2_, O = H161n3, _R165:23 = H161n4, R165:24 = H161n5, R167:2-i = H161n6, R167:2-ii = H161n7, R167:2-iii = H161n6, R167:3 = H161n8, R167:6 = H161n9_, R167:8* = H161n10, R167:10 = N, _R167:13 = H163n1, R167:14-i = H163n2, R167:14-ii = H167n3, R167:15–16 = H163n4, R167:16_[15] _= H163n5, R167:20 = H163n6_, O = H163n7, R167:21 = N (cf. H163n8), O = H163n9, _R167:22-i = H163n10, R167:22-ii = H163:11_, O = H163n12, R167:24 = CrH163:18, _R169:1 = H165n1, R169:2 = H165n2, R169:4 = H165n3, R169:8 = H165n4, R169:10 = H165n5_, R169:11 = CrH165:18, R169:13* = H165n6, R169:14 = N (cf. H165n7, H165:22), _R169:15-i = H165n8, R169:15-ii = H165n9, R169:17 = H167n1_, R169:19–20 = CrH167:4–5, R169:20 = NegH167:5, R169:22-i = N, R169:22-ii = N, R169:22-iii = N, R169:23-i = N, _R169:23-ii = H167n2, R169:24 = H167n3–4, R171:2-i = H167n5_, R171:2-ii = N, _R171:2-iii = H167n6, R171:3 = H167n7_, O = H167n8, _R171 : 4 = H167n9._

Proposition 6: _R171:10 = H169n1, R171:11 = H169n2, R171:12-i = H169n3, R171:12-ii = H169n4, R171:15 = H169n5, R173:1 = H169n6, R173:3 = H169n7, R173:4 = H171n1, R173:8-i = H171n2, R173:8-ii = H171n3, R173:11 = H171n4_, R173:12-i = CrH171:13, _R173:12-ii = H171n5, R173:15 = H171n6, R171:16 = H171n7,_

[15] The line number should be R167:17.

R173:17 = H171n8, R173:18–19 = H171n9, R173:18 = H171n10, R173:19 = H171n11, R173:20 = H173n1, R173:21* = H173n2, *R173:22-i = H173n3, R173:22-ii = H173n4.*

Proposition 7: R175:1 = N, *R175:2-i = H173n5, R175:2-ii = H173n6, R175:4 = H173:8**, R175:7 = N (cf. H175n1), *R175:9 = H175n2*, O = H175n3, *R175:11 = +H175n4*, R177:4 = N (cf. H175n5), R177:6 = CrH175:15, *R177:8 = H175n6*, R175:10 = CrH177n1, *R177:11-i = H177n2, R177:11-ii = H177n3, R177:13 = H177n4, R177:15 = H177n5, R177:17-i = H177n6, R177:17-ii = H177n7, R177:18-i = H177n8, R177:18-ii = H177n9, R177:24 = H177n10*, O = H177n11, *R179:3 = H179n1, R179:4 = H179n2, R179:6 = H179n3, R179:7 = H179n4*, R179:9-i = N, *R179:9-ii = H179n5, R179:9-iii = H179n6, R179:9–10 = H179n5, R179:10-i = H179n7, R179:10-ii = H179n8, R179:11-i = H179n9, R179:11-ii = H179n10*, R181:1* = H181n1, *R181:3 = H181n2*, O = H181n3, R181:6 = N, O = H181n4, *R181:9-i = H181n5, R181:9-ii = H181n6*, O = H181n7, *R181:11 = H181n8, R181:12 = H181n9, R181:13 = H181n10, R181:15-i = H181n11, R181:15-ii = H181n12*, R181:15-iii = N, R181:17 = CrH181:20, *R181:19 = H181n13*, O = H181n14.

Proposition 8: *R181:22 = H183n1, R183:6 = H183n2, R183:7 = H183n3, R183:13 = H185n1.*

Proposition 9: *R183:18 = H185n2, R185:3-i = H185n3, R185:3-ii = H185n4,* O = H185n5 (cf. R185:6), O = H185n6, O = H185n7, *R185:11 = H185n8, R185:12-i = H185n9, R185:12-ii = H185n10*, R185:13 = CrH187:2.

Proposition 10: R185:18 = CrH187:7, *R187:3-i = H187n1, R187:3-ii = H187n2, R187:4 = H189n1, R187:5-i = H189n2, R187:5-ii = H189n3, R187:6-i = H189n4, R187:6-ii = H189n5, R187:7 = H189n6, R187:8 = H189n7*, R187:9 = N, *R187:12 = H189n8*, O = H189n9, *R187:15-i = H189n10, R187:15-ii = H189n11*, R187:16 = CrH189:16, *R187:17 = H189n12.*

Proposition 11: R189:3* = H189n13, *R189:3 = H189n14, R189:6-i = H189n15, R189:6-ii = H191n1, R189:11 = H191n2, R189:13-i = H191n3*, R189:13-ii = N (cf. H191n4), R189:15* = H191n5, *R189:17 = H191n6*, R191:3 = CrH191:19.

Proposition 12: *R191:11 = H193n1, R191:14 = +H193n2, R193:3-i = H195n1, R193:3-ii = H195n2, R193:4 = H195n3, R193:5 = H195n4, R193:6 = H195n5*, R193:10* = H195n6, *R193:13 = H195n7*, R193:15 = N, *R193:17-i = H197n1, R193:17-ii = H197n2, R195:4 = H197n3, R195:7 = H197n4, R195:9 = H197n5, R195:11–12 = H197n6,*[16] *R195:12 = H197n7, R195:13 = H199n1, R195:14 = H199n2, R195:15 = H199n3, R195:16 = H199n4*, R195:16–17 = CrH, but incorrectly so, compare H199:5–6 and H199n5, R195:18 = CrH199:9, *R195:20 = H199n6, R195:22 = H199n7*, O = H199n8, O = H199n9, O = H199n10, O = H199n11, O = H199n12.

Proposition 13: *R197:2 = H201:1**, R197:3–i = CrH201:3, *R197:3-ii = H201n1, R197:4 = H201n2, R197:5-i = H201n3, R197:5-ii = H201n4, R197:7 = H201n5, R197:12-i = H203n1*, R197:12-ii = CrH203:7–8, *R197:13 = H203n2, R197:14 = H203n3*, R197:14–15 cf. H203:10, *R197:15 = H203n4, R197:16 = H203n5, R199:1 = H203n6*, O = H203n7, *R199:4-i = H203n8, R199:4-ii = H203n9, R199:6-i = H203:21*, R199:6-ii = H203n10, R199:7 = H203n11, R199:8-i = H205n1, R199:8-ii = H205n2, R199:9-i = H205n3, R199:9-ii = H205:2*, R199:10 = H205:4*, R199:11 = H205n4, R199:12 = H205n5,* R199:14* = H205n6, R199:15* = H205n7, *R199:15 = H205n8*, R199:16* = H205n9, R199:17* = H205n10, R199:18* = H205n11, R199:18* = H205n12, *R199:19 = H205n13, R199:21-i = H205:18*, R199:21-ii = H205:19**, R199:23* = H205n14, *R199:23 = H205n15, R201:2-i = H207n1, R201:2-ii = H207n2, R201:4 = H207n3*, R201:8* = H207n4, *R201:15 = H207n5*, R203:1 = CrH207:22, *R203:2 = H209n1, R203:4 = H209n2*, R203:9 = CrH209:11, R203:10* = H209n3, R203:10* = H209n4, [R209:10], R203:11* = H211n1, *R203:17-i = H211n2, R203:17-ii = H211n3*, R203:19 = CrH211:10, O = H211n4, O = H211n5, *R205:2 = H213n1*, R205:5 = N, R205:7 = CrH213:10, *R205:9 = H213n2*, O = H213n3, *R205:16 = H215n1, R205:18-i = H215n2, R205:18-ii = H215n3, R205:19-i = H215n4, R205:19-ii = H215n5, R205:22 = H215n6*, O = H215n7, O = H215n8, R207:2* = H215n9.

Proposition 14: R207:3 = N, *R207:5 = H217n1, R207:12 = H217n2, R207:14-i = H217n3*, O = H217n4, *R207:14-ii = H217n5.*

Proposition 15: *R207:16–17 = H219n1, R209:1 = H219n2, R209:5-i = H219n3, R209:5-ii = H219n4, R209:6 = H219n5, R209:7 = H219n6.*

Proposition 16: *R209:9 = H221n1, R209:11 = H221n2, R211:2-i = H221n3, R211:2-ii = H221n4, R211:4 = H221n5, R211:5 = H223n1, R211:6 = H223n2*, O = H223n3, *R211:8 = H223n4, R211:9 = H223n5, R211:10 = H223n6, R211:11 = H223n7, R211:12 = H223n8*, R211:13 = CrH223:12, *R211:16 = H223n9*, R211:17-i = N, *R211:17-ii = H223n10, R213:2 = H223n11, R213:6 = H225n1.*

[16] Note 6 should be placed on sīn-ḥā’ in H197:18; in H197n6 change dāl-bā’ to sīn-ḥā’; in Rashed’s critical apparatus R195:11–12 change sīn-^{c}ayn to sīn-ḥā’.

Proposition 17: *R213:9 = H225n2, R213:10 = H225n3, R213:13 = H225n4*, R213:14 = CrH225:11, *R213:16 = H225n5, R215:2 = H227:2*, R215:4 = H227n1, R215:5 = H227n2, R215:6-i = H227n3, R215:6-ii = H227n4, R215:6-iii = H227n5, R215:9-i = H227n6*, R215:9-ii = CrH227:11, *R215:10 = H227n7, R215:16 = H229n1*, R215:17–18 = CrH229:2–3, *R215:19 = H227n2*, R217:3* = H229n3, O = H229n4, *R217:5 = H229n5, R217:8 = H229n6*.

Proposition 18: R217:11 = N, *R217:17 = H231n1, R217:20-i = H231n2, R217:20-ii = H231n3*, O = H231n4.

Proposition 19: *R219:13-i = H235n1, R219:13-ii = H235n2, R219:16 = H235n3, R219:20 = H235n4*, R221:1 = CrH235:11, *R221:8 = H235n5, R221:9 = H235n6, R221:11 = H235n7*, R221:12 = CrH237:1, *R221:16 = H237n1, R223:2 = H237n2*, R223:4* = H237n3, R223:6 = CrH237:12, R223:10* = H237n4, R223:11-i = N, *R223:11-ii = H237n5*, R223:12 = N, R223:13* = H237n6, R223:14 = CrH237:22, R225:2* = H237n7.

Proposition 20: R225:5 = N, R225:4* = H239n1, R225:7 = CrH239:5, *R225:9 = H239n2, R225:11 = H239n3*, O = H239n4.

Proposition 21: R227:1-i = CrH241:1, *R227:1-ii = H241n1*, R227:1-iii = CrH241:1, R227:2 = N, *R227:4 = H241n2, R227:5 = H241n3, R227:6 = H241n4, R227:7 = H241n5*, R227:8* = H243n1, *R227:13 = H241:6**, R229:3 = CrH243:11, *R229:8 = H243n2, R229:10 = R243n3*.

Proposition 22: R231:2 = CrH245:2, *R231:4 = H245n1, R231:5 = H245n2*, R231:8 = N, O = H245n3, O = H245n4, R231:12 = N, R231:12* = H245n5, *R231:14-i = H245n6*, R231:14-ii = N, *R231:15 = H245n7*, R231:17 = N (cf. H245n8), R231:18 = N, R231:19 = N, R233:1 = N, O = H247n1, R233:4-i = N, R233:4-ii = N, R233:5 = N, R233:6-i = N, R233:6-ii = N, R233:7 = N, *R233:8 = H247n2, R233:14 = H249n1, R233:15 = H249n2, R233:16 = H249n3*, R235:1 = CrH249:8, *R235:2 = H249n4, R235:11 = H249n5*, R235:13 = N.

Proposition 23: R235:16* = H251n1, R235:17 = CrH251:7, *R235:20 = H251n2*, R235:21 = N (cf. H251n3), O = H251n4, R235:22 = N (cf. H251n5), *R235:23 = H251n6, R235:24 = H251n7*, R237:3 = N, *R237:4 = H251:15*, R237:5 = H251n8, R237:8-i = H251n9, R237:8-ii = H251n10, R237:10 = H251n11*, O = H255n1, *R239:19 = H255n2, R241:5 = H257n1, R241:6 = H257n2, R241:7 = H257n3*, O = H257n4–n8, *R241:11 = H257n9*, O = H257n10–n12, *R241:14 = H257n13, R241:21 = H259n1*, R241:22 = CrH259:8, *R241:25 = H259n2, R243:2 = H259n3*, R243:2* = H259n4, R243:5* = H259n5, R243:5* = H259n6.

Proposition 24: R243:8-i = CrH261:3, *R243:8-ii = H261n1, R243:9 = H261n2, R243:11 = H261n3, R243:12 = H261n4, R243:13-i = H261:9*, R243:13-ii = H261n5, R245:1 = H261:13*, R245:3 = H261n6.*

Proposition 25: R245:4 = CrH263:1, *R245:7 = H263n1, R245:11 = H263n2, R245:12 = H265n1, R245:14 = H265n2*, R247:1* = H265n3, *R247:1 = H265n4*, R247:6 = CrH265:11–12, *R247:7 = H265n5*, R247:13* = H265n6, *R247:14 = H267n1, R247:15 = H267n2*, R247:16* = H267n3, *R247:16 = H267n4, R249:2 = H267n5*, R249:4 = N (cf. H267n6), O = H267n7, R249:5-i = N, *R249:5-ii = H267n8*, R249:7* = H267n9.

Proposition 26: R249:9 = CrH269:2, *R249:11 = H269n1, R249:13 = H269n2*, O = H269n3, *R249:14 = H269n4, R249:16 = H269n5.*

Proposition 27: *R251:1 = H271n1, R251:2 = H271n2*, R251:2–3 = CrH271:2–3, *R251:5 = H271n3, R251:5–6 = H271n4, R251:6 = H271n5, R251:9 = H271n6, R251:10-i = H271n7*, R251:10-ii = CrH271:11–12, R251:11 = CrH271:13, *R251:14-i = H273n1, R251:14-ii = H273n2*, R253:1* = H273n3, O = H273n4, *R253:11 = H273n5*, R253:12* = H273n6, R253:17 = N, R253:18 = CrH275:6, R253:21* = H275n1, [R253:22], *R255:2-i = H277n1, R255:2-ii = H277n2*, R255:3 = CrH271:6, *R255:12 = H277n3*, O = H277n4, *R255:15 = H277n5*.

Proposition 28: O = H279n1, *R255:17 = H279n2*, R257:3 = N, *R257:4 = H279:9*, R257:5 = H279n3, R257:6 = H279n4*, R257:6* = H279n5, *R257:8-i = H279n6, R257:8-ii = H279:14*, R257:10 = H281n1, R257:13-i = H281n2, R257:13-ii = H281n3, R257:13-iii = H281n4*, R257:14* = H281n5, *R257:15 = H281n6, R257:16 = H281n7, R257:17 = H281n8*, R257:19 = CrH271:14, *R257:20 = H281n9*, R257:21* = H281n10, *R257:23 = H281n11*.

Proposition 29: *R259:1 = H283n1*, R259:3* = H283n2, *R259:4 = H283n3*, R259:8* = H283n4, *R259:13 = H283n5, R259:14-i = H283n6, R259:14-ii = H283n7, R259:15 = H283n8*, O = H283n9, R257:18 = CrH283:21.

Proposition 30: *R259:20 = H285n1, R261:4 = H285n2, R261:6 = H287n1, R261:7-i = H287n2, R261:7-ii = H287:2*, R261:8 = H287n3, R261:9 = H287n4*, R263:1* = H287n5, *R263:4-i = H287n6, R263:4-ii = H287n7, R263:7 = H289n1, R263:8 = H289n2*, R263:9-i = CrH289:7, *R263:9-ii = H289n3*, R263:17* = H289n4, *R263:19 = H289n5, R263:20 = H289n6, R263:21 = H289n7, R263:22 = H289n8, R263:23 = H289n9*, R263:23–24 = CrH291:1–2, *R263:24 = H291n1-n2, R265:1-i + R265:1-ii = H291n3, R265:2 = H291n4*.

Proposition 31: *R265:5 = H291n5, R265:8-i = H291n6, R265:8-ii = H291n7, R265:11 = H291n8, R265:15-i = H293n1, R265:15-ii = H293n2, R267:1-i = H293n3, R267:1-ii = H293n4, R267:2 = H295n1*, R267:2–3 = CrH295:1–2, R267:4 = CrH295:3, *R267:5 = H295n2, R267:6 = H295n3, R267:7-i = H295n4-n5,* *R267:7–ii* = *H295n6*, *R267:8* = *H295n7*, R267:9* = H295n8, *R267:11* = *H295n9*, *R267:12-i* = *H295:12–13**, *R267:12-ii* = *H295n10*, *R267:15-i* = *H295:16**, *R267:15-ii* = *H295n11*, *R267:15-iii* = *H295n12*, *R267:17* = *H295n13*, R265:21 = CrH297:6, *R267:22-i* = *H297n1*, *R267:22-ii* = *H297n2*, *R269:1* = *H297n3*, *R269:3-i* = *H297n4*, *R269:3-ii* = *H297n5*, *R269:3-iii* = *H297n6*, R269:4 = N, R269:5* = H297n7, *R269:6* = *H297n8*, *R269:13* = *H299n1*, R269:16 = N, R269:20* = H299n2, *R269:22* = *H299n3*.

In the edited texts, Hogendijk and Rashed sometimes put square brackets around words in the manuscript that have to be deleted from the text. In the following cases, Rashed follows Hogendijk without acknowledgement: R147:16 [bihi] = H135:17, R149:3 [yumāss] = H137:4, R163:11 [wa-]yakūn = H157:1, R169:12 [aw] = H165:19.

APPENDIX 2

In this Appendix, all nontrivial differences between Hogendijk's and Rashed's translations of Ibn al-Haytham's text are listed, which correspond to Rashed's corrections (R23–26) to Hogendijk's edition, and which have not yet been mentioned in Section 4 of this paper. For sake of brevity, angular brackets are inserted in translations of Rashed to indicate words that he has added to the manuscript. The reader can form his own opinion about the plausibility or implausibility of these corrections. For easy reference, H and R are used as abbreviations of Hogendijk and Rashed, and the notation H136h:8 is used for line 8 of paragraph h on p. 136 of H's translation.

The following differences will not be discussed in detail because the reader can identify them for himself: Passages in H164r:10, H164s:1–4, H166v:6, H198o:1, H198o:2, H254h:7–8, H260a:1, H276p:6–7, which H adds in angular brackets ⟨ . . . ⟩ to the manuscript and translation, and which are rejected by R, and changes where H has "so" in the translation and R "et" (or conversely, H has "and" and R has "donc"). The two corresponding Arabic words *wa-* and *fa-* are sometimes indistinguishable in the manuscript and scribal errors are likely.

Differences between the Two Translations

R148:10 (Apollonius dit que) . . . "dans le livre huit, lequel comprend des problèmes qui concernent les diamètres," cf. H136f:6–8 (Apollonius says that . . .) "in the eighth book. ⟨So the eighth book⟩ contained problems connected with the diameters" (i.e., this is not what Apollonius said but what Ibn al-Haytham thought).

R148:22 "les livres qui le précèdent," cf. H136h:8 "the books which were transmitted."

R148:25 "Comme cette idée s'est imposée à notre conviction," cf. H138i:1 "Since in our opinion this state of things is impossible." R excised the word *lam*, meaning "not," from the manuscript. The interpretation is the same, because R's *idée* (that Apollonius dealt with certain geometrical problems) is the opposite of H's *state of things* (that Apollonius did not deal with these problems).

R154:8 "Mais le rapport" is R's emendation for the manuscript text, which H translates as "for the ratio" in H144c:3.

R154:14–15 cf. H144e:3 ⟨de la droite homologue plus AD à AD qui est égal au rapport⟩ is R's addition to the text. Following H, R deleted an incomprehensible passage in the manuscript; see H315 note 3.5.

R158:15–16, H150f:6 H excised "mais le rapport du produit de MI par IA au carré de AS est égal au rapport de ME à EA" as a mechanical repetition of an earlier passage; see R158:11–12.

R160:11, H152k:4, H excised "La parabole passe par le point A" as a mechanical repetition of an earlier passage, see R160:8.

R172:3 "les moitiés des autres droites": a more literal translation is "halves of those lines" H168c:5–6; the passage has been discussed in Section 2 above. This difference is not due to a correction by R to H's Arabic text. H uses his more precise translation as an argument that the preceding sentence is an interpolation.

R178:11 "C'est pourquoi" is R's emendation for a word in the manuscript; cf. H178l:1 "Then, similarly." H argues that the preceding passage is an interpolation; see H333 note 7.13.

R180:11 "l'angle qui est ⟨au-delà de⟩ la section apres le point," cf. H180p:3 "the angle beyond the point." H excises "la section" as a scribal error.

R180:22–23 "de quelque manière " is emended by H to H180q:3 "how it can be found."

R184:10 H184d excised "et le rapport de EP à PI est égal au rapport de IP à PA," as a mechanical scribal repetition (cf. R184:8–9); the following passage ⟨et au rapport de EI à IA⟩ is R's own addition to the text.

R194:22–23 "c'est-à-dire le rapport de ⟨AD à⟩H⟨S⟩, et est égal au . . . " corresponds to H198o:1–2 "⟨. . . ⟩ that is the ratio of HN ⟨. . . ⟩ is equal to . . . ." The manuscript has HW, which H emends to HN, and which R interprets as H *wa-*, meaning "H and."

R194:30–31 “Si la droite NH, ... et si la section HA ...,” cf. H198q:1 “If line NH, ... (then) (conic) section HA ....”

R216:7–8 “l’hyperbole AN se rapproche indéfiniment de la droite sur laquelle tombe ⟨la section EN⟩; cf. H228j:4 “the hyperbola approaches its asymptote continuously.” H adds “⟨not⟩” to the text; the “line at which the conic section does not fall” is the technical term for the asymptote of a hyperbola.

R222:15,17 “intérieur ... extérieur, cf. H236i:4,6 “exterior ... interior.” R emends the text for mathematical sense, but H translates the manuscript text because he considers the passage to be an interpolation which does not necessarily make mathematical sense; cf. H359 note 19.15.

R226:3 “qu’elles soient $\langle E\rangle H$ et $\langle D\rangle I$,” cf. H240:3, “let them be H and T.”

R230:18–20 “L’ordonnée menée du point M au pourtour de la parabole ⟨est égale à l’ordonnée menée du point M au pourtour de l’ellipse,⟩” cf. H244e:4–5 “So the ordinate drawn from point M to the boundary ⟨of the ellipse is equal to the ordinate drawn from point M to the boundary⟩ of the parabola.” In his list of corrections to H, R does not mention this alternative.

R232:2–3 “le rapport du produit de HO par OI ⟨au produit de OP par OI⟩ est donc égal au rapport de HI à IK, cf. H246g:1 “[t]hen the ratio of $\langle HO$ to $OF\rangle$ is equal to the ratio of HT to TK.” Rashed translates the manuscript. More passages may be missing at this point.

R238:7 ⟨est égal au rapport de HO à OP qui est égal au rapport de $HI\rangle$ à IK, cf. H254g:6 ⟨is equal to the ratio of $HT\rangle$ to TK.”

R238:20, cf. H254h:11. Here H and R both excise a passage as a mechanical scribal error. H removes a longer passage than R, including the words “comme l’ordonnée” in R’s translation.

R244:17 “diamètre ⟨droit⟩ qui est son conjugué,” cf. H264d:4 “its conjugate diameter.”

R246:20 ⟨la moitié du⟩ diamètre ... plus grande que l’axe,” cf. H266i:3–4 “the diameter ... longer than twice the axis.” R removes the word “twice” from the manuscript.

R248:5 “ou si” is R’s emendation for “and”; H’s emendation is “then” in H266k:4. The meaning of the passage remains the same.

R268:5 “au rapport de QK à QL,” cf. H296l:7–8 “to the ratio of QK to $\langle QL \dots\rangle$ KL.” R emends KL to QL.

APPENDIX 3: FIGURES

Photographic reproductions of the figures in the manuscript are to be found in H301–310.

Hogendijk and Rashed use different systems for transcribing the labels of points in the figures. To make the systems correspond to each other, the following substitutions have to be made:

Hogendijk ↔ Rashed: $G \leftrightarrow C$, $W \leftrightarrow F$, $Z \leftrightarrow G$, $T \leftrightarrow I$, $I \leftrightarrow J$, $F \leftrightarrow P$, $C \leftrightarrow U$, $J \leftrightarrow X$, $t \leftrightarrow T$, $h \leftrightarrow W$, $X \leftrightarrow V$, $V \leftrightarrow D'$, $d \leftrightarrow U'$, $Y \leftrightarrow I'$, $g \leftrightarrow O'$, $\lambda \leftrightarrow L_a$.

In the following cases, Hogendijk and Rashed read one or more labels of points differently:

Proposition 5, H156 = Fig. 5, R162 (Hogendijk ↔ Rashed): $I \leftrightarrow T$, $t \leftrightarrow W$, $h \leftrightarrow J$.

Proposition 7, H173 = Fig. 7.1 R174; H174 = Fig. 7.2 R178; Proposition 23, H250 = Fig. 23.1 R236; H255 = Fig. 23.2 R238: $R \leftrightarrow G$.

Proposition 12: H193 = Fig. 12.1 R190; H196 = Fig. 12.2 R192, H255 = Fig. 23.2 R238: $d \leftrightarrow U$. It is not necessary to assume with Rashed (R236f50) that Ibn al-Haytham indicated different points by the same letter.

In Proposition 22, the *latus rectum* of the parabola appears two times (in Hogendijk’s transcription) as WA and 13 times as W. Hogendijk draws the *latus rectum* as a separate segment W (H247). In Fig. 22.1 (R230), Fig. 22.2 (R232), Rashed draws the *latus rectum* as segment FA.

The following notes to Rashed’s figures will clarify his translation and commentary. Figure 3.1 (R152), Fig. 4.1 (R156): the hyperbola does not in general pass through O. Figure 4.2 (R158): T is incorrectly located on axis NO, and Q should be placed inside the ellipse. Figure 7.1 (R174): Q should be placed outside the ellipse. In Fig. 12.1 (R190): points O and S should be added to the figure; the auxiliary hyperbola $\mathcal{H}_1$ is similar and similarly situated to the hyperbola conjugate to $\mathcal{H}$, so the hyperbolas $\mathcal{H}_1$ and $\mathcal{H}$ cannot be tangent at A (Rashed states that NH is the transverse axis of the auxiliary hyperbola $\mathcal{H}_1$ (R66), and that $\mathcal{H}_1$ is similar to the original hyperbola $\mathcal{H}$ (R67f9, R78). As a matter of fact, NH is the transverse axis of $\mathcal{H}_1$ only if B is on the axis of $\mathcal{H}$; and $\mathcal{H}_1$ is similar to $\mathcal{H}$ only if $\mathcal{H}$ is orthogonal). In Fig. 12.2 (R65), which corresponds to Fig. 12.1 (R 190), the line through S parallel to KA should be drawn through the intersection D of the tangents at A and at B. Rashed was probably misled by the manuscript figure, see H304. In Proposition 13, the manuscript has four figures: one original and one auxiliary figure for the hyperbola and for the ellipse. Rashed’s Fig. 13.1 (R196) is the auxiliary figure for the hyperbola. Figure 13.2 on R200 should be replaced by the auxiliary figure for the ellipse, which appears as Fig. 13.2 on page R69. Rashed used the figures for Proposition 12 as the original figures for Proposition 13 although these figures are not the same in the Manisa manuscript. The original figures for Proposition 12 and the auxiliary figures for Proposition 13 do not correspond to each other; see below. Figure 16 (R210) delete segment BA; Rashed fails to mention that the three figures Fig. 19.1 (R218), Fig. 19.7 (R224), and Fig. 20 (R224) are not in the manuscript; in Fig. 19.6 (R222) delete the small circle, the point B on it, and the radius through H intersecting BE; the conic section in Fig. 29 (R258) is actually an ellipse, not a parabola; in Fig. 30 (R260) and Fig. 31 (R264) the point of contact G of the tangent through H to the hyperbola is incorrectly identified with the point of intersection of the diameter EKO and the hyperbola.

The reader of Rashed's translation and commentary has to keep in mind that Rashed's figures are not always mathematically consistent. In Proposition 31, for example, Rashed's Fig. 31 (R264) displays a hyperbola with eccentricity $\varepsilon \approx 1.1$, considerably flatter than the orthogonal hyperbola ($\varepsilon = \sqrt{2} \approx 1.4$) used by Hogendijk in the corresponding figure (H293). Ibn al-Haytham uses in his proof an auxiliary point C and an auxiliary segment t (in Hogendijk's transcription) such that $EC : CH = 1 : (\varepsilon^2 - 1)$ and $t : W = 1 : 2\varepsilon$, where W is a given segment. In Hogendijk's figure $\varepsilon \approx 1.4$ so $EC : CH \approx 1 : 1$ and $t : W \approx 1 : 2.8$. Mathematical consistency would require $EU : UH \approx 5 : 1$ and $T : F \approx 1 : 2.2$ in Rashed's figures, using Rashed's system of transcription. However, in Rashed's Fig. 31, also $EU : UH \approx 1 : 1$ and $T : F \approx 1 : 2.8$. The figure for Proposition 31 is missing in the Arabic manuscript.

In Propositions 12–13, the mathematical proof rests on the similarity of $NHKOAP$ in Fig. 12.1 (R190) to $MLVXRJ$ in Fig. 13.1 (R196) and on the similarity of $NKHOAP$ in Fig. 12.2 (R192) to $MVLXRJ$ in Fig. 13.2 (as printed on p. R69). Thus one would expect $NH : HK = ML : LV$ in Figs. 12.1–13.1 and $NK : KH = MV : VL$ in Figs. 12.2–13.2. The figures have been drawn in such a way that in Figs. 12.1–13.1, $NH : HK \approx 1 : 2$ and $ML : LV \approx 10 : 1$, and in Figs. 12.2–13.2, $MV : VL \approx 9 : 1$ and $NK : KH \approx 1 : 3$. These errors can partly be explained as follows. In Fig. 13.1, we have $\angle XRM = \angle JLM \approx 120^\circ$ but the mathematical context requires $\angle XRM = \angle AKN = \angle JLM \approx 60^\circ$. Thus Rashed used the "wrong" part of the circle for the construction of R in Fig. 13.1; we can construct the "correct" point R by intersecting the part of the circle above line ML with the same hyperbola through L. In Fig. 13.2 (R69), the ellipse and circular arc are incorrectly drawn. If these curves are drawn accurately (cf. H385), the part of the ellipse above line LM is inside the circle, and the ellipse and circle intersect in a second point R' under line LM close to M. For the construction of point V in Fig. 13.2 and hence K and A in Fig. 12.2, this point R' should be used rather than Rashed's point R. Using R, we obtain a different tangent $A'D'$ with A' on the arc of the ellipse between A and B. Figures $NKHOAP$ and $MVLXRJ$ appear correctly in Rashed's commentary in Fig. 13.13 (R76), which resembles the figures in H89–91.

Propositions 12 and 13 are closely related to Ibn al-Haytham's *Chapter on the Lemma for the Side of the Heptagon*, which is also discussed in [Rashed 2000a, 386–396, 438–453]. In the "lemma for the side of the heptagon," one considers a square $ABCD$ and a line $BGHE$ which intersects the diagonal AC at G, the side CD at H and the rectilinear extension of AD at E. Solving the lemma means finding $BGHE$ in such a way that the two triangles BGC and DHE have equal area. If the lemma is solved, the regular heptagon can be constructed as is shown in Propositions 17–18 of an Arabic treatise attributed to Archimedes. These propositions were edited in [Hogendijk 1984, 285–290] and again in [Rashed 2000a, 653–654, 686–691] without reference to the earlier edition.

Ibn al-Haytham's *Chapter* begins with a solution of the lemma of Archimedes which will now be summarized in modern notation. Put $AD = x_1$, $AE = y_1$. Ibn al-Haytham shows that the equality of the areas of triangles BGC and DHE implies $y_1^2/(y_1 - x_1)^2 = x_1/(2x_1 - y_1)$. Let s be a line segment of arbitrary length. Ibn al-Haytham shows that x_1 and y_1 can be found by means of a point of intersection of the two parabolas $\mathcal{P}_1 : y^2 = sx$ and $\mathcal{P}_2 : (y - x)^2 = s(2x - y)$. This construction produces the square $ABCD$ and the line $BGHE$ at the same time. If one starts with a *given* square $A'B'C'D'$, the construction will produce an auxiliary figure $ABCDEGH$ (compare Proposition 13 above) similar to the desired figure $A'B'C'D'E'G'H'$. In the *Chapter*, Ibn al-Haytham provides an analysis of his construction but not a synthesis. In his analysis, Ibn al-Haytham does not assume the size of the square AD to be known in advance.

The figures in the German translation [Schoy 1927, 84] and in the Arabic edition and French paraphrase [Rashed 1979, 382, 315] are unclear because the parabola $\mathcal{P}_2$ is not drawn or appears as a straight line. In his 1979 French paraphrase, Rashed incorrectly assumed that the size of AD must be known. He then stated: "L'examen attentif de cette analyse d'I.H. montre qu'elle ne mène pas à la solution du problème d'Archimède [i.e., the lemma]. Sans doute est-ce en raison de cette difficulté qu'I.H. n'a jamais repris la synthèse de sa propre analyse" [Rashed 1979, 314].

Ibn al-Haytham's construction was explained in [Hogendijk 1984, 226–231], with two parabolas in the figure, and with a detailed synthesis in order to show that the construction is correct. The new paraphrase of Ibn al-Haytham's works on the heptagon [Rashed 2000a, 386–419] is essentially a reprint of the earlier paraphrase

[Rashed 1979, 311–340], but the three pages [Rashed 1979, 314–316] about the "difficulté" in Ibn al-Haytham's construction were replaced by a new passage [Rashed 2000a, 390–391] with a sketch of the synthesis, without reference to [Hogendijk 1984]. The two parabolas occur in the figures in [Rashed 2000a, 389, 440, 441] but $\mathcal{P}_2$ incorrectly appears to be tangent at $\mathcal{P}_1$ at (0,0); note that Fig. 57 in [Rashed 2000a, 390] resembles part of Fig. 23 in [Hogendijk 1984, 230].

In [Rashed 2000a, 391 note 68] the following is said about the previous edition [Rashed 1979] (italics mine): "On nous a reproché d'avoir affirmé que l'analyse d'Ibn al-Haytham était 'erronée'; nous n'avons jamais rien écrit de tel, mais seulement que 'l'analyse ne mène pas à la solution du problème *tel qu'il était posé* ,' c'est-à-dire avec un segment AD donné; voir "La construction de l'heptagone" [i.e., [Rashed 1979]], p. 314. Peut-être faut-il d'ailleurs y voir la raison pour laquelle Ibn al-Haytham n'a pas jugé utile de rédiger une synthèse, *laquelle ne présente du reste aucune difficulté*. D'autres critiques qui nous ont été faites, à propos de la suite de notre édition et de notre commentaire, n'ont pas davantage de portée que la précedente. Nous ne nous y arrêterons donc pas." In the new edition [Rashed 2000a, 438–489] of Ibn al-Haytham's two treatises on the heptagon, Rashed adopted, without noticing this, more than half of the corrections to his earlier edition [Rashed 1979] which had been listed in [Hogendijk 1984, 327–330]. Rashed did not make the following crucial correction: In "Si donc la droite AD est de position connue" [Rashed 2000a, 444:3, 445:3], "AD" has to be corrected to the manuscript reading "LD"; see Rashed's critical apparatus and [Hogendijk 1984, 228].

In [Rashed 2000a], a single reference is made to [Hogendijk 1984] in connection with Ibn al-Haytham's work on the heptagon. In the second part of the *Chapter on the Lemma for the Side of the Heptagon*, Ibn al-Haytham presents another construction of the regular heptagon by means of a parabola and a hyperbola. Rashed says: "D'autre part, on a cru voir dans la deuxième partie de ce traité qu'Ibn al-Haytham procède par 'analyse aussi bien que par synthèse,' et que 'l'analyse a été rendu d'une manière confuse.'[67] La simple étude du texte montre cependant qu'il ne contient pas d'analyse et qu'Ibn al-Haytham procède seulement par synthèse" [Rashed 2000a, 391]. Footnote 67 is a reference to [Hogendijk 1984, 234]. This should be compared to [Hogendijk 1984, 234]: "In the *Chapter*, there is only a synthesis of IH2. In the *Treatise*, there is an analysis as well as a synthesis but the analysis is rendered in a confused manner." IH2 and *Treatise* are Hogendijk's abbreviations of the construction and of Ibn al-Haytham's *Treatise on the Construction of the Heptagon in the circle*, another work on the heptagon, also edited in [Rashed 2000a, 454–489]; the confusion between analysis and synthesis is mentioned in [Rashed 2000a, 484 note 12].

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REFERENCES

Berggren, J. L. 1996. Al-Kūhī's "Filling a lacuna in Book II of Archimedes" in the version of Naṣīr al-Dīn al-Ṭūsī. *Centaurus* **38**, 140–207.

Heath, T. L. 1896. *Apollonius of Perga, Treatise on Conic Sections*, edited in modern notation. Cambridge, UK: Cambridge Univ. Press.

Heiberg, J. L. 1891–1893. *Apollonii Pergaei quae Graece exstant com commentariis antiquis*. Leipzig: Teubner.

Hermelink, H., and Kennedy, E. S. 1962. Transcriptions of Arabic letters in geometrical figures. *Journal of the American Oriental Society* **82** (1962), 204, and also in *Sudhoffs Archiv* **45** (1969), 85; reprinted in E. S. Kennedy *et al., Studies in the Islamic Exact Sciences*, Beirut, 1983, 745.

H: see Hogendijk, J. P. 1985.

Hogendijk, J. P. 1984. Greek and Arabic constructions of the regular heptagon. *Arch. Hist. Exact Sci.* **35**, 197–330.

——— 1985. *Ibn al-Haytham's Completion of the Conics*, Sources in the History of Mathematics and Physical Sciences, Vol. 7. New York: Springer-Verlag.

——— 2002. The burning mirrors of Diocles: Reflections on the methodology and purpose of the history of pre-modern science. *Early Science and Medicine*, 2002, to appear.

Jaouiche, Kh. Review of [Rashed 1984], *Ann. Sci.* **44** (1987), 308–311.

Lorch, R. 1986. Abū Ja^{c}far al-Khāzin on isoperimetry and the Archimedean tradition, *Z. Geschichte Arabisch-Islamischen Wiss.* **3**, 150–229.

R: see Rashed, R. 2000a.

Rashed, R. 1975. *Ṣinā^{c}at al-jabr li-Dīyūfanṭis, tarjamat Qusṭā ibn Lūqā*, ed. R. Rāshid, al-turāth al-^{c}ilmī al-^{c}arabī, No. 1. Cairo, al-hay'a al-miṣriyya al-^{c}āmma li l-kitāb.

——— 1979. La construction de l'heptagone régulier par Ibn al-Haytham. *J. Hist. Arabic Sci.* **3**, 309–387.

——— 1984. *Diophante: Les arithmétiques*, 2 vols. Paris: Les Belles Lettres.

——— 1996. *Les mathématiques infinitésimales du IXe au XIe siècle. Vol. 1. Fondateurs et commentateurs.* London: Al-Furqan Foundation.

——— 2000a. *Les mathématiques infinitésimales du IXe au XIe siècle. Vol. 3. Ibn al-Haytham, Théorie des Coniques, Constructions Géométriques et Géométrie Pratique.* London: Al-Furqan Foundation.

———2000b. *Les catoptriciens grecs I.* Paris: Les Belles Lettres.

———1927. *Die trigonometrischen Lehren des persischen Astronomen Abu'l-Raiḥān Muḥ. ibn Aḥmad al-Bīrūnī. Nach dem Tode des Verfassers herausgegeben von Julius Ruska und Heinrich Wieleitner.* Hannover.

Schoy, C. 1926. Graeco-Arabische Studien. *Isis* **8**, 21–40.

Sesiano, J. 1975. *The Arabic Text of Books IV to VII of Diophantus' Arithmetica in the Translation of Qusṭā ibn Lūqā*. Providence: Brown University (Ph.D. thesis, available through University Microfilms International, Ann Arbor, Mich, No. DCJ 76-15709).

——— 1982. *Books IV to VII of Diophantus' Arithmetica in the Arabic Translation Attributed to Qusṭā ibn Lūqā*, Sources in the History of Mathematics and Physical Sciences, Vol. 3. New York: Springer-Verlag.

Terzioğlu, N. 1974. *Das achte Buch zu den "Conica" des Apollonius von Perge. Rekonstruiert von Ibn al-Hayṯam, Herausgegeben und eingeleitet von Nazım Terzioğlu.* Istanbul.

Toomer, G. J. 1976. *Diocles on Burning Mirrors*, Sources in the History of Mathematics and Physical Sciences, Vol. 1. New York: Springer-Verlag.

——— 1985. Réponse, *Revue des Questions Scientifiques* **156**, 237–241.

——— 1990. *Apollonius, Conics Books V to VII: The Arabic Translation of the Lost Greek Original in the Version of the Banū Mūsā*, Sources in the History of Mathematics and Physical Sciences, Vol. 9. New York: Springer-Verlag.

Ver Eecke, P. 1922. *Les Coniques d'Apollonius de Perge: Oeuvres traduites pour la première fois du Grec en Français*. Bruges.