Applied Mathematics in Eleventh Century Al-Andalus: Ibn Mu’adh al-Jayyānī and his Computation of Astrological Houses and Aspects

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Introduction

Ancient and medieval astrologers believed that the sun, moon and planets influence events on earth in at least three ways: (1) by their positions in the twelve signs of the zodiac (Aries, Taurus etc.); (2) by their positions in the twelve so-called houses, which will be defined below; (3) by their aspects: i.e., certain geometrical configurations which will be explained in Section 2 of this paper.

Most medieval astrologers defined the twelve houses in the following way. First divide the celestial sphere into four quadrants by the horizon and meridian planes. Then divide each quadrant into three houses. The first house begins directly under the Eastern horizon, and the houses are numbered counter-clockwise, for an observer in the temperate regions of the Northern hemisphere facing South. The result resembles the division of an orange into twelve parts, not necessarily of equal size. The different houses were related to friends, marriage, family, death, enemies, etc.

In the course of history, the astrologers developed various geometrical systems to divide the four quadrants of the celestial sphere into three houses each (Kennedy

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1996, North 1986, part 1). The differences between these systems were important, because a planet could be in the 12th house (of ‘enemies’) according to one system and at the same time in the 11th house (of ‘friends’) according to another system. There has been no agreement among astrologers on the choice of a best system.

This paper is devoted to the system which has often been named after Regiomontanus (1436-1476). North has shown (1986, pp. 35-36) that the system was already used by the 11th-century Andalusi mathematician Abū ʿAbdallāh ibn Muʿādh al-Jayyānī (Sezgin 1974, p. 109; Samsó 1992, pp. 137-144; Rosenfeld and Ihsanoğlu 2003, p. 140). I will call it the ‘system of Ibn Muʿādh’, although it is possible that it was used before him, as we will see in Section 3 of this paper. The system of Ibn Muʿādh is as follows. The horizon and meridian planes divide the celestial equator into four arcs of 90 degrees each. Now divide these arcs into three equal parts of 30 degrees each, and draw great circles through the division points and the North and South points of the horizon. These circles are the boundaries of the houses.

Figure 1 displays the horizon SENW, with the South point S, the East point E, the North point N, the upper half of the meridian SMMN, and the parts EM and E'M' of the celestial equator and the ecliptic between the Eastern horizon and the upper half of the meridian. The quadrant EM of the celestial equator is divided into three arcs of 30° at points B and A, and the great semicircles SBN and SAN intersect the ecliptic at points B* and A*. Then arcs M'B*, B'A* and A'E* define the tenth, eleventh and twelfth house. The three other quadrants are subdivided in a similar way.

Fig. 1.
The astrologer who wanted to use this system had to compute the ecliptical longitudes of so-called ‘cusps of the houses’, i.e., the points of intersection $B', A'$ etc. These longitudes depend on the geographical latitude of the locality and the ecliptical longitude of the ascendent, that is the intersection $E'$ of the ecliptic and the Eastern horizon in Figure 1. The mathematics is not easy, but traces of a method of computation have survived in the Tabulae Jahen, the medieval Latin translation of the instructions to a set of astronomical tables which Ibn Mu'adh compiled for the city of Jaén, where he lived (Samsó 1992, pp. 152-166). The Latin text of the Tabulae Jahen is so obscure that Ibn Mu'adh’s method of computation has been a mystery thus far (North 1986, p. 37; Samsó 1992, p. 158 note 74). In the present paper, the method will be compared with a hitherto unpublished Arabic text by Ibn Mu'adh, entitled On the Projection of Rays. This is the special work on projection of rays to which Ibn Mu'adh refers in his Tabulae Jahen.1 In On the Projection of Rays, Ibn Mu'adh explains the computation of the houses in connection with the computation of the astrological aspects, which is even more interesting from a mathematical point of view. It turns out that Ibn Mu'adh’s solutions are mathematically exact.

The structure of the rest of this paper is as follows. Section 2 is an introduction to the astrological aspects and their history. In Section 3 I explain the details of Ibn Mu'adh’s computations, in his own notations and also in modern formulas. The interested reader can use these formulas to write a computer program for the computations. Section 4 contains the Arabic text with an English translation of the relevant part of Ibn Mu'adh’s On the Projection of Rays. In this part, Ibn Mu'adh explains his method of computation of the houses (according to his own system) and the aspects (according to the equatorial theory) with reference to a geometrical figure. The figure itself is missing in the manuscript but has been reconstructed as Figure 7 below. Other parts from the same Arabic text by Ibn Mu'adh have been discussed in (Kennedy 1994). Section 5 contains the Latin text and an English paraphrase of the chapter of the Tabulae Jahen on the houses and aspects. This chapter includes the mysterious Latin passages which have been studied by North. These passages will be clarified by a comparison with the Arabic text in Section 4.

Ibn Mu'adh’s mathematical work on the houses and aspects is an example where Andalusi mathematicians reached the same level as their colleagues in the Eastern Islamic world. Although Eastern Islamic mathematicians such as al-Birūnī were able to do the same computations, Ibn Mu'adh’s method was probably his own invention, because it is based on a rather complicated preliminary computation in his book On the Unknown Arcs of the Sphere, as we will see below. Ibn
Muṣṭafā’s work shows that mathematical astrology was a field of intense research in Al-Andalus. Ibn Muṣṭafā improved on his predecessor al-Majrūṭ (ca. 1000), as we will see below, and his work was continued in the thirteenth century by Ibn al-Raqqām (Kennedy 1996). There is an intriguing connection between Ibn Muṣṭafā and Regiomontanus. Hitherto the oldest known tables for Ibn Muṣṭafā’s system of houses were the tables by Regiomontanus (North 1986, pp. 27-30). Although Ibn Muṣṭafā’s tables of houses have not been found, it now appears that he was in the possession of a correct method of computation. Tracing the direct or indirect influence of Ibn Muṣṭafā on Regiomontanus could be an interesting subject for future research.

The astrological aspects

In this section we assume for sake of simplicity that the moon and all planets are in the ecliptic.

![Diagram of astrological aspects](image)

In ancient Greek astrology, the seven planets (including the sun and moon) were each believed to cast seven visual rays to the ecliptic (Bouché-Leclercq 1899, pp. 247-251). According to the simplest doctrine, which is used by modern astrologers, these rays hit the ecliptic at points whose ecliptical longitudes are obtained by adding or subtracting 60°, 90°, 120° or 180° from the ecliptical longitude of the planet $P$ in Figure 2. Six of the rays are called ‘right sextile ray’ (subtract 60°), right quartile ray (subtract 90°), right trine ray (subtract 120°), left sextile ray (add 60°), left quartile ray (add 90°), and left trine ray (add 120°). If another planet happens to be near a point where one of the rays hits the ecliptic, the second planet is supposedly ‘looked at’ (Latin: adspectus) by the first planet,
and the planets are said to form an ‘aspect’. For example, if the second planet is sufficiently close to the left sextile ray, the two planets form a sextile aspect; then the second planet also looks with its right sextile ray at the first planet.

Ibn Muʿadh defined the astrological aspects by a method which I call the equatorial system, following (Kennedy and Krikorian 1972). In this system, the arcs of 60°, 90° etc., which define the aspects, have to be measured on the celestial equator rather than the ecliptic. As in Ibn Muʿadh’s system of houses, great circles through the North and South points of the horizon are used to connect points on the ecliptic to points on the equator. To find the rays of a planet at $P^*$ in the ecliptic (Figure 3, compare Figure 1), we draw a semicircle from the North point $N$ through $P^*$ to the south point $S$ of the horizon. This semicircle meets the equator at a point $P$. Then we measure the arcs $PQ = 60^\circ$ etc. on the equator and draw through points $Q$ etc. semicircles $NQS$ etc. to meet the ecliptic at points $Q^*$ etc. The planet will now cast its rays to the points $Q^*$ etc. which we have thus obtained. Figure 3 displays the case where the planet $P^*$ is a little above the Eastern horizon, and $Q^*$ is the point where the right sextile ray hits the ecliptic. The size of the arcs $P^*Q^*$, etc. depends on the ascendent, the position of $P^*$ and the geographical latitude, but the computation of these aspects is even more complex than that of the houses according to Ibn Muʿadh’s system.

![Figure 3](image-url)

**Fig. 3.**

The equatorial system of aspects was known at least two centuries before Ibn Muʿadh, and it seems to have been widespread in the medieval Islamic world. Medieval Islamic astrologers often said that the equatorial theory of aspects is found in Ptolemy’s *Tetrabiblos*, even though this is not the case. An approximate method of computation for the equatorial aspects was given by a number
of Islamic authors, including al-Bīrūnī and the Andalusi mathematician Ibn al-Samh (979-1035) (Kennedy and Krikorian 1972; Viladrich 1986, pp. 69, 147-149; Hogendijk 1988, p. 178, where formula (2.6) should read $P \cdot Q' = A_x(\lambda Q) - A_x(\lambda P)$.) The famous al-Khwārizmī (ca. 830) compiled a crude table for the equatorial aspects for the latitude of Baghdad (33°). The Andalusi astronomer al-Majrūtī (ca. 1000) computed a more complicated table for the latitude of Cordoba (38°30′) in his revision of the astronomical tables of al-Khwārizmī. A mathematical analysis of these tables shows that the underlying method of computation is only a rough approximation to the geometric definition of the aspects (Hogendijk 1988). Below we will see that Ibn Muʿādh computed the equatorial aspects in a mathematically correct way. Ibn Muʿādh then says that approximate computations of the aspects can be used in almost all cases. He presents the same approximate method as Ibn al-Samh and al-Bīrūnī in a lengthy passage which I have not edited and translated in the present paper. Thus Ibn Muʿādh’s exact mathematical method was inspired by (astrological) applications, but it was so sophisticated that it was only rarely used. This situation is not unusual in the history of medieval Islamic mathematics.

Ibn Muʿādh’s lost tables for the houses from the Tabulae Jahen were probably computed according to his exact method. In a table of houses, the only independent variable is the ecliptical longitude of the ascendent, but in a table of aspects, the ecliptical longitude of the planet is a second independent variable. Thus a gigantic amount of computation would be necessary for the compilation of mathematically exact tables for the aspects according to the equatorial system. Ibn Muʿādh says that the mathematically exact computation is rarely necessary, so it is very unlikely that he compiled tables according to his own mathematically exact method. Because personal computers are now generally available, the time is perhaps ripe for a rediscovery of the equatorial aspects by modern astrologers.

Ibn Muʿādh’s computation of aspects and houses

In the following overview of Ibn Muʿādh’s computations, I use the same notation as Ibn Muʿādh in the relevant section in On the Projection of Rays. Ibn Muʿādh’s figure in the manuscript is missing, and my reconstruction appears in Section 4 of this paper as Figure 7, which has been drawn according to medieval conventions. Figures 4, 5 and 6 in the present section have been extracted from Figure 7, but have been drawn according to modern conventions. In my overview
I use a few standard concepts of Ptolemaic spherical astronomy, which are explained in (Pedersen 1974). I then present the entire computation in modern formulas without reference to ancient astronomical concepts. These formulas can be used to write a computer program for the computations.

Figure 4 displays the celestial sphere. Points B, T, D and W are the North, East, South and West points of the horizon. The meridian is DHGB, the equator is THW, the ecliptic is AGZ. Point A is the ascendent or rising point and point Z is the descendent or setting point.

I first discuss Ibn Muʿadh’s computation of the aspects. Ibn Muʿadh assumes that the planet is at point E on the ecliptic, and he explains the computation of the right quartile ray of E. The figure is drawn for the case where point E is above the horizon in the Eastern half of the celestial sphere, but the method of computation is general, and can also be used for the other rays.

In order to find the desired point M where the right quartile ray hits the ecliptic, Ibn Muʿadh draws a great semicircle DEB which meets the celestial equator at point K. He then constructs arc KL=90° on the celestial equator. Then the great semicircle BLD meets the ecliptic at M. Thus the problem is essentially to determine the ecliptical longitude λₘ of M from the ecliptical longitude λₑ of E, the geographical latitude φ, and the ecliptical longitude λₐ of the ascendent.

Ibn Muʿadh’s solution consists of two parts. In the first part (Figure 5), he computes the right ascension αₖ of point K from λₐ, φ and λₑ. In medieval Islamic astronomy, the quantity αₖ is called the “ray ascension” of point E. Then the right ascension αₗ of point L can be found because the difference αₗ − αₖ is
±60°, 90°, 120° depending on the type of ray. In the second part of the solution (Figure 6), he computes $\lambda_M$ from $\lambda_A, \phi$ and $\alpha_L$. Again, $\alpha_L$ is called the “ray ascension” of point $M$.

Ibn Mu¯adh draws two more great semicircles through the celestial North pole $N$, namely circle $NOE$ (Figure 5), which meets the celestial equator at $O$ near $K$, and circle $NQL$ (Figure 6), which meets the ecliptic at $Q$ near $L$.

![Figure 5](image)

The first and second part of the solution are based on the theorem of Menelaus for arcs of great circles on a sphere (Pedersen 1974, pp. 72-75).

In Figure 5 we have, by the theorem of Menelaus,

$$\frac{\sin(OK)}{\sin(KH)} = \frac{\sin(OE)}{\sin(EN)} \frac{\sin(ND)}{\sin(DH)}.$$ 

The arcs on the right-hand side are known: $OE = |\delta_E|$, $EN = 90^o \pm |\delta_E|$, $ND = 180^o - \phi$, $DH = 90^o - \phi$. Here $\delta_E$ is the declination of $E$, which can be found from $\lambda_E$.

Therefore $\sin OK / \sin KH$ is known.

Arc $OH$ can be found from $\lambda_E, \lambda_A$ and $\phi$. Thus we have to solve the following problem: to compute two arcs $OK$ and $KH$ from their sum or difference $OH$ and the ratio $\sin OK / \sin KH$. Ibn Mu¯adh solved this problem in his work *On the Unknown Arcs of the Sphere* (fi majhūlat qis¯ı al-kura). The work is extant, and an edition of the Arabic text with Spanish translation can be found in (Villuendas 1979). The details of the computation of $KH$ will be presented below in modern
formulas. Thus $\alpha_K$ is known. We have $\alpha_L = \alpha_K - 90^\circ$, because we are computing the right quartile ray.

**Fig. 6.**

In Figure 6 we have by the Theorem of Menelaus:

$$\frac{\sin QM}{\sin MG} = \frac{\sin QL}{\sin LN} \cdot \frac{\sin ND}{\sin DG}.$$

The arcs on the right-hand side can be found: $QL$ can be determined from $\alpha_L$, $LN = 90^\circ$, $ND = 180^\circ - \phi$, and $DG$ can be determined from $\lambda_A$. Therefore the ratio $\sin QM/\sin MG$ is known. Arc $QG$ can be found from $\alpha_L$. Hence our problem is to compute two arcs $QM, MG$ from their sum or difference $QG$ and the ratio $\sin QM/\sin MG$ between their sines. For the solution, Ibn Muâdh explicitly refers to his work *On the Unknown Arcs of the Sphere*.

For the astrological houses we only need the second part of the computation. In Figure 6, if $HL = 30^\circ$, point $L$ is the beginning of the ninth house; if $HL = 60^\circ$, point $L$ is the beginning of the eighth house, and so on.

I now present Ibn Muâdh’s computation of the aspects and houses in modern formulas, and in real numbers, for a locality in the temperate regions of the Northern hemisphere. I have added formulas for the preliminary computations which Ibn Muâdh does not explain. He probably used standard tables, which were computed on the basis of the formulas which I have presented or by mathematically equivalent methods.

We assume that the following quantities are known: the obliquity of the ecliptic $\epsilon (\approx 23^\circ 30')$, the geographical latitude $\phi$ with $0^\circ < \phi < 90^\circ - \epsilon$, the ecliptical
longitude $\lambda_A$ of the ascendent $A$, and the ecliptical longitude $\lambda_E$ of the planet $E$, with $0 \leq \lambda_A < 360^\circ$, $0 \leq \lambda_E < 360^\circ$. Ibn Mu'adh ignores the latitude of the planet.

For sake of clarity I have divided the computation into the following three parts: preliminaries, computation of $K$, computation of $M$.

**Preliminaries**

1. Find the declination $\delta_A$ of the ascendent from $\sin \delta_A = \sin \lambda_A \cdot \sin \varepsilon$, $-\varepsilon \leq \delta_A \leq \varepsilon$.
2. Find the right ascension $\alpha_A$ of the ascendent from e.g.,
   \[ \tan \alpha_A = \cos \varepsilon \cdot \tan \lambda_A, \]
   such that $\lambda_A$ and $\alpha_A$ are in the same quadrant. This is to say that $0^\circ \leq \alpha_A < 90^\circ$ if $0^\circ \leq \lambda_A < 90^\circ$, $90^\circ \leq \alpha_A < 180^\circ$ if $90^\circ \leq \lambda_A < 180^\circ$, $180^\circ \leq \alpha_A < 270^\circ$ if $180^\circ \leq \lambda_A < 270^\circ$, $270^\circ \leq \alpha_A < 360^\circ$ if $270^\circ \leq \lambda_A < 360^\circ$.
3. Find the ascensional difference $\Delta_A$ of the ascendent from \[ \sin \Delta = \tan \phi \cdot \tan \delta_A, \]
   $-90^\circ <\Delta < 90^\circ$.
4. Find the oblique ascension $\eta_A$ of the ascendent from $\eta_A = \alpha_A - \Delta$.
5. Find the right ascension $\alpha_G$ of $G$ from $\alpha_G = \eta_A - 90^\circ \mod 360^\circ$.
6. Find the declination $\delta_G$ of $G$ from, for example, $\tan \varepsilon \cdot \sin \alpha_G = \tan \delta_G$, $-\varepsilon \leq \delta_G \leq \varepsilon$.
7. Find the ecliptical longitude $\lambda_G$ of $G$ from $\tan \alpha_G = \cos \varepsilon \cdot \tan \lambda_G$, such that $\lambda_G$ and $\alpha_G$ are in the same quadrant (see above, step 2).
8. Find the altitude $a$ of point $G$ from $a = 90^\circ - \phi + \delta_G$.

**Computation of point $K$ (see Figure 5)**

1. Find the declination $\delta_E$ of point $E$ from $\sin \delta_E = \sin \lambda_E \cdot \sin \varepsilon$, $-\varepsilon \leq \delta \leq \varepsilon$.
2. Find the right ascension $\alpha_E$ of point $E$ from e.g.,
   \[ \tan \alpha_E = \cos \varepsilon \cdot \tan \lambda_E, \]
   such that $\lambda_E$ and $\alpha_E$ are in the same quadrant.
3. Find the quantity $c = \alpha_E - \alpha_G$. Since Ibn Mu'adh reduces all computations to a situation where point $E$ is above the horizon (see the Latin text below), we can assume $-90^\circ < c < 90^\circ$.
4. Determine the quantity $k = -\tan \delta_E \cdot \tan \phi$. Note that $-1 < k < 1$ since $0 < \phi < 90^\circ - \varepsilon$ and $-\varepsilon \leq \delta_E \leq \varepsilon$. 
5. Solve \( x = \pm \arccos k \) from \( \sin x = k \sin(x + c) \) with \(-90^\circ < x < 90^\circ\).

According to Ibn Mu\u0101adh’s book *On the Unknown Arcs of the Sphere* (Villuendas 1979, pp. 11-30, 106-118), \( x \) can be computed directly using

\[
\tan \left( \frac{c}{2} + x \right) = \frac{1 + k}{1 - k} \cdot \tan \frac{c}{2}.
\]

6. Find the desired right ascension of point \( K \) from \( \alpha_K = \alpha_E + x \).

**Computation of point \( M \) (see Figure 6)**

1. Compute the right ascension \( \alpha_L \) of point \( L \) in the following way: For the right sextile ray: \( \alpha_L = \alpha_K - 60^\circ \mod 360^\circ \); for the right quartile ray: \( \alpha_L = \alpha_K - 90^\circ \mod 360^\circ \); for the right trine ray: \( \alpha_L = \alpha_K - 120^\circ \mod 360^\circ \); for the left sextile ray: \( \alpha_L = \alpha_K + 60^\circ \mod 360^\circ \); for the left quartile ray: \( \alpha_L = \alpha_K + 90^\circ \mod 360^\circ \); for the right trine ray: \( \alpha_L = \alpha_K + 120^\circ \mod 360^\circ \). It is sufficient to consider points \( L \) above the horizon.

If we want to compute the houses, we do not use point \( E \) but we put for the ninth house \( \alpha_L = \alpha_G - 30^\circ \mod 360^\circ \); for the eighth house \( \alpha_L = \alpha_G - 60^\circ \mod 360^\circ \); for the eleventh house \( \alpha_L = \alpha_G + 30^\circ \mod 360^\circ \), and for the twelfth house \( \alpha_L = \alpha_G + 60^\circ \mod 360^\circ \).

2. Put \( \alpha_Q = \alpha_L \). Compute the ecliptical longitude \( \lambda_Q \) of \( Q \) from tan \( \alpha_Q = \cos \epsilon \tan \lambda_Q \); \( \lambda_Q \) is in the same quadrant as \( \alpha_Q \).

3. Find the declination \( \delta_Q \) of \( Q \) from, e.g., tan \( \delta_Q = \tan \epsilon \sin \alpha_Q \).

4. Find \( k' = + \sin \phi \cdot \sin \delta_Q / \sin a \).

This is (apart from a possible negative sign) the right hand term in the Theorem of Menelaus. Because Ibn Mu\u0101adh uses the medieval Sine (which is 60 times the modern sine), he has to divide by an extra factor 60.

5. Find \( c' = \lambda_Q - \lambda_G \). We can assume \(-90^\circ < c' < 90^\circ \) because it is sufficient to explain the computation for points \( Q \) above the horizon.

6. Solve \( x' \) from the equation \( \sin(x') = k' \sin(x' + c') \). Ibn Mu\u0101adh finds the solution \( \tan \left( \frac{c'}{2} + x' \right) = \left[ \frac{1 + k'}{1 - k'} \right] \cdot \tan \frac{c'}{2}, -90^\circ < x < 90^\circ \).

7. Find the ecliptical longitude \( \lambda_M \) from \( \lambda_M = \lambda_Q + x' \). Point \( M \) is the cusp of the house or the place where the ray hits the ecliptic. The computation is now complete.
Worked example

The following worked example for the right quartile ray does not correspond to Figure 5, but to a case where the simple theory of rays and the equatorial theory produce very different results. Assume $\varepsilon = 23.5^\circ$, $\phi = 52^\circ$, $\lambda_A = 30^\circ$, $\lambda_E = 300^\circ$. Then

\begin{align*}
\delta_A &= 11.50^\circ, & \alpha_A &= 27.90^\circ, & \Delta &= 15.09^\circ, \\
\eta_A &= 12.81^\circ, & \alpha_G &= 282.81^\circ, & \delta_G &= -22.98^\circ, \\
\lambda_G &= 281.78^\circ, & a &= 15.02^\circ, & \delta_E &= -20.20^\circ, \\
\alpha_E &= 302.19^\circ, & c &= 19.38^\circ, & k &= +0.47, \\
x &= 15.74^\circ, & \alpha_K &= 317.93^\circ, & \alpha_L = \alpha_Q = 227.93^\circ, \\
\lambda_Q &= 230.38^\circ, & \delta_Q &= -17.89^\circ, & c' &= -51.40^\circ, \\
k' &= -0.927 & x' &= 24.66^\circ, & \lambda_M &= 255.04^\circ.
\end{align*}

Note that $\lambda_E - \lambda_M = 44.96^\circ$. In the simple theory of aspects, this difference would be $90^\circ$. Thus the equatorial system of aspects is very different from the simple theory.

Ibn Mu'ādadh notes that the right trine ray is opposite to the left sextile ray, etc., so one only has to compute the three rays which hit the ecliptic at points $L$ above the horizon, and the other rays can be found by adding $180^\circ$. Similarly, once the cusps of the eighth, ninth, eleventh and twelfth house have been computed, the diametrically opposite points are the cusps of the second, third, fifth and sixth house respectively. In a similar vein, if point $E$ is under the horizon, Ibn Mu'ādadh performs the computation for the diametrically opposite point $E'$ above the horizon.

I now return to the history of Ibn Mu'ādadh’s system of houses. North (1986, p. 35, 64) has collected medieval texts in which the system is attributed to Ibn Mu'ādadh, but also an astrolabe plate on which the system is called “the method of al-Ghāfiqī.” As North remarks, the reference is probably to Abu al-Qasim Ahmad ibn 'Abdallāh ibn 'Umar al-Ghāfiqī, also called Ibn al-Ṣaffār, who died in 1035 (Sezgin 1974, p. 356). Since Ibn Mu'ādadh died in 1093 (Samsó 1992, p. 138), it is possible that the ‘system of Ibn Mu'ādadh’ was already known to Ibn al-Ṣaffār, if not before.

In the beginning of the passage translated in Section 4, Ibn Mu'ādadh quotes a lost work of the Arabic philosopher al-Kinār (ninth century) on the relationship between the houses and the aspects. According to Ibn Mu'ādadh, al-Kinār said that
if a planet is at the ascendent, it casts its sextile and trine rays to the cusps of the eleventh and third house, its quartile rays to the points of lower and upper culmination, and its trine rays to the cusps of the ninth and the fifth house. Thus al-Kindī probably did not accept the simple theory of aspects, which prescribes that the distance between a planet and its quartile ray is 90°, measured along the ecliptic. The above-mentioned worked example shows that the distance between ascendent and upper or lower culmination is in general not equal to 90 degrees. If al-Kindī believed in the equatorial theory of aspects, his system of houses must have been the same as the one of Ibn Muṭāridī. It is conceivable that al-Kindī believed in a slightly different system of houses which the Arabic astrologers attributed to Hermes, and which most Latin sources attribute to Campanus (North 1986). In this system, the prime vertical is divided into equal arcs of 30°, and the great circles through the North and South points of the horizon and these division points are the boundaries of the houses. If this was al-Kindī’s system of houses, he must have used a theory of aspects in which the defining arcs of 60°, 90° etc. were located in the prime vertical. In (Woepcke 1858 pp. 29-32, 37) we find an astrolabe plate for the aspects according to this system³ for the latitudes 38°30’ and 42°, so the system of Hermes for aspects must have been used at some point in history. Note that al-Birūnī (973-1048) also claims to be the inventor of the system (Samsó p. 589). In conclusion, we do not know in which system of aspects al-Kindī believed, and we cannot be sure that Ibn Muṭāridī was the person who first developed ‘his’ system of houses. But we do know that Ibn Muṭāridī was the author of a mathematically correct computation of the houses according to ‘his’ system.

Arabic text and translation of part of the Projection of Rays of Ibn Muṭāridī

The following Arabic text and translation are based on Ms. Florence, Bibliotheca Medicae-Laurenziana Or. 152, ff. 77b:8 - 78b:22 (Sabra 1977, pp. 276-283). According to the colophon, this manuscript was completed in the middle decade of March in the year 1303 of the Spanish Era (A.D. 1265), in the city of Toledo. The manuscript was probably written in the court of king Alfonso X (1252-1284) (Samsó p. 252), more than two centuries after the city of Toledo had been conquered by the Christians. The identity of the scribe is unknown. The manuscript also contains a copy of Ibn Muṭāridī’s above-mentioned work On the Unknown
Arches of the Sphere.

Several mistakes in the Arabic manuscript of Ibn Mu‘adh’s treatise were corrected in the margin of the manuscript. In my edited version, these marginal corrections appear in pointed brackets < >. I have not indicated the incorrect words in the main text which were replaced by these corrections. In order to preserve some of the flavour of the original, I have not corrected all grammatical and stylistic inconsistencies. My own explanatory additions to the translation appear in parentheses.

Arabic text

ومنا يفْعَلُ رَأْيًا ومَذْهِبًا في الْجُمُعَ بين تَسْوِيَة الْبَيْوِت ومَطْرَح الشُّعُاع أنَّ الْكَنْدَيَ الْكَيْمِيَ الْمَلَائِمَة في صُنُفُهذَه العُلُم حَتَّى لا يِجَلِهُ إِنَّهُ يَقُولُ في بعَض تَوَقَّعُه في الشُّعُاعات إِنَّ الْكَوْكِبِ إِذَا كَانَ عِلِّيًّا لَّيْكَة الأَقْفُ في حَزْوَة الطَّالِع فإنَّ درَجَة الْبَيْتِ الثَّنَانِي عَشْرَة ودِرَجَة الْبَيْتِ الثَّالثَة هَمَا بِشُتَّاه إِنَّ درَجَة وَسَط السَّماء ووَتَد الأرض هَمَا يَرْتَعَانَهُ إِنَّ دِرَجَة الْبَيْتِ التَّاسِعَة ودِرَجَة الْبَيْتِ الْخَانَسِ هَمَا بِثِّيَانِهِ وَهَذَا
لَا يِتَبَّأُ في المَذْهِب الذي نَسَبَ إِلَى بِطْلِيْمِس بُوْجُ وَقَد بِيَنَا فَضِلَ ذلك < المَذْهِب >< إِخْرَاح إِلَى تَلَكِ الأَقْسَام بِدُواْرَة مِنَ الطَّفْقَانِ الْمَشْتَكِيَنِ لَدِرَجَة الأَقْف لَدِرَجَة نُصَ النَّهَارَ. وَقَلِبَ الْدُوْارَاتَ تَلَقَّى فَلَكَ الْبِرْحُ إِلَى بِيوُتِهِ الأَلْثَي عَشْرَة وَمَا وَاقِفْ مِن الْكَوْكَابِ عِلَى دِرَجَة مِنَ تلكَ الدُوْارَاتِ فَإِنَّ تُلِيْثِهِ وَتَرْيِمُهُ وَتَسْدِيْسُهُ لِكُلٍّ إِلَيْهِ أَرْبَعَة أَبَيَاتِ وَثَلَاثَاتِ وَثَالِثِينَ.
وَلَا مَا يَكُن عَلَى حَقِيقَة دِرَجَة مِنَ تلكَ الدُوْارَاتِ فَإِنَّ تَرْيِمُهُ وَتَسْدِيْسُهُ وَتَلِيْثُهُ لِكُلٍّ بَعْلِ بَلْ اِخْرَحُ مِنَ الطَّفْقَانِ الْمَشْتَكِيَنِ الْمَذْكُورِيَنَّ قَوْنًا مِنْ مَعْوِضَة الْكَوْكِبِ فَمَا قَطَعَت مِن مَعْدَل الْتَلِيّ مُرْيِ عَلِيًّا لِلْقَدِيْسَ سِتِّينَ دِرَجَة مِن مَعْدَل الْتَلِيّ ثُمَّ أَخْرِجَ مِن حِيْثَ اَتْبَتَ السِّتُونَ دِرَجَة فُيُوَّتُ فِي الْتَلِيّ مُشْتَكِيَنَّ كَمَا قَطَعَت مِنْهَا الْقُوْسُ مِنْهَا فَلَكَ الْبِرْحُ فَهُوَ مَوْضُوِعُ تَسْدِيْسِهِ الأَسْمُ وَكَذَٰلِكَ بِمَعْرُوف تَرْيِمُهُ الأَسْمُ وَتَلِيْثُهُ الأَسْمُ وَيُوْقِعُ تَسْدِيْسِهِ الأَسْمُ مَقَالَ بَيْنِهِ الأَسْمِ وَالْتَرْيِمٍ مَتَابِيَانَ مَقَالَ وَتَلِيْثِهِ الأَسْمِ مَقَالَةْ
تَسْدِيْسِهِ الأَسْم وَإِنَّ شَتَّى أَنَّهُ تَقَدِّمَ لأَكْمَل وَتَصَلُّ مَثَل ذَلِكَ.
وريد أن نذكر كيف العمل في استخراج موضع الشماع. فنطلق ذلك الأفق عليه
أب ونوقاع فيه نصف تلك الريح كيف ما اتقن في جهن الطالب عليه أجز الشماع
منه في وسط السماء والألف الطالب وازاوي الغار.
ووضع أن كوكبا على نقطة هاء ونريد أن نعمل مطرح الشماع. فنستخرج معدل
النبر وهو قوس وخطপ تمّ تمّ بالكوكب قوسا خرجها من نقطة ب وهو قوس
بكره. تطفل معدل النبر على الله ونريد موضع ترابك < الكوكب.
فجعل من معدل النبر ض جزء من نقطة الله وهو قوس دليل تم خرج من نقطة
ب دлина تمّ على نقطة اللام وهو قوس بلغ تطفل تلك الريح على الله فإذا
أردنا أن نعلم نقطة ميم هو من تلك الريح فنولأ نستخرج نقطة
ثاني درجة هي من دائرة معدل الريح تلك الدربة هي مطالع الكوكب الشماعية
واستخراجها على ما أصف.
نعمل طلب معدل النبر نقطة ن خرج منها قوسا إلى نقطة الهاء موضع
الكوكب وهي قوس الله. ومن أجل أن نقطة هاء معلومة تكون نقطة ع معلومة
لأنها مطالعها في الفلك المستخدم متقطعة موضع < العلة
> قوسه الهاء عين معلومة لأنه تبت في نقطة هاء. ويكون نسبة جب قوس جم إلى
جب قوس جم هو مطلقة من نسبة جب قوس دح ومن نسبة جب
قوس هاء عين إلى جب قوس دح. فإذا شرنا جيب قوس تد وهو جيب
عرس البلد في جيب قوس هو قوس ونفسنا الحاصل على جيب ذو جيب
قوس دح في جيب قوس هو جزء الخارجة النسبة الموجودة من جيب قوس جم إلى
جب قوس جم. وكنا التكوين مهونا إلا أن نفصل إما هذه أو الأخرى معلوم
وهو جيب دح. فنعمل حينئذ قوس جم وهو مطالع جزء الكوكب في الدائرة التي هو
عليها فإذا علمنا ذلك علمنا نقطة < من دائرة معدل الريح.
فإذا أردنا أن نعلم بها نقطة < من تلك الريح فإننا نستخرجها على ما أصف.
وبهذا العمل الذي نستتألف نستخرج مراكز الألوان الثلاثة عشر على ما نقدم

*ة: the ms. has زحة.
Translation of the Arabic text

Among the things which strengthen our view and our doctrine in combining the equalization (i.e. determination) of the houses and the projections of the rays is the following: Al-Kindi, who was among the brilliant people in the branches of

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\[\text{among the brilliant people in the branches of}\]

\[\text{Al-Kindi, who was among the brilliant people in the branches of}\]
these sciences, in such a way that he was not ignorant (of anything), said in one of his writings on the rays: If the star (i.e. any celestial body) is on the horizon at the degree of the ascendent, the degree of the eleventh house and the degree of the third house are the sextiles (i.e. sextile rays) of it, and the degree of midheaven and lower culmination are the quartiles (i.e. quartile rays) of it, and the degree of the ninth house and the degree of the fifth house are the trines (i.e. trine rays) of it. This does not occur in any way in the doctrine attributed to Ptolemy, and we have demonstrated the absurdity of that doctrine. But this occurs if the equator is divided into equal parts, as we have mentioned, and if circles are drawn to these division points from the two common points of the horizon and the meridian. So those circles divide the ecliptic into its twelve houses. If any star is on any of these circles, its trines, quartiles and sextiles are at a distance of four, three and two houses.

If (a star) is not exactly on one of those circles, its quartiles, sextiles and trines are known as follows: We draw from the two above-mentioned common points an arc passing through the position of the star. To its point of intersection with the equator, one adds for the sextile sixty equatorial degrees. Then one draws from where the (arc of) sixty degrees end an arc to the two common points (of the horizon and the meridian). The point of intersection of that arc with the ecliptic is the position of the left sextile. We proceed in the same way for the left quartile and the left trine. The right sextile is in opposition with the left trine, and the two quartiles are in opposition, and the right trine is in opposition with the left sextile. If you wish to subtract (60°, 90° or 120°) for the right (rays), you can also proceed in that way.

We want to explain the procedure for finding the position of the rays. We draw the horizon $ABD$, and we let fall in it half of the ecliptic, for an arbitrary position, in the domain of the ascendent. It is $AGZ$, point $G$ is on the meridian, $A$ is the ascendent, and $Z$ is the descendent. We assume that our star is at point $E$, and we wish to construct the projection of the rays. We draw the equator, namely arc $WHT$. Then we let pass through the star an arc which we draw through points $B$ and $D$, namely arc $BKED$, which intersects the equator at $K$. We want the position of the (right) quartile (ray) of the star.

Thus we find on the equator (an arc of) 90 degrees from point $K$, namely arc $KHL$. Then we draw from points $B, D$ an arc passing through point $L$, namely arc $BLD$, which intersects the ecliptic at $M$. If we want to know the ecliptical degree of point $M$, we first determine the equatorial degree of point $K$. This degree is the ray-ascension of the star. It is determined as I will now describe.
We make the pole of the equator point $N$, and we draw from it an arc to point $E$, the position of the star, namely arc $NOE$. Since point $E$ is known, point $O$ is known, because it is its right ascension. Point $H$ is known and arc $EO$ is known since it is the declination of point $E$. The ratio of the sine of arc $KO$ to the sine of arc $KH$ is composed of the ratio of the sine of arc $ND$ to the sine of arc $DH$ and the ratio of the sine of arc $EO$ to the sine of arc $NE$. If we multiply the sine of arc $ND$, that is the sine of the (geographical) latitude of the locality, with the sine of arc $EO$, and divide the product by the product of the sine of arc $DH$ and the sine of arc $NE$, the result is the ratio between the sine of arc $KO$ and the sine of arc $KH$, which (ratio) has now been found. Each of these arcs is unknown, but the excess of one of them over the other is known, that is arc $HO$. Then we know arc $KH$, and this is the ascension of the (ecliptical) degree of the star in the circle on which it is located. If we know this, we know point $L$ of the equator.

If we want to know (i.e. determine) from it (i.e. point $L$) point $M$ of the ecliptic, we determine it as I now describe. By this procedure, which we begin anew, we determine the cusps of the twelve houses according to our doctrine, which we have mentioned above, because we make arc $LH$ thirty degrees if we want the ninth house, and sixty degrees if we want the eighth house, and similarly for the
eastern side (of the celestial sphere). That is to say, we draw from the pole $N$ an arc to the known point $L$, namely arc $NQL$. Then point $Q$ on the ecliptic is known, because the ecliptical arc $GQ$ has right ascension $HL$. But $LQ$ is known because it is the declination of the degree of $Q$. The ratio of the sine of arc $MQ$ to the sine of arc $MG$ is composed of the ratio of the sine of arc $ND$ to the sine of arc $DG$ and the ratio of the sine of arc $LQ$ to the sine of arc $NL$. All these arcs are known except the two arcs $MQ$ and $QG$. They are unknown, but the excess of the one over the other is known, and the ratio between them is known. Therefore they can be known as we have explained in our book On the Extraction of the Unknown Arcs.

Among the things which this procedure verifies is the fact that the two quartiles (i.e. the quartile rays) are diametrically opposite in it (i.e. in this system), and similarly the left trine and the right sextile, and similarly the left sextile and the right trine. The same is true in the doctrine of Hermes.\textsuperscript{7}

Simplification of the procedure for finding the (ray) ascension of any degree you wish: You find the ascendant, and the (ecliptical) degree of midheaven, and the right ascension of the degree of midheaven, and the right ascension of the position of the star. You take the “distance” of the star from midheaven as follows: you subtract the right ascension of its position from the right ascension of the degree of midheaven, if the position of the star is in the Western quadrant. If its position is in the Eastern quadrant, you subtract the (right) ascension of the degree of midheaven from the (right) ascension of the position of the star. The remainder is the distance, God willing.

Concerning the projection of rays in the most accurate and correct way: what we have mentioned (above) is in the way which is necessary for someone who wants to determine it exactly, for a special anniversary or a matter which deserves an exact investigation. As for its simplification...\textsuperscript{8}

\textit{The Tabulae Jahen on houses and projections of rays}

The Arabic original of most of the Tabulae Jahen is lost, including the passage dealing with the astrological houses. I have paraphrased the passage in the medieval Latin translation because it is difficult to translate literally.

Samsó and Mielgo have recently published the Arabic original of the chapter of the Tabulae Jahen on the qibla, which has come down to us in the \textit{Zīj} of Ibn Ishāq (Samsó and Mielgo 1994). It turns out that the medieval translator of the Tabulae
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Jahren translated the Arabic word \textit{bucd} (distance) as “longitudo” (meaning: ecliptical longitude). I have therefore translated “longitudo” as distance. This distance is almost always the distance of a point from the meridian, measured along the celestial equator or ecliptic. Once this is understood, the obscure Latin passage on the astrological houses becomes easier to understand.

The chapters of the \textit{Tabulae Jahen} translated here do not contain proofs and there is no figure. For sake of clarity I have added references to Figures 4-7 above and explanatory remarks in parentheses. Angular brackets contain words which I have restored to the text for mathematical sense. I have put in square brackets all words which I have omitted in order to restore the mathematical sense.

\textit{Latin text}

Qualiter autem extrahantur proiectiones radiorum secundum rem veriorem in eo quod credimus, de hoc est ut extrahamus in primo\textit{[s]} ascensiones stellae radiales secundum quod narro.


Si ergo fuerit declinatio loci cum quo operaris (septentrionalis), tunc extrahe arcum \textit{[septentrionalis,]}\textsuperscript{9} longitudinis ascensionum radialis, sicut extrahis ex proportione nota eveniente inter duos sinus duorum arcuum separatorum ignotorum, quorum aggregatio est nota cum aggregamus duos arcus separatos, secundum quod ostendimus illud in libro nostro de extractione ignotorum arcuum Sphaerae ex notis eius. Arcus enim longitudinis ascensionum radialis, cum aggregantur ipse, et arcus sequens eum in proportione sunt simul arcus longitudinis...
notae.

Et si fuerit declinatio loci cum quo operaris meridana, tunc extrahe arcum longitudinis ascensionum radialium, sicut extrahis ex proportione nota accidente inter duos sinus duorum arcurum compositorum ignorum quorum unius super altius est nota, duos arcos compositos, secundum quod declaravimus illud illic. Quoniam arcus longitudinis ascensionum radialium, addit super arcum sequentem eum in proportione per aequalitatem arcus longitudinis notae.

Quicunque ergo fuerit arcus longitudinis ascensionum radialium, adde cum super ascensiones medi coeli, si fuerit pars cum qua operaris in quarta orientali. Et minue de ascensionibus medi coeli, si fuerit in quarta occidentali, et quod fuerit ex hoc, erunt ascensiones radiales. Cum ergo egrediuntur ascensiones radiales, tunc adde super eas sextilitatem sinistram sexaginta graduum, et quadraturam sinistram nonaginta et tripliciitatem sinistram, centum et viginti graduum, et minue radii dextrari, quantum pro sinistro addidisti, et melius in hoc est, ut scias ascensiones radiales. Tunc si fuerint in quarta orientali, operare per quadraturam dextram, et non opereris per sinistram. Et si fuerint in occidentali, tunc operare per quadraturam sinistram. Et iterum si fuerit arcus longitudinis ascensionum radialium minor triginta ad partem orientis, aut ad partem occidentis, tunc operare sextilitatem sinistram, et sextilitatem dextram simul, et ne opereris aliquam tripliciitatem. Et si fuerit arcus longitudinis ascensionum radialium plus triginta ad orientem, tunc fac sextilitatem, et tripliciitatem dextram, et non facias sextilitatem, et tripliciitatem sinistram, et si fuerit plus triginta ad partem occidentis, tunc fac sextilitatem et tripliciitatem sinistram, et facies eas secundum quod narro, et per similitudinem illius aequabis domos secundum modum quem aestimo convenientem rectitudini, et est illud quod promisi me dicturum in Capitolo projectionum radiorum.

Rememoratio aequationis domorum duodecim.


Quascunque ergo harum ascensionum habueris, aut ascensiones radiales post additionem laterum sextilitatis aut quadraturae, aut tripliciitatis, et diminutionem earum. Tunc fac cum eis totum, quod narro tibi, et scias quod unaqueaeque earum
est super terram. Et propter illud acceimus has domos quae sunt supra terram, et acceimus radios qui cadunt supra terram, et convertimus illas ascensiones in ascensionibus orbis recti ad gradus aequales, et quod fuit scivimus quod fuit inter illud et inter partem medii coeli, ita quod minuimus dextrum ambo eum, et propinquius ipsorum ad orizontem occidentis de sinistro ambo eum, et est longinquus ipsorum ab orizonte occidentis, et quod est, est longitudo a medio coeli.

Deinde accipe declinationem illius partis, ad quam convertimus ascensiones, et accipe sinum eius, et est sinus declinationis. Et scias partem eius, et scias altitudinem partis decimae in medio coeli, et accipe sinus illius altitudinis, et multiplica ipsum in sexaginta, et divide aggregatum per illud quod egreditur de multiplicatione sinus declinationis partis in sinus latitudinis regionis, tunc quod egreditur est proportio sinus longituninis gradus, ad sinus arcus qui est inter longituninem gradus quaesiti, et inter longituninem servatam a medio coeli. Si ergo fuerit declinatio partis multiplicata prius septentrionalis, tunc extrae sinus longituninis gradus quaesiti, sicut extrahis ex proportione nota eveniente inter duos sinus duorum arcuum ignotiorum compositorum, quorum unus addit super alterum, cum arcu noti duos arcus compositos. Quoniam arcus longituninis gradus quaesiti, tunc addit super arcum sequentem eum in proportione, cum arcu noti qui est longituveh servata a medio coeli. Et si fuerit declinatio partis multiplicata meridiana, tunc extrae sinus longituninis gradus quaesiti, sicut extrahis ex proportione nota accidente inter duos sinus duorum arcuum ignotiorum separatorum, quorum aggregatio est nota cu aggregat duos arcus separatos. Quoniam arcus longituninis gradus quaesiti, tunc cum aggregatur ipse et arcus sequens eum in proportione, sunt simul arcus [quaesitus] notus, qui est longituveh servata a medio coeli. Cum ergo scieris longituninem arcus noti a medio coeli, tunc adde eam super partem medii coeli, si fuerint in quarta orientali ascensiones quas convertisti, et si fuerint in quarta occidentali, tunc minue eam de medio coeli. Quod ergo est post illud totum, est gradus quaesitus, ad quem operatus es proieotionem quidem radiorem, si es operatus ad eam, aut initium alciuSex sex dominorum, quae sunt supra terram. Cum ergo feceris domos sex, quae sunt supra terram, tunc illae sex quae sunt sub terra, sunt earum nadir in oppositione earum. Et similiter sextiltas sinistra cum triplicatione dextra sunt oppositae, et duae quadraturea sunt oppositae, et similiter sextiltas dextra cum triplicatione sinistra sunt oppositae, et radii omnium duorum partium oppositorum in orbe, quos operaris (sic) sunt convenientes radii partis alterius conveniencia contraria, et quadratura cuiusque ambarum sinistra est quadratura alterius dextra, et sextiltas cuiusque duarum sinistra[, est triplicitas alterius dextra, et triplicitas cuiusque duarum sinistra est sextiltas alterius dext-

**Paraphrase of part of Chapters 37 and 38 of the Tabulae Jahen**

As to the derivation of the projections of rays in a more correct way, according to what we believe, we first derive the *radial ascension* of the star (i.e., sun, moon or planet) as I will explain. If the star is above the horizon, we work with its position, and if it is under the horizon, we work with the point diametrically opposite to it (thus we always work with point $E$ above the horizon in Figure 7). Whichever of the two (positions) is above the horizon, you take its right ascension ($O$), and also the right ascension of the tenth house ($H$). We subtract the right one from the left one, and the remainder is the arc of the distance ($HO$). Take its sine, it is the sine of the distance ($\sin HO$). Then take the declination of the position of the star ($EO$), or the declination of the point diametrically opposed to it, whichever you work with. And you know the sine of it ($\sin EO$), and its degree (i.e. the magnitude of arc $EO$); then it is a sine of the degree of the declination. Then multiply the cosine of the degree (i.e., $\cos EO = \sin NE$) by the cosine of the latitude of the locality ($\cos NB = \sin HD$), and divide the product by the product of the sine of the declination of the degree ($\sin EO$) and the cosine of the latitude of the locality ($\sin DN$). The quotient is the ratio between the sine of the distance of the radial ascension ($\sin HK$) and the sine of the difference ($\sin KO$) between the radial ascension ($KH$) and the arc of the distance ($HO$). (We have $\frac{\sin HK}{\sin KO} = \frac{\sin HD}{\sin DN} \cdot \frac{\sin NE}{\sin EO}$ by the theorem of Menelaus.) Keep this ratio in mind.

If the declination of the position with which you work is northern, then extract the arc of the distance of the radial ascension ($KH$), just as you extract two separate unknown arcs, whose sum is known, from the known ratio between their sines, as we have shown in our book *On the Extraction of the Unknown Arcs of the Sphere from Known Arcs*. For the arc of the distance of the radial ascension ($KH$) plus the arc following it in the ratio ($KO$) are equal to an arc of known length ($HO$).

And if the declination of the position with which you work is southern (as in Figure 7), then extract the arc of the distance of the radial ascension ($KH$), just as you extract two combined arcs, whose difference is known, from the known
ratio between their sines, as we explained there. For the arc of the distance of the radial ascension ($KH$) exceeds the arc which follows it in the ratio ($KO$), by (a magnitude) equal to an arc of known length ($HO$).

Add the arc of the distance of the radial ascension ($KH$) to the ascension of midheaven, if the degree with which you work ($E$) is in the Eastern quadrant. Subtract it from the ascension of midheaven if it is in the Western quadrant. The result is the radial ascension. Since the radial ascension has been determined, add to it $60'$ for the left sextile (ray), $90'$ for the left quartile, $120'$ for the left trine, and subtract for the right ray what you have added for the (corresponding) left ray. But a better way in this is to look at the radial ascension. Then if it is in the Eastern quadrant (i.e. point $K$), work with the right quartile, not with the left. If it is in the Western quadrant (i.e. point $L$), work with the left quartile. Again, if the arc of the distance of the radial ascension ($KH$) is less than $30'$ on the Eastern side or on the Western side, then work with the left sextile and the right sextile, and do not work with any trine ray. And if the arc of the length of the radial ascension ($KH$) is more than $30'$ on the Eastern side (of the meridian), make the right sextile and trine (ray) and do not make the left sextile and trine ray, and if it is more than $30'$ on the Western side, make the left sextile and trine ray. Make them as I explain (below), and in a similar way you equalize the houses in the way which I consider to be in agreement with correctness. This is what I promised to say in the chapter on the projection of rays.

Description of the computation of the twelve houses.

This is as follows. You take the oblique ascension of the ascendent for the (latitude of the) locality, and subtract $30'$ from it, and the remainder is the ascension of the twelfth house. (You subtract $60'$ from the ascension of the ascendent, the remainder is the ascension of the eleventh house.) You subtract $120'$ from the ascension of the ascendent, and the remainder is the ascension of the ninth house. You subtract $150'$ from the ascension of the ascendent, and the remainder is the ascension of the eighth house.

Whichever of these ascensions (i.e. point $L$ in Figure 7) you have obtained, (either the ascensions for the houses) or radial ascensions after addition or subtraction of the sides (sc. of the regular polygons) for the sextile, quartile or trine (rays), do with them all I tell you. You know that each of them is above the horizon; this is why we have taken the houses above the horizon and the rays which
fall above the horizon. We take the ascension as a right ascension and we convert it to equal (i.e., ecliptical) degrees. We find the (difference) between the result \(Q\) and the degree of midheaven \(G\), by subtracting the right one, (which is) closer to the Western horizon, from the left one, (which is) farther from the Western horizon. The result is the distance (in longitude) from the meridian.

Then take the declination of this (ecliptical) degree \(Q\) to which we have converted the ascension, and take its sine, this is the sine of the declination \(\sin QL\). You know the degree of it, and the altitude of the (ecliptical) degree of the tenth house, which is midheaven, and you take the sine of this altitude \(\sin DG\), and multiply it by 60 (=\(\sin NL\)), and divide the result by the product of the sine of the declination of the degree \(\sin QL\) and the sine of the latitude of the locality \(\sin DN\). Then the quotient is the ratio between the sine of the \(\langle\) arc of the\(\rangle\) distance of the degree \(\sin MG\) and the sine \(\sin MQ\) of the arc \(MQ\) between the distance of the desired degree \(MG\) and the distance from midheaven which was kept in mind \(QG\). (We have \(\sin GM/\sin MQ = (\sin GD/\sin DN) \cdot (\sin NL/\sin LQ)\) by the theorem of Menelaus.)

If the declination of the degree \(Q\) which (i.e., whose sine) has been multiplied before is northern, then extract the sine of the distance of the desired degree \(\sin MG\) as you extract two combined unknown arcs whose difference is known from the known proportion between their sines. For the arc of the distance of the desired degree \(MG\) exceeds the arc following it in the proportion \(MQ\) by a known arc \(QG\), which is the distance from midheaven, which has been kept in mind.

And if the declination of the degree \(Q\) which (i.e., whose sine) has been multiplied is southern, extract the sine of the distance of the desired degree \(\sin MG\), as you extract two separate unknown arcs whose sum is known from the known proportion between their sines. For the arc of the distance of the desired degree \(MG\) plus the arc following it in the proportion \(MQ\) are equal to the [sought] known arc \(QG\), which is the distance from midheaven which has been kept in mind.

Since you know the distance of a known arc from midheaven, add it to the (ecliptical) degree of midheaven, if the ascension which you converted \(L\) is in the Eastern quadrant, and if it \(L\) is in the Western quadrant, subtract it from (the ecliptical degree of) midheaven. The result of all this is the desired degree \(M\), which is the projection of a ray, if you wanted to construct this, or the beginning (cusp) of any of the six houses above the earth.

When you have made (the computation for) the six houses above the horizon,
the six under the horizon are the corresponding diametrically opposed (points). Similarly, the left sextile ray and the right trine ray are diametrically opposed, and the two quartile rays are diametrically opposed, and the right sextile ray and the left trine ray are opposed in the same way.

The rays of any of two diametrically opposed degrees in the ecliptic, which you constructed (i.e. the rays of a point above the horizon), coincide with the rays of the other point (under the horizon) in a converse way: The left quartile ray of one is the right quartile ray of the other. The left sextile ray of one is the right trine ray of the other, and the left trine ray of one is the right sextile ray of the other.

This is the most correct (method) which we have found for the determination of the rays, best established by all kinds of experience, and most in agreement with the rules of geometrical form.

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NOTES

1. “…et iam complexi illud cum eo quod adiungitur ipsi de eo, quod assimilatur ei ex aquatio
tione domorum in libro singulari in expositione proiectionum radiorum.” (Ibn Muʿādh 1549, cap. 26.)

2. In the *Tetrabiblos*, Ptolemy computes the astrological progressions (Arabic: *tasyrīr*) according to an equatorial method, see (Hogendijk 1988, p. 179).

3. In Woepcke’s Figure 12, the equatorial arcs between the East point and the boundary curves of the twelfth and the eleventh house are not 30° and 60°, as Woepcke seems to suggest, but approximately 36° and 65°. This agrees well with the theoretical values for the system of Hermes for geographical latitude \(\phi = 38°30'\), namely \(\arctan(\tan(30')/\cos(\phi)) = 36.4°\) and \(\arctan(\tan(60')/\cos(\phi)) = 65.7°\).

4. Ibn Muʿādh probably means the celestial hemisphere above the horizon. Earlier in the treatise, Ibn Muʿādh calls the Eastern quadrant above the horizon the “domain of rise, advance, and ascent” (ḥayyiz al-maṣhriq wa l-iqba l wa l-ṣūrūd), and the Western quadrant the “do-

5. The manuscript has *ZHT*. I have emended the text because the descendant Z is in general not the same as the West point \(W\) on the horizon.
6. Points $B$ and $D$ are the North and South points of the horizon.

7. In this system, the prime vertical is divided into arcs of 30 degrees and the boundaries of the houses are the great semicircles through the division points and the North and South points of the horizon. See Kennedy 1994.

8. Ibn Muʿadh continues with a lengthy description of an approximate method for determining the 'ray ascension' of any star, as $(1 - w)$ times its right ascension plus $w$ times its oblique ascension (or descension). The weight $w$ is the “distance” of the star from the meridian, which has just been determined, divided by the difference between the right ascensions of ascendent and midheaven, or midheaven and descendent.

9. This word is displaced in the text.

10. Here a passage was left out by scribal error.

11. This term corresponds to the Arabic $māt ʿādl shuʿāʿīyya$.

12. The text means that the smaller arc is part of the greater arc.

13. Ibn Muʿadh likes to work with points above the horizon.

14. To “equalize the houses” is a technical term for finding the houses.