Applied Mathematics in Eleventh Century Al-Andalus: Ibn Mu^{c}ādh al-Jayyānī and his Computation of Astrological Houses and Aspects

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Introduction

Ancient and medieval astrologers believed that the the sun, moon and planets influence events on earth in at least three ways: (1) by their positions in the twelve signs of the zodiac (Aries, Taurus etc.); (2) by their positions in the twelve so-called houses, which will be defined below; (3) by their aspects: i.e., certain geometrical configurations which will be explained in Section 2 of this paper.

Most medieval astrologers defined the twelve houses in the following way. First divide the celestial sphere into four quadrants by the horizon and meridian planes. Then divide each quadrant into three houses. The first house begins directly under the Eastern horizon, and the houses are numbered counter-clockwise, for an observer in the temperate regions of the Northern hemisphere facing South. The result resembles the division of an orange into twelve parts, not necessarily of equal size. The different houses were related to friends, marriage, family, death, enemies, etc.

In the course of history, the astrologers developed various geometrical systems to divide the four quadrants of the celestial sphere into three houses each (Kennedy

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1996, North 1986, part 1). The differences between these systems were important, because a planet could be in the 12th house (of 'enemies') according to one system and at the same time in the 11th house (of 'friends') according to another system. There has been no agreement among astrologers on the choice of a best system.

This paper is devoted to the system which has often been named after Regiomontanus (1436-1476). North has shown (1986, pp. 35-36) that the system was already used by the 11th-century Andalusi mathematician Abū ^{c}Abdallāh ibn Mu^{c}ādh al-Jayyānī (Sezgin 1974, p. 109; Samsó 1992, pp. 137-144; Rosenfeld and Ihsanoğlu 2003, p. 140). I will call it the 'system of Ibn Mu^{c}ādh', although it is possible that it was used before him, as we will see in Section 3 of this paper. The system of Ibn Mu^{c}ādh is as follows. The horizon and meridian planes divide the celestial equator into four arcs of 90 degrees each. Now divide these arcs into three equal parts of 30 degrees each, and draw great circles through the division points and the North and South points of the horizon. These circles are the boundaries of the houses.

Figure 1 displays the horizon $SENW$, with the South point S, the East point E, the North point N, the upper half of the meridian SMM^*N, and the parts EM and E^*M^* of the celestial equator and the ecliptic between the Eastern horizon and the upper half of the meridian. The quadrant EM of the celestial equator is divided into three arcs of 30^o at points B and A, and the great semicircles SBN and SAN intersect the ecliptic at points B^* and A^*. Then arcs M^*B^*, B^*A^* and A^*E^* define the tenth, eleventh and twelfth house. The three other quadrants are subdivided in a similar way.

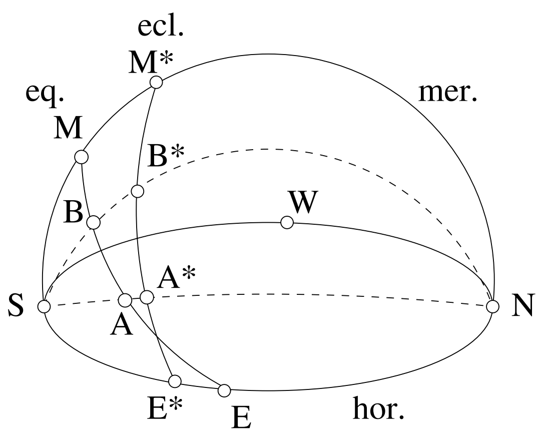

Fig. 1.

The astrologer who wanted to use this system had to compute the ecliptical longitudes of so-called 'cusps of the houses', i.e., the points of intersection B^*, A^* etc. These longitudes depend on the geographical latitude of the locality and the ecliptical longitude of the ascendent, that is the intersection E^* of the ecliptic and the Eastern horizon in Figure 1. The mathematics is not easy, but traces of a method of computation have survived in the *Tabulae Jahen*, the medieval Latin translation of the instructions to a set of astronomical tables which Ibn Muᶜādh compiled for the city of Jaén, where he lived (Samsó 1992, pp. 152-166). The Latin text of the *Tabulae Jahen* is so obscure that Ibn Muᶜādh's method of computation has been a mystery thus far (North 1986, p. 37; Samsó 1992, p. 158 note 74). In the present paper, the method will be compared with a hitherto unpublished Arabic text by Ibn Muᶜādh, entitled *On the Projection of Rays*. This is the special work on projection of rays to which Ibn Muᶜādh refers in his *Tabulae Jahen*.[1] In *On the Projection of Rays*, Ibn Muᶜādh explains the computation of the houses in connection with the computation of the astrological aspects, which is even more interesting from a mathematical point of view. It turns out that Ibn Muᶜādh's solutions are mathematically exact.

The structure of the rest of this paper is as follows. Section 2 is an introduction to the astrological aspects and their history. In Section 3 I explain the details of Ibn Muᶜādh's computations, in his own notations and also in modern formulas. The interested reader can use these formulas to write a computer program for the computations. Section 4 contains the Arabic text with an English translation of the relevant part of Ibn Muᶜādh's *On the Projection of Rays*. In this part, Ibn Muᶜādh explains his method of computation of the houses (according to his own system) and the aspects (according to the equatorial theory) with reference to a geometrical figure. The figure itself is missing in the manuscript but has been reconstructed as Figure 7 below. Other parts from the same Arabic text by Ibn Muᶜādh have been discussed in (Kennedy 1994). Section 5 contains the Latin text and an English paraphrase of the chapter of the *Tabulae Jahen* on the houses and aspects. This chapter includes the mysterious Latin passages which have been studied by North. These passages will be clarified by a comparison with the Arabic text in Section 4.

Ibn Muᶜādh's mathematical work on the houses and aspects is an example where Andalusi mathematicians reached the same level as their colleagues in the Eastern Islamic world. Although Eastern Islamic mathematicians such as al-Bīrūnī were able to do the same computations, Ibn Muᶜādh's method was probably his own invention, because it is based on a rather complicated preliminary computation in his book *On the Unknown Arcs of the Sphere*, as we will see below. Ibn

Mu^{c}ādh's work shows that mathematical astrology was a field of intense research in Al-Andalus. Ibn Mu^{c}ādh improved on his predecessor al-Majrīṭī (ca. 1000), as we will see below, and his work was continued in the thirteenth century by Ibn al-Raqqām (Kennedy 1996). There is an intriguing connection between Ibn Mu^{c}ādh and Regiomontanus. Hitherto the oldest known tables for Ibn Mu^{c}ādh's system of houses were the tables by Regiomontanus (North 1986, pp. 27-30). Although Ibn Mu^{c}ādh's tables of houses have not been found, it now appears that he was in the possession of a correct method of computation. Tracing the direct or indirect influence of Ibn Mu^{c}ādh on Regiomontanus could be an interesting subject for future research.

The astrological aspects

In this section we assume for sake of simplicity that the moon and all planets are in the ecliptic.

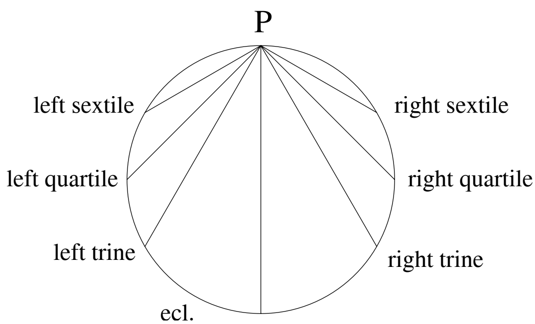

Fig. 2.

In ancient Greek astrology, the seven planets (including the sun and moon) were each believed to cast seven visual rays to the ecliptic (Bouché-Leclercq 1899, pp. 247-251). According to the simplest doctrine, which is used by modern astrologers, these rays hit the ecliptic at points whose ecliptical longitudes are obtained by adding or subtracting 60^o, 90^o, 120^o or 180^o from the ecliptical longitude of the planet *P* in Figure 2. Six of the rays are called 'right sextile ray' (subtract 60^o), right quartile ray (subtract 90^o), right trine ray (subtract 120^o), left sextile ray (add 60^o), left quartile ray (add 90^o), and left trine ray (add 120^o). If another planet happens to be near a point where one of the rays hits the ecliptic, the second planet is supposedly 'looked at' (Latin: adspectus) by the first planet, and the planets are said to form an 'aspect'. For example, if the second planet is sufficiently close to the left sextile ray, the two planets form a sextile aspect; then the second planet also looks with its right sextile ray at the first planet.

Ibn Mu^{c}ādh defined the astrological aspects by a method which I call the equatorial system, following (Kennedy and Krikorian 1972). In this system, the arcs of 60^o, 90^o etc., which define the aspects, have to be measured on the celestial equator rather than the ecliptic. As in Ibn Mu^{c}ādh's system of houses, great circles through the North and South points of the horizon are used to connect points on the ecliptic to points on the equator. To find the rays of a planet at P^* in the ecliptic (Figure 3, compare Figure 1), we draw a semicircle from the North point N through P^* to the south point S of the horizon. This semicircle meets the equator at a point P. Then we measure the arcs $PQ = 60^o$ etc. on the equator and draw through points Q etc. semicircles NQS etc. to meet the ecliptic at points Q^* etc. The planet will now cast its rays to the points Q^* etc. which we have thus obtained. Figure 3 displays the case where the planet P^* is a little above the Eastern horizon, and Q^* is the point where the right sextile ray hits the ecliptic. The size of the arcs P^*Q^*, etc. depends on the ascendent, the position of P^* and the geographical latitude, but the computation of these aspects is even more complex than that of the houses according to Ibn Mu^{c}ādh's system.

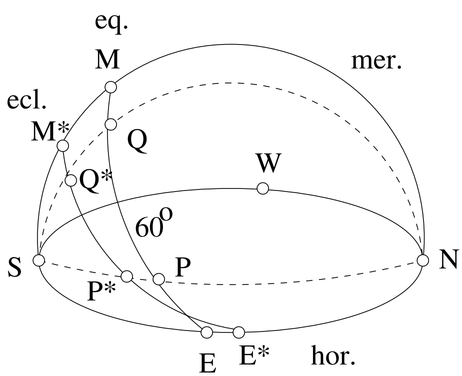

Fig. 3.

The equatorial system of aspects was known at least two centuries before Ibn Mu^{c}ādh, and it seems to have been widespread in the medieval Islamic world. Medieval Islamic astrologers often said that the equatorial theory of aspects is found in Ptolemy's *Tetrabiblos*, even though this is not the case.[2] An approximate method of computation for the equatorial aspects was given by a number of Islamic authors, including al-Bīrūnī and the Andalusi mathematician Ibn al-Samḥ (979-1035) (Kennedy and Krikorian 1972; Viladrich 1986, pp. 69, 147-149; Hogendijk 1988, p. 178, where formula (2.6) should read $P^*Q^* = A_\xi(\lambda_Q) - A_\xi(\lambda_P)$.) The famous al-Khwārizmī (ca. 830) compiled a crude table for the equatorial aspects for the latitude of Baghdad (33^o). The Andalusi astronomer al-Majrīṭī (ca. 1000) computed a more complicated table for the latitude of Cordoba ($38^o30'$) in his revision of the astronomical tables of al-Khwārizmī. A mathematical analysis of these tables shows that the underlying method of computation is only a rough approximation to the geometric definition of the aspects (Hogendijk 1988). Below we will see that Ibn Mu^{c}ādh computed the equatorial aspects in a mathematically correct way. Ibn Mu^{c}ādh then says that approximate computations of the aspects can be used in almost all cases. He presents the same approximate method as Ibn al-Samḥ and al-Bīrūnī in a lengthy passage which I have not edited and translated in the present paper. Thus Ibn Mu^{c}ādh's exact mathematical method was inspired by (astrological) applications, but it was so sophisticated that it was only rarely used. This situation is not unusual in the history of medieval Islamic mathematics.

Ibn Mu^{c}ādh's lost tables for the houses from the *Tabulae Jahen* were probably computed according to his exact method. In a table of houses, the only independent variable is the ecliptical longitude of the ascendent, but in a table of aspects, the ecliptical longitude of the planet is a second independent variable. Thus a gigantic amount of computation would be necessary for the compilation of mathematically exact tables for the aspects according to the equatorial system. Ibn Mu^{c}ādh says that the mathematically exact computation is rarely necessary, so it is very unlikely that he compiled tables according to his own mathematically exact method. Because personal computers are now generally available, the time is perhaps ripe for a rediscovery of the equatorial aspects by modern astrologers.

Ibn Mu^{c}ādh's computation of aspects and houses

In the following overview of Ibn Mu^{c}ādh's computations, I use the same notation as Ibn Mu^{c}ādh in the relevant section in *On the Projection of Rays*. Ibn Mu^{c}ādh's figure in the manuscript is missing, and my reconstruction appears in Section 4 of this paper as Figure 7, which has been drawn according to medieval conventions. Figures 4, 5 and 6 in the present section have been extracted from Figure 7, but have been drawn according to modern conventions. In my overview

I use a few standard concepts of Ptolemaic spherical astronomy, which are explained in (Pedersen 1974). I then present the entire computation in modern formulas without reference to ancient astronomical concepts. These formulas can be used to write a computer program for the computations.

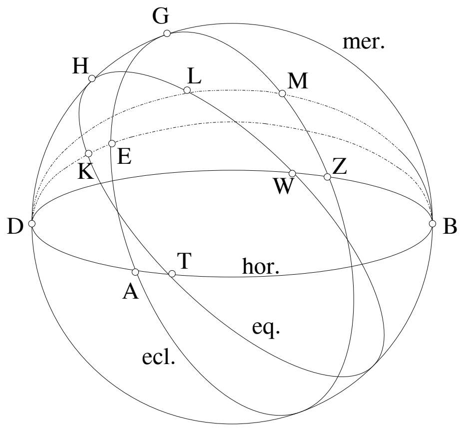

Fig. 4.

Figure 4 displays the celestial sphere. Points B, T, D and W are the North, East, South and West points of the horizon. The meridian is $DHGB$, the equator is THW, the ecliptic is AGZ. Point A is the ascendent or rising point and point Z is the descendent or setting point.

I first discuss Ibn Mu^{c}ādh's computation of the aspects. Ibn Mu^{c}ādh assumes that the planet is at point E on the ecliptic, and he explains the computation of the right quartile ray of E. The figure is drawn for the case where point E is above the horizon in the Eastern half of the celestial sphere, but the method of computation is general, and can also be used for the other rays.

In order to find the desired point M where the right quartile ray hits the ecliptic, Ibn Mu^{c}ādh draws a great semicircle DEB which meets the celestial equator at point K. He then constructs arc $KL=90^\circ$ on the celestial equator. Then the great semicircle BLD meets the ecliptic at M. Thus the problem is essentially to determine the ecliptical longitude λ_M of M from the ecliptical longitude λ_E of E, the geographical latitude ϕ, and the ecliptical longitude λ_A of the ascendent.

Ibn Mu^{c}ādh's solution consists of two parts. In the first part (Figure 5), he computes the right ascension α_K of point K from λ_A, ϕ and λ_E. In medieval Islamic astronomy, the quantity α_K is called the "ray ascension" of point E. Then the right ascension α_L of point L can be found because the difference $\alpha_L - \alpha_K$ is

$\pm 60^o, 90^o, 120^o$ depending on the type of ray. In the second part of the solution (Figure 6), he computes λ_M from λ_A, ϕ and α_L. Again, α_L is called the "ray ascension" of point M.

Ibn Mu^{c}ādh draws two more great semicircles through the celestial North pole N, namely circle NOE (Figure 5), which meets the celestial equator at O near K, and circle NQL (Figure 6), which meets the ecliptic at Q near L.

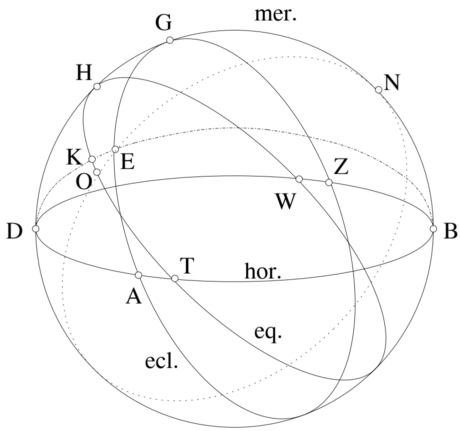

Fig. 5.

The first and second part of the solution are based on the theorem of Menelaus for arcs of great circles on a sphere (Pedersen 1974, pp. 72-75).

In Figure 5 we have, by the theorem of Menelaus,

$$\frac{\sin(OK)}{\sin(KH)} = \frac{\sin(OE)}{\sin(EN)} \cdot \frac{\sin(ND)}{\sin(DH)}.$$

The arcs on the right-hand side are known: $OE = |\delta_E|$, $EN = 90^o \pm |\delta_E|$, $ND = 180^o - \phi$, $DH = 90^o - \phi$. Here δ_E is the declination of E, which can be found from λ_E.

Therefore $\sin OK / \sin KH$ is known.

Arc OH can be found from λ_E, λ_A and ϕ. Thus we have to solve the following problem: to compute two arcs OK and KH from their sum or difference OH and the ratio $\sin OK / \sin KH$. Ibn Mu^{c}ādh solved this problem in his work *On the Unknown Arcs of the Sphere* (*fī majhūlāt qisī al-kura*). The work is extant, and an edition of the Arabic text with Spanish translation can be found in (Villuendas 1979). The details of the computation of KH will be presented below in modern formulas. Thus α_K is known. We have $\alpha_L = \alpha_K - 90^o$, because we are computing the right quartile ray.

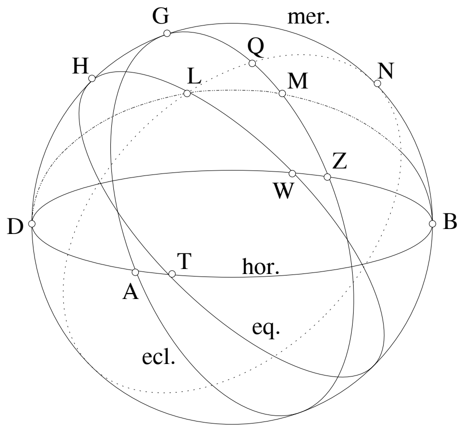

Fig. 6.

In Figure 6 we have by the Theorem of Menelaus:

$$\frac{\sin QM}{\sin MG} = \frac{\sin QL}{\sin LN} \cdot \frac{\sin ND}{\sin DG}.$$

The arcs on the right-hand side can be found: QL can be determined from α_L, $LN = 90^o$, $ND = 180^o - \phi$, and DG can be determined from λ_A. Therefore the ratio $\sin QM / \sin MG$ is known. Arc QG can be found from α_L. Hence our problem is to compute two arcs QM, MG from their sum or difference QG and the ratio $\sin QM / \sin MG$ between their sines. For the solution, Ibn Muᶜādh explicitly refers to his work *On the Unknown Arcs of the Sphere*.

For the astrological houses we only need the second part of the computation. In Figure 6, if $HL = 30^o$, point L is the beginning of the ninth house; if $HL = 60^o$, point L is the beginning of the eighth house, and so on.

I now present Ibn Muᶜādh's computation of the aspects and houses in modern formulas, and in real numbers, for a locality in the temperate regions of the Northern hemisphere. I have added formulas for the preliminary computations which Ibn Muᶜādh does not explain. He probably used standard tables, which were computed on the basis of the formulas which I have presented or by mathematically equivalent methods.

We assume that the following quantities are known: the obliquity of the ecliptic $\varepsilon\,(\approx 23^o30')$, the geographical latitude ϕ with $0^o < \phi < 90^o - \varepsilon$, the ecliptical longitude λ_A of the ascendent A, and the ecliptical longitude λ_E of the planet E, with $0 \leq \lambda_A < 360^o, 0 \leq \lambda_E < 360^o$. Ibn Mu$^c\bar{a}$dh ignores the latitude of the planet. For sake of clarity I have divided the computation into the following three parts: preliminaries, computation of K, computation of M.

Preliminaries

1. Find the declination δ_A of the ascendent from $\sin\delta_A = \sin\lambda_A \cdot \sin\varepsilon$, $-\varepsilon \leq \delta_A \leq \varepsilon$.
2. Find the right ascension α_A of the ascendent from e.g.,
   $\tan\alpha_A = \cos\varepsilon \cdot \tan\lambda_A$, such that λ_A and α_A are in the same quadrant. This is to say that $0^o \leq \alpha_A < 90^o$ if $0^o \leq \lambda_A < 90^o$, $90^o \leq \alpha_A < 180^o$ if $90^o \leq \lambda_A < 180^o$, $180^o \leq \alpha_A < 270^o$
   if $180^o \leq \lambda_A < 270^o$, $270^o \leq \alpha_A < 360^o$ if $270^o \leq \lambda_A < 360^o$.
3. Find the ascensional difference Δ of the ascendent from
   $\sin\Delta = \tan\phi \cdot \tan\delta_A$, $-90^o < \Delta < 90^o$.
4. Find the oblique ascension η_A of the ascendent from $\eta_A = \alpha_A - \Delta$.
5. Find the right ascension α_G of G from $\alpha_G = \eta_A - 90^o \bmod 360^o$.
6. Find the declination δ_G of G from, for example, $\tan\varepsilon \cdot \sin\alpha_G = \tan\delta_G$,
   $-\varepsilon \leq \delta_G \leq \varepsilon$.
7. Find the ecliptical longitude λ_G of G from $\tan\alpha_G = \cos\varepsilon \cdot \tan\lambda_G$, such that λ_G and α_G are in the same quadrant (see above, step 2).
8. Find the altitude a of point G from $a = 90^o - \phi + \delta_G$.

Computation of point K (see Figure 5)

1. Find the declination δ_E of point E from $\sin\delta_E = \sin\lambda_E \cdot \sin\varepsilon$,
   $-\varepsilon \leq \delta \leq \varepsilon$.
2. Find the right ascension α_E of point E from e.g.,
   $\tan\alpha_E = \cos\varepsilon \cdot \tan\lambda_E$, such that λ_E and α_E are in the same quadrant.
3. Find the quantity $c = \alpha_E - \alpha_G$. Since Ibn Mu$^c\bar{a}$dh reduces all computations to a situation where point E is above the horizon (see the Latin text below), we can assume $-90^o < c < 90^o$.
4. Determine the quantity $k = -\tan\delta_E \cdot \tan\phi$. Note that $-1 < k < 1$ since
   $0 < \phi < 90^o - \varepsilon$ and $-\varepsilon \leq \delta_E \leq \varepsilon$.

5. Solve x (= $\pm$ arc OK) from $\sin x = k \sin(x+c)$ with $-90^o < x < 90^o$.
   According to Ibn Mu$^c\bar{a}$dh's book *On the Unknown Arcs of the Sphere* (Villuendas 1979, pp. 11-30, 106-118), x can be computed directly using

$$\tan(\frac{c}{2}+x) = \frac{1+k}{1-k} \cdot \tan\frac{c}{2}.$$

6. Find the desired right ascension of point K from $\alpha_K = \alpha_E + x$.

Computation of point M (see Figure 6)

1. Compute the right ascension α_L of point L in the following way: For the right sextile ray: $\alpha_L = \alpha_K - 60^o \bmod 360^o$; for the right quartile ray: $\alpha_L = \alpha_K - 90^o \bmod 360^o$; for the right trine ray: $\alpha_L = \alpha_K - 120^o \bmod 360^o$; for the left sextile ray: $\alpha_L = \alpha_K + 60^o \bmod 360^o$; for the left quartile ray: $\alpha_L = \alpha_K + 90^o \bmod 360^o$; for the right trine ray: $\alpha_L = \alpha_K + 120^o \bmod 360^o$. It is sufficient to consider points L above the horizon.
   If we want to compute the houses, we do not use point E but we put for the ninth house $\alpha_L = \alpha_G - 30^o \bmod 360^o$; for the eighth house $\alpha_L = \alpha_G - 60^o \bmod 360^o$; for the eleventh house $\alpha_L = \alpha_G + 30^o \bmod 360^o$, and for the twelfth house $\alpha_L = \alpha_G + 60^o \bmod 360^o$.
2. Put $\alpha_Q = \alpha_L$. Compute the ecliptical longitude λ_Q of Q from $\tan\alpha_Q = \cos\varepsilon \tan\lambda_Q$; λ_Q is in the same quadrant as α_Q.
3. Find the declination δ_Q of Q from, e.g., $\tan\delta_Q = \tan\varepsilon \sin\alpha_Q$.
4. Find $k' = +\sin\phi \cdot \sin\delta_Q / \sin a$.
   This is (apart from a possible negative sign) the right hand term in the Theorem of Menelaus. Because Ibn Mu$^c\bar{a}$dh uses the medieval Sine (which is 60 times the modern sine), he has to divide by an extra factor 60.
5. Find $c' = \lambda_Q - \lambda_G$. We can assume $-90^o < c' < 90^o$ because it is sufficient to explain the computation for points Q above the horizon.
6. Solve x' from the equation $\sin(x') = k' \sin(x'+c')$. Ibn Mu$^c\bar{a}$dh finds the solution $\tan(\frac{c'}{2}+x') = [(1+k)/(1-k)] \cdot \tan\frac{c'}{2}$, $-90^o < x < 90^o$.
7. Find the ecliptical longitude λ_M from $\lambda_M = \lambda_Q + x'$. Point M is the cusp of the house or the place where the ray hits the ecliptic. The computation is now complete.

Worked example

The following worked example for the right quartile ray does not correspond to Figure 5, but to a case where the simple theory of rays and the equatorial theory produce very different results. Assume $\varepsilon = 23.5^o$, $\phi = 52^o$, $\lambda_A = 30^o$, $\lambda_E = 300^o$. Then

$$\begin{array}{lll} \delta_A = 11.50^o, & \alpha_A = 27.90^o, & \Delta = 15.09^o, \\ \eta_A = 12.81^o, & \alpha_G = 282.81^o, & \delta_G = -22.98^o, \\ \lambda_G = 281.78^o, & a = 15.02^o, & \delta_E = -20.20^o, \\ \alpha_E = 302.19^o, & c = 19.38^o, & k = +0.47, \\ x = 15.74^o, & \alpha_K = 317.93^o, & \alpha_L = \alpha_Q = 227.93^o, \\ \lambda_Q = 230.38^o, & \delta_Q = -17.89^o, & c' = -51.40^o, \\ k' = -0.927 & x' = 24.66^o, & \lambda_M = 255.04^o. \end{array}$$

Note that $\lambda_E - \lambda_M = 44.96^o$. In the simple theory of aspects, this difference would be 90^o. Thus the equatorial system of aspects is very different from the simple theory.

Ibn Mu^{c}ādh notes that the right trine ray is opposite to the left sextile ray, etc., so one only has to compute the three rays which hit the ecliptic at points L above the horizon, and the other rays can be found by adding 180^o. Similarly, once the cusps of the eighth, ninth, eleventh and twelfth house have been computed, the diametrically opposite points are the cusps of the second, third, fifth and sixth house respectively. In a similar vein, if point E is under the horizon, Ibn Mu^{c}ādh performs the computation for the diametrically opposite point E' above the horizon.

I now return to the history of Ibn Mu^{c}ādh's system of houses. North (1986, p. 35, 64) has collected medieval texts in which the system is attributed to Ibn Mu^{c}ādh, but also an astrolabe plate on which the system is called "the method of al-Ghāfiqī." As North remarks, the reference is probably to Abū al-Qāsim Aḥmad ibn ^{c}Abdallāh ibn ^{c}Umar al-Ghāfiqī, also called Ibn al-Ṣaffār, who died in 1035 (Sezgin 1974, p. 356). Since Ibn Mu^{c}ādh died in 1093 (Samsó 1992, p. 138), it is possible that the 'system of Ibn Mu^{c}ādh' was already known to Ibn al-Ṣaffār, if not before.

In the beginning of the passage translated in Section 4, Ibn Mu^{c}ādh quotes a lost work of the Arabic philosopher al-Kindī (ninth century) on the relationship between the houses and the aspects. According to Ibn Mu^{c}ādh, al-Kindī said that if a planet is at the ascendent, it casts its sextile and trine rays to the cusps of the eleventh and third house, its quartile rays to the points of lower and upper culmination, and its trine rays to the cusps of the ninth and the fifth house. Thus al-Kindī probably did not accept the simple theory of aspects, which prescribes that the distance between a planet and its quartile ray is 90^o, measured along the ecliptic. The above-mentioned worked example shows that the distance between ascendent and upper or lower culmination is in general not equal to 90 degrees. If al-Kindī believed in the equatorial theory of aspects, his system of houses must have been the same as the one of Ibn Mu^{c}ādh. It is conceivable that al-Kindī believed in a slightly different system of houses which the Arabic astrologers attributed to Hermes, and which most Latin sources attribute to Campanus (North 1986). In this system, the prime vertical is divided into equal arcs of 30^o, and the great circles through the North and South points of the horizon and these division points are the boundaries of the houses. If this was al-Kindī's system of houses, he must have used a theory of aspects in which the defining arcs of 60^o, 90^o etc. were located in the prime vertical. In (Woepcke 1858 pp. 29-32, 37) we find an astrolabe plate for the aspects according to this system[3] for the latitudes $38^o30'$ and 42^o, so the system of Hermes for aspects must have been used at some point in history. Note that al-Bīrūnī (973-1048) also claims to be the inventor of the system (Samso 1996, p. 589). In conclusion, we do not know in which system of aspects al-Kindī believed, and we cannot be sure that Ibn Mu^{c}ādh was the person who first developed 'his' system of houses. But we do know that Ibn Mu^{c}ādh was the author of a mathematically correct computation of the houses according to 'his' system.

Arabic text and translation of part of the Projection of Rays *of Ibn Mu^{c}ādh*

The following Arabic text and translation are based on Ms. Florence, Bibliotheca Medicae-Laurenziana Or. 152, ff. 77b:8 - 78b:22 (Sabra 1977, pp. 276-283). According to the colophon, this manuscript was completed in the middle decade of March in the year 1303 of the Spanish Era (A.D. 1265), in the city of Toledo. The manuscript was probably written in the court of king Alfonso X (1252-1284) (Samsó p. 252), more than two centuries after the city of Toledo had been conquered by the Christians. The identity of the scribe is unknown. The manuscript also contains a copy of Ibn Mu^{c}ādh's above-mentioned work *On the Unknown*

Arcs of the Sphere.

Several mistakes in the Arabic manuscript of Ibn Muʿādh's treatise were corrected in the margin of the manuscript. In my edited version, these marginal corrections appear in pointed brackets < >. I have not indicated the incorrect words in the main text which were replaced by these corrections. In order to preserve some of the flavour of the original, I have not corrected all grammatical and stylistic inconsistencies. My own explanatory additions to the translation appear in parentheses.

Arabic text

وممّا يقوّي رأينا ومذهبنا في الجمع بين تسوية البيوت ومطارح الشعاع أنّ الكندي وكان من الجلالة في صنوف هذه العلوم حيث لا يجهل فإنه يقول في بعض تواليفه في الشعاعات إن الكوكب إذا كان على دائرة الأفق في جزء الطالع فإن درجة البيت الحادي عشر ودرجة البيت الثالث هما يسدّسانه وإن درجة وسط السماء ووتد الأرض هما يربّعانه وإن درجة البيت التاسع ودرجة البيت الخامس هما يثلّثانه. وهذا لا يتهيّأ في المذهب الذي نسب إلى بطلميوس بوجهٍ وقد بيّنا فساد ذلك < المذهب > . وإنما يتهيّأ ذلك على ما ذكرناه من قسمة دائرة معدل النهار بأقسام متساوية ثم إخراج إلى تلك الأقسام بدوائر من النقطتين المشتركتين لدائرة الأفق ودائرة نصف النهار . < فتلك الدوائر > تقسم فلك البروج إلى بيوته الاثني عشر وما وافق من الكواكب على دائرة من تلك الدوائر فإن تثليثه وتربيعه وتسديسه يكون على بُعد أربعة أبيات وثلاثة واثنين.

وما لم يكن على حقيقة دائرة من تلك الدوائر فإن تربيعه وتسديسه وتثليثه يعلم بأن نخرج من النقطتين المشتركتين المذكورتين قوسًا تمرّ بموضع الكوكب فما قطعت من معدل النهار زيد عليه للتسديس ستين درجة من معدل النهار ثم أخرج من حيث انتهت الستون درجة قوسًا إلى النقطتين المشتركتين فما قطعت تلك القوس من فلك البروج فهو موضع تسديسه الأيسر وكذلك يعمل بتربيعه الأيسر وبتثليثه الأيسر ويكون تسديسه الأيمن مقابل بتثليثه الأيسر والتربيعان متقابلان وتثليثه الأيمن مقابلة تسديسه الأيسر وإن شئت أن تنقص للأيمن وتعمل مثل ذلك.

ونريد أن نذكر كيف العمل في استخراج موضع الشعاع. فنخطّ فلك الأفق عليه ابد ونوقع فيه نصف فلك البروج كيف ما اتفق في حيز الطالع عليه اجز نقطة الجيم منه في وسط السماء والألف الطالع والزاي الغارب.

ونضع أن كوكبنا على نقطة هاء ونريد أن نعمل مطرح الشعاع. فنستخرج معدل النهار وهو قوس وحط[a] ثمّ نمرّ بالكوكب قوسًا نخرجها من نقطتي ب د وهي قوس بكهد تقطع معدل النهار عل ك ونريد موضع < تربيع > الكوكب.

فنجد من معدل النهار ض جزءًا من نقطة ك وهي قوس كل ثم نخرج من نقطتي ب د قوسًا تمرّ على نقطة < اللّام > وهي قوس بلد تقطع فلك البروج على م فإذا أردنا أن نعلم نقطة < ميم > أي جزء هو من فلك البروج فإن أولًا نستخرج نقطة ك أي درجة هي من دائرة معدل النهار فتلك الدرجة هي مطالع الكوكب الشعاعيّة واستخراجها على ما أصف.

نجعل قطب معدل النهار نقطة ن ونخرج منها قوسًا إلى نقطة < الهاء > موضع الكوكب وهي قوس نعه . ومن أجل أن نقطة هاء معلومة تكون نقطة ع معلومة لانّها مطالعها في الفلك المستقيم ونقطة ح معلومة
< وقوس هاء عين معلومة > لأنّها ميل نقطة هاء. ويكون نسبة جيب قوس كع إلى جيب قوس كح مؤلّفة من نسبة جيب قوس ند إلى جيب قوس دح ومن نسبة جيب قوس < هاء عين > إلى جيب قوس نه . فإذا ضربنا جيب قوس ند وهو جيب عرض البلد في جيب قوس < هع > وقسمنا الحاصل على المجتمع من ضرب جيب قوس دح في جيب قوس نه كان الخارجة النسبة الموجودة من جيب قوس كع إلى جيب قوس كح . وكلتا القوسين مجهولتان إلّا أن فضل إحداهما على الأخرى معلوم وهو قوس حع . فنعلم حينئذ قوس كح وهي مطالع جزء الكوكب في الدائرة التي هو عليها فإذا علمنا ذلك علمنا نقطة ل من دائرة معدل النهار.

فإذا أردنا أن نعلم بها نقطة م من فلك البروج فإنا نستخرجها على ما أصف. وبهذا العمل < الذي > نستأنف نستخرج مراكز البيوت الاثني عشر على ما تقدم

[a] وحط : the ms. has زحط.

من مذهبنا فيها لأن قوس لح نجعلها إذا أردنا البيت التاسع ثلاثين جزءًا وإذا أردنا الثامن نجعلها ستين جزءًا وكذلك نجعلها في الجهة الشرقية. وذلك أننا[b] نخرج من قطب < النون > قوسًا إلى نقطة ل المعلومة وهي قوس نقل فنقطة ق من فلك البروج معلومة لأن قوس جق من فلك البروج هو ما يجب لمطالع حل في الفلك المستقيم و لق معلومة لانّها ميل جزء ق ونسبة جيب قوس مق إلى جيب قوس مج مؤلّفة من نسبة < جيب قوس ند إلى جيب قوس دج ومن نسبة > جيب قوس لق إلى جيب قوس نل . وكل هذه القسي معلومة إلّا قوسي مق و قج فإنّهما مجهولتان لكن فضل إحداهما على الأخرى معلوم والنسبة التي بينهما معلومة فهما يعلمان على ما بينا في كتابنا في استخراج مجهولات القسي.

وممّا[c] يحقّق هذا العمل أن التربيعين يخرجان فيه متقابلين وكذلك التثليث الأيسر مع التسديس الأيمن وكذلك التسديس الأيسر مع التثليث الأيمن وكذلك هو في مذهب هرمس.

فأمّا تجريد العمل في استخراج مطالع أي جزء شئت هو أن تعلم الطالع وجزء وسط السماء ومطالع جزء وسط السماء في الفلك المستقيم ومطالع موضع الكوكب في الفلك المستقيم وتأخذ بعد الكوكب من وسط السماء بأن تنقص مطالع موضعه في الفلك المستقيم من مطالع جزء وسط السماء في الفلك المستقيم إن كان موضع الكوكب في الربع الغربي وإن كان موضعه في الربع الشرقي فتنقص مطالع جزء وسط السماء من مطالع موضع الكوكب فيبقى البعد إن شاء اللّه

فأمّا مطرح الشعاع على الأمر الأوجب الأحقّ فما ذكرناه على ما ينبغي لمن أراد تحقيقه في مولد خاصّ أو لأمرٍ معتنى بتحقيقه. وأما تقريبه . . .

Translation of the Arabic text

Among the things which strengthen our view and our doctrine in combining the equalization (i.e. determination) of the houses and the projections of the rays is the following: Al-Kindī, who was among the brilliant people in the branches of

[b] أننا: the ms. has انا.
[c] The manuscript has وما.

these sciences, in such a way that he was not ignorant (of anything), said in one of his writings on the rays: If the star (i.e. any celestial body) is on the horizon at the degree of the ascendent, the degree of the eleventh house and the degree of the third house are the sextiles (i.e. sextile rays) of it, and the degree of midheaven and lower culmination are the quartiles (i.e. quartile rays) of it, and the degree of the ninth house and the degree of the fifth house are the trines (i.e. trine rays) of it. This does not occur in any way in the doctrine attributed to Ptolemy, and we have demonstrated the absurdity of that doctrine. But this occurs if the equator is divided into equal parts, as we have mentioned, and if circles are drawn to these division points from the two common points of the horizon and the meridian. So those circles divide the ecliptic into its twelve houses. If any star is on any of these circles, its trines, quartiles and sextiles are at a distance of four, three and two houses.

If (a star) is not exactly on one of those circles, its quartiles, sextiles and trines are known as follows: We draw from the two above-mentioned common points an arc passing through the position of the star. To its point of intersection with the equator, one adds for the sextile sixty equatorial degrees. Then one draws from where the (arc of) sixty degrees end an arc to the two common points (of the horizon and the meridian). The point of intersection of that arc with the ecliptic is the position of the left sextile. We proceed in the same way for the left quartile and the left trine. The right sextile is in opposition with the left trine, and the two quartiles are in opposition, and the right trine is in opposition with the left sextile. If you wish to subtract (60^o, 90^o or 120^o) for the right (rays), you can also proceed in that way.

We want to explain the procedure for finding the position of the rays. We draw the horizon ABD, and we let fall in it half of the ecliptic, for an arbitrary position, in the domain of the ascendent.[4] It is AGZ, point G is on the meridian, A is the ascendent, and Z is the descendent. We assume that our star is at point E, and we wish to construct the projection of the rays. We draw the equator, namely arc WHT.[5] Then we let pass through the star an arc which we draw through points B and D,[6] namely arc $BKED$, which intersects the equator at K. We want the position of the (right) quartile (ray) of the star.

Thus we find on the equator (an arc of) 90 degrees from point K, namely arc KHL. Then we draw from points B, D an arc passing through point L, namely arc BLD, which intersects the ecliptic at M. If we want to know the ecliptical degree of point M, we first determine the equatorial degree of point K. This degree is the ray-ascension of the star. It is determined as I will now describe.

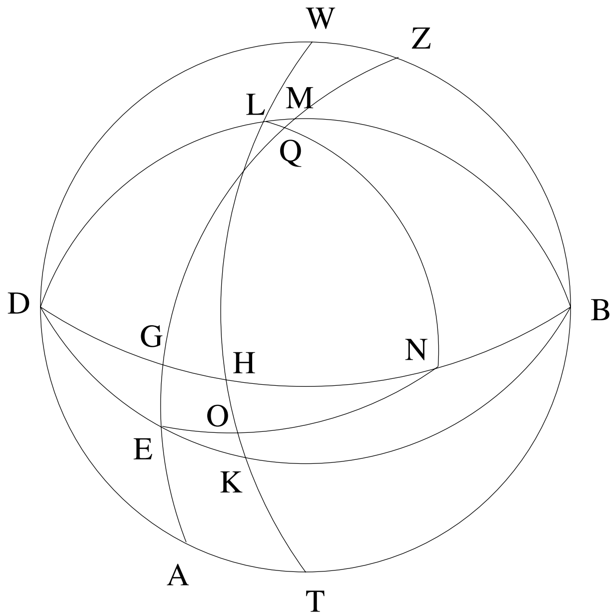

Fig. 7.

We make the pole of the equator point *N*, and we draw from it an arc to point *E*, the position of the star, namely arc *NOE*. Since point *E* is known, point *O* is known, because it is its right ascension. Point *H* is known and arc *EO* is known since it is the declination of point *E*. The ratio of the sine of arc *KO* to the sine of arc *KH* is composed of the ratio of the sine of arc *ND* to the sine of arc *DH* and the ratio of the sine of arc *EO* to the sine of arc *NE*. If we multiply the sine of arc *ND*, that is the sine of the (geographical) latitude of the locality, with the sine of arc *EO*, and divide the product by the product of the sine of arc *DH* and the sine of arc *NE*, the result is the ratio between the sine of *KO* and the sine of *KH*, which (ratio) has now been found. Each of these arcs is unknown, but the excess of one of them over the other is known, that is arc *HO*. Then we know arc *KH*, and this is the ascension of the (ecliptical) degree of the star in the circle on which it is located. If we know this, we know point *L* of the equator.

If we want to know (i.e. determine) from it (*L*) point *M* of the ecliptic, we determine it as I now describe. By this procedure, which we begin anew, we determine the cusps of the twelve houses according to our doctrine, which we have mentioned above, because we make arc *LH* thirty degrees if we want the ninth house, and sixty degrees if we want the eighth house, and similarly for the eastern side (of the celestial sphere). That is to say, we draw from the pole *N* an arc to the known point *L*, namely arc *NQL*. Then point *Q* on the ecliptic is known, because the ecliptical arc *GQ* has right ascension *HL*. But *LQ* is known because it is the declination of the degree of *Q*. The ratio of the sine of arc *MQ* to the sine of arc *MG* is composed of the ratio of the sine of arc *ND* to the sine of arc *DG* and the ratio of the sine of arc *LQ* to the sine of arc *NL*. All these arcs are known except the two arcs *MQ* and *QG*. They are unknown, but the excess of the one over the other is known, and the ratio between them is known. Therefore they can be known as we have explained in our book *On the Extraction of the Unknown Arcs*.

Among the things which this procedure verifies is the fact that the two quartiles (i.e. the quartile rays) are diametrically opposite in it (i.e. in this system), and similarly the left trine and the right sextile, and similarly the left sextile and the right trine. The same is true in the doctrine of Hermes.[7]

Simplification of the procedure for finding the (ray) ascension of any degree you wish: You find the ascendant, and the (ecliptical) degree of midheaven, and the right ascension of the degree of midheaven, and the right ascension of the position of the star. You take the "distance" of the star from midheaven as follows: you subtract the right ascension of its position from the right ascension of the degree of midheaven, if the position of the star is in the Western quadrant. If its position is in the Eastern quadrant, you subtract the (right) ascension of the degree of midheaven from the (right) ascension of the position of the star. The remainder is the distance, God willing.

Concerning the projection of rays in the most accurate and correct way: what we have mentioned (above) is in the way which is necessary for someone who wants to determine it exactly, for a special anniversary or a matter which deserves an exact investigation. As for its simplification...[8]

The Tabulae Jahen *on houses and projections of rays*

The Arabic original of most of the *Tabulae Jahen* is lost, including the passage dealing with the astrological houses. I have paraphrased the passage in the medieval Latin translation because it is difficult to translate literally.

Samsó and Mielgo have recently published the Arabic original of the chapter of the *Tabulae Jahen* on the qibla, which has come down to us in the Z̄ij of Ibn Isḥāq (Samsó and Mielgo 1994). It turns out that the medieval translator of the *Tabulae*

Jahen translated the Arabic word *buʿd* (distance) as “longitudo" (meaning: ecliptical longitude). I have therefore translated “longitudo" as distance. This distance is almost always the distance of a point from the meridian, measured along the celestial equator or ecliptic. Once this is understood, the obscure Latin passage on the astrological houses becomes easier to understand.

The chapters of the *Tabulae Jahen* translated here do not contain proofs and there is no figure. For sake of clarity I have added references to Figures 4-7 above and explanatory remarks in parentheses. Angular brackets contain words which I have restored to the text for mathematical sense. I have put in square brackets all words which I have omitted in order to restore the mathematical sense.

Latin text

Qualiter autem extrahantur proiectiones radiorum secundum rem veriorem in eo quod credimus, de hoc est ut extrahamus in primo[s] ascensiones stellae radiales secundum quod narro.

Si fuerit stella supra terram operabimus cum loco eius. Et si fuerit sub terra operabimus cum eius nadir. Quodcunque ergo duorum fuerit supra terram, accipies ascensiones eius in orbe meridiei, et ascensiones decimi in orbe meridiei iterum, et proiiciemus dextrum illarum ambarum de sinistro eorum, et quod remanet est arcus longitudinis. Accipe ergo sinum eius, et est sinus longitudinis. Deinde accipe declinationem loci stellae, aut declinationem nadir eius cum quocunque eorum operaris. Et scias sinum eius, et partem ipsius, et est sinus declinationis partis, deinde multiplica sinum complementi partis, in sinum complementi latitudinis regionis, et divide aggregatum per illud quod egreditur de multiplicatione sinus declinationis partis, in sinum [complementi] latitudinis regionis, et quod egreditur est proportio sinus longitudinis ascensionum radialium, ad sinum eius quod est inter ascensiones radiales, et inter arcum longitudinis. Serva ergo hanc proportionem.

Si ergo fuerit declinatio loci cum quo operaris ⟨septentrionalis⟩, tunc extrahe arcum [septentrionalis,][9] longitudinis ascensionum radialium, sicut extrahis ex proportione nota eveniente inter duos sinus duorum arcuum separatorum ignotorum, quorum aggregatio est nota cum aggregamus duos arcus separatos, secundum quod ostendimus illud in libro nostro de extractione ignotorum arcuum Sphaerae ex notis eius. Arcus enim longitudinis ascensionum radialium, cum aggregantur ipse, et arcus sequens eum in proportione sunt simul arcus longitudinis notae.

Et si fuerit declinatio loci cum quo operaris meridana, tunc extrahe arcum longitudinis ascensionum radialium, sicut extrahis ex proportione nota accidente inter duos sinus duorum arcuum compositorum ignotorum quorum unius superfluitas super alium est nota, duos arcos compositos, secundum quod declaravimus illud illic. Quoniam arcus longitudinis ascensionum radialium, addit super arcum sequentem eum in proportione per aequalitatem arcus longitudinis notae.

Quicunque ergo fuerit arcus longitudinis ascensionum radialium, adde cum super ascensiones medii coeli, si fuerit pars cum qua operaris in quarta orientali. Et minue de ascensionibus medii coeli, si fuerit in quarta occidentali, et quod fuerit ex hoc, erunt ascensiones radiales. Cum ergo egrediuntur ascensiones radiales, tunc adde super eas sextilitatem sinistram sexaginta graduum, et quadraturam sinistram nonaginta et triplicitatem sinistram, centum et viginti graduum, et minue radii dextri, quantum pro sinistro addidisti, et melius in hoc est, ut scias ascensiones radiales. Tunc si fuerint in quarta orientali, operare per quadraturam dextram, et non opereris per sinistram. Et si fuerint in occidentali, tunc operare per quadraturam sinistram. Et iterum si fuerit arcus longitudinis ascensionum radialium minor triginta ad partem orientis, aut ad partem occidentis, tunc operare sextilitatem sinistram, et sextilitatem dextram simul, et ne opereris aliquam triplicitatem. Et si fuerit arcus longitudinis ascensionum radialium plus triginta ad orientem, tunc fac sextilitatem, et triplicitatem dextram, et non facias sextilitatem, et triplicitatem sinistram, et si fuerit plus triginta ad partem occidentis, tunc fac sextilitatem et triplicitatem sinistram, et facies eas secundum quod narro, et per similitudinem illius aequabis domos secundum modum quem aestimo convenientem rectitudini, et est illud quod promisi me dicturum in Capitolo proiectionum radiorum.

Rememoratio aequationis domorum duodecim.

Et illud est, ut accipias ascensiones ascendentis in regione, et minuas ex eis triginta, et remanebunt ascensiones duodecimae.[10] Et minuas ex ascensionibus ascendentis centum viginti, et remanebunt ascensiones nonae, et minuas ex ascensionibus ascendentis centum quinquaginta, et remanebunt ascensiones octavae.

Quascunque ergo harum ascensionum habueris, aut ascensiones radiales post additionem laterum sextilitatis aut quadraturae, aut triplicitatis, et diminutionem earum. Tunc fac cum eis totum, quod narro tibi, et scias quod unaquaeque earum est super terram. Et propter illud accepimus has domos quae sunt supra terram, et accepimus radios qui cadunt supra terram, et convertimus illas ascensiones in ascensionibus orbis recti ad gradus aequales, et quod fuit scivimus quod fuit inter illud et inter partem medii coeli, ita quod minuimus dextrum amborum, et propinquius ipsorum ad orizontem occidentis de sinistro amborum, et est longinquius ipsorum ab orizonte occidentis, et quod est, est longitudo a medio coeli.

Deinde accipe declinationem illius partis, ad quam convertimus ascensiones, et accipe sinum eius, et est sinus declinationis. Et scias partem eius, et scias altitudinem partis decimae in medio coeli, et accipe sinum illius altitudinis, et multiplica ipsum in sexaginta, et divide aggregatum per illud quod egreditur de multiplicatione sinus declinationis partis in sinum latitudinis regionis, tunc quod egreditur est proportio sinus longitudinis gradus, ad sinum arcus qui est inter longitudinem gradus quaesiti, et inter longitudinem servatam a medio coeli. Si ergo fuerit declinatio partis multiplicata prius septentrionalis, tunc extrahe sinum longitudinis gradus quaesiti, sicut extrahis ex proportione nota eveniente inter duos sinus duorum arcuum ignotorum compositorum, quorum unus addit super alterum, cum arcu noto duos arcus compositos. Quoniam arcus longitudinis gradus quaesiti, tunc addit super arcum sequentem eum in proportione, cum arcu noto qui est longitudo servata a medio coeli. Et si fuerit declinatio partis multiplicata meridiana, tunc extrahe sinum longitudinis gradus quaesiti, sicut extrahis ex proportione nota accidente inter duos sinus duorum arcuum ignotorum separatorum, quorum aggregatio est nota cum aggregant duos arcos separatos. Quoniam arcus longitudinis gradus quaesiti, tunc cum aggregatur ipse et arcus sequens eum in proportione, sunt simul arcus [quaesitus] notus, qui est longitudo servata a medio coeli. Cum ergo sciveris longitudinem arcus noti a medio coeli, tunc adde eam super partem medii coeli, si fuerint in quarta orientali ascensiones quas convertisti, et si fuerint in quarta occidentali, tunc minue eam de medio coeli. Quod ergo est post illud totum, est gradus quaesitus, ad quem operatus es proiectionem quidem radiorum, si es operatus ad eam, aut initium alicuius sex domorum, quae sunt supra terram. Cum ergo feceris domos sex, quae sunt supra terram, tunc illae sex quae sunt sub terra, sunt earum nadir in oppositione earum. Et similiter sextilitas sinistra cum triplicitate dextra sunt oppositae, et duae quadraturae sunt oppositae, et similiter sextilitas dextra cum triplicitate sinistra sunt oppositae, et radii omnium duarum partium oppositarum in orbe, quos operaris (sic) sunt convenientes radiis partis alterius convenientia contraria, et quadratura cuiusque ambarum sinistra est quadratura alterius dextra, et sextilitas cuiusque duarum sinistra[,] est triplicitas alterius dextra, et triplicitas cuiusque duarum sinistra est sextilitas alterius dextra. Hoc est ergo verius quod invenimus in opere radiorum, et firmius in omnibus modis experientiae, et magis quadrans secundum canones formae geometricae.

Paraphrase of part of Chapters 37 and 38 of the Tabulae Jahen

As to the derivation of the projections of rays in a more correct way, according to what we believe, we first derive the *radial ascension*[11] of the star (i.e., sun, moon or planet) as I will explain.

If the star is above the horizon, we work with its position, and if it is under the horizon, we work with the point diametrically opposite to it (thus we always work with point E above the horizon in Figure 7). Whichever of the two (positions) is above the horizon, you take its right ascension (O), and also the right ascension of the tenth house (H). We subtract the right one from the left one, and the remainder is the arc of the distance (HO). Take its sine, it is the sine of the distance ($\sin HO$). Then take the declination of the position of the star (EO), or the declination of the point diametrically opposed to it, whichever you work with. And you know the sine of it ($\sin EO$), and its degree (i.e. the magnitude of arc EO); then it is a sine of the degree of the declination. Then multiply the cosine of the degree (i.e., $\cos EO = \sin NE$) by the cosine of the latitude of the locality ($\cos NB = \sin HD$), and divide the product by the product of the sine of the declination of the degree ($\sin EO$) and the [co]sine of the latitude of the locality ($\sin DN$). The quotient is the ratio between the sine of the distance of the radial ascension ($\sin HK$) and the sine of the difference ($\sin KO$) between the radial ascension (KH) and the arc of the distance (HO). (We have $(\sin HK/\sin KO = (\sin HD/\sin DN)\cdot(\sin NE/\sin EO)$ by the theorem of Menelaus.) Keep this ratio in mind.

If the declination of the position with which you work is northern, then extract the arc of the distance of the radial ascension (KH), just as you extract two separate unknown arcs, whose sum is known, from the known ratio between their sines, as we have shown in our book *On the Extraction of the Unknown Arcs of the Sphere from Known Arcs*. For the arc of the distance of the radial ascension (KH) plus the arc following it in the ratio (KO) are equal to an arc of known length (HO).

And if the declination of the position with which you work is southern (as in Figure 7), then extract the arc of the distance of the radial ascension (KH), just as you extract two combined[12] arcs, whose difference is known, from the known ratio between their sines, as we explained there. For the arc of the distance of the radial ascension (KH) exceeds the arc which follows it in the ratio (KO), by (a magnitude) equal to an arc of known length (HO).

Add the arc of the distance of the radial ascension (KH) to the ascension of midheaven, if the degree with which you work (E) is in the Eastern quadrant. Subtract it from the ascension of midheaven if it is in the Western quadrant. The result is the radial ascension. Since the radial ascension has been determined, add to it 60^o for the left sextile (ray), 90^o for the left quartile, 120^o for the left trine, and subtract for the right ray what you have added for the (corresponding) left ray.

But a better way in this[13] is to look at the radial ascension. Then if it is in the Eastern quadrant (i.e. point K), work with the right quartile, not with the left. If it is in the Western quadrant (i.e. point L), work with the left quartile. Again, if the arc of the distance of the radial ascension (KH) is less than 30^o on the Eastern side or on the Western side, then work with the left sextile and the right sextile, and do not work with any trine ray. And if the arc of the length of the radial ascension (KH) is more than 30^o on the Eastern side (of the meridian), make the right sextile and trine (ray) and do not make the left sextile and trine ray, and if it is more than 30^o on the Western side, make the left sextile and trine ray. Make them as I explain (below), and in a similar way you equalize[14] the houses in the way which I consider to be in agreement with correctness. This is what I promised to say in the chapter on the projection of rays.

Description of the computation of the twelve houses.

This is as follows. You take the oblique ascension of the ascendent for the (latitude of the) locality, and subtract 30^o from it, and the remainder is the ascension of the twelfth house. ⟨You subtract 60^o from the ascension of the ascendent, the remainder is the ascension of the eleventh house.⟩ You subtract 120^o from the ascension of the ascendent, and the remainder is the ascension of the ninth house. You subtract 150^o from the ascension of the ascendant, and the remainder is the ascension of the eighth house.

Whichever of these ascensions (i.e. point L in Figure 7) you have obtained, (either the ascensions for the houses) or radial ascensions after addition or subtraction of the sides (sc. of the regular polygons) for the sextile, quartile or trine (rays), do with them all I tell you. You know that each of them is above the horizon; this is why we have taken the houses above the horizon and the rays which fall above the horizon. We take the ascension as a right ascension and we convert it to equal (i.e., ecliptical) degrees. We find the (difference) between the result (Q) and the degree of midheaven (G), by subtracting the right one, (which is) closer to the Western horizon, from the left one, (which is) farther from the Western horizon. The result is the distance (in longitude) from the meridian.

Then take the declination of this (ecliptical) degree (Q) to which we have converted the ascension, and take its sine, this is the sine of the declination ($\sin QL$). You know the degree of it, and the altitude of the (ecliptical) degree of the tenth house, which is midheaven, and you take the sine of this altitude ($\sin DG$), and multiply it by 60 ($=\sin NL$), and divide the result by the product of the sine of the declination of the degree ($\sin QL$) and the sine of the latitude of the locality ($\sin DN$). Then the quotient is the ratio between the sine of the $\langle$ arc of the$\rangle$ distance of the degree ($\sin MG$) and the sine ($\sin MQ$) of the arc (MQ) between the distance of the desired degree (MG) and the distance from midheaven which was kept in mind (QG). (We have $\sin GM/\sin MQ = (\sin GD/\sin DN)\cdot(\sin NL/\sin LQ)$ by the theorem of Menelaus.)

If the declination of the degree (Q) which (i.e., whose sine) has been multiplied before is northern, then extract the sine of the distance of the desired degree ($\sin MG$) as you extract two combined unknown arcs whose difference is known from the known proportion between their sines. For the arc of the distance of the desired degree (MG) exceeds the arc following it in the proportion (MQ) by a known arc (QG), which is the distance from midheaven, which has been kept in mind.

And if the declination of the degree (Q) which (i.e., whose sine) has been multiplied is southern, extract the sine of the distance of the desired degree ($\sin MG$), as you extract two separate unknown arcs whose sum is known from the known proportion between their sines. For the arc of the distance of the desired degree (MG) plus the arc following it in the proportion (MQ) are equal to the [sought] known arc (QG), which is the distance from midheaven which has been kept in mind.

Since you know the distance of a known arc from midheaven, add it to the (ecliptical) degree of midheaven, if the ascension which you converted (L) is in the Eastern quadrant, and if it (L) is in the Western quadrant, subtract it from (the ecliptical degree of) midheaven. The result of all this is the desired degree (M), which is the projection of a ray, if you wanted to construct this, or the beginning (cusp) of any of the six houses above the earth.

When you have made (the computation for) the six houses above the horizon, the six under the horizon are the corresponding diametrically opposed (points). Similarly, the left sextile ray and the right trine ray are diametrically opposed, and the two quartile rays are diametrically opposed, and the right sextile ray and the left trine ray are opposed in the same way.

The rays of any of two diametrically opposed degrees in the ecliptic, which you constructed (i.e. the rays of a point above the horizon), coincide with the rays of the other point (under the horizon) in a converse way: The left quartile ray of one is the right quartile ray of the other. The left sextile ray of one is the right trine ray of the other, and the left trine ray of one is the right sextile ray of the other.

This is the most correct (method) which we have found for the determination of the rays, best established by all kinds of experience, and most in agreement with the rules of geometrical form.

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NOTES

1. "...et iam compleui illud cum eo quod adiungitur ipsi de eo, quod assimilatur ei ex aequatione domorum in libro sungulari in expositione proiectionum radiorum." (Ibn Mu ^{c}ādh 1549, cap. 26.)
2. In the *Tetrabiblos*, Ptolemy computes the astrological progressions (Arabic: *tasyīr*) according to an equatorial method, see (Hogendijk 1988, p. 179).
3. In Woepcke's Figure 12, the equatorial arcs between the East point and the boundary curves of the twelfth and the eleventh house are not 30^o and 60^o, as Woepcke seems to suggest, but approximately 36^o and 65^o. This agrees well with the theoretical values for the system of Hermes for geograpical latitude $\phi = 38^o30'$, namely $\arctan(\tan 30^o/\cos\phi) = 36.4^o$ and $\arctan(\tan 60^o/\cos\phi) = 65.7^o$.
4. Ibn Mu^{c}ādh probably means the celestial hemisphere above the horizon. Earlier in the treatise, Ibn Mu^{c}ādh calls the Eastern quadrant above the horizon the "domain of rise, advance, and ascent" (ḥayyiz al-mashriq wa l-iqbāl wa l-ṣu^{c}ūd), and the Western quadrant the "domain of setting, descent and retreat" (ḥayyiz al-maghrib wa l-hubūṭ wa l-idbār) (f. 73b lines 9-10).
5. The manuscript has ZHT. I have emended the text because the descendant Z is in general not the same as the West point W on the horizon.

6. Points *B* and *D* are the North and South points of the horizon.
7. In this system, the prime vertical is divided into arcs of 30 degrees and the boundaries of the houses are the great semicircles through the division points and the North and South points of the horizon. See Kennedy 1994.
8. Ibn Muᶜādh continues with a lengthy description of an approximate method for determining the 'ray ascension' of any star, as $(1 - w)$ times its right ascension plus w times its oblique ascension (or descension). The weight w is the "distance" of the star from the meridian, which has just been determined, divided by the difference between the right ascensions of ascendent and midheaven, or midheaven and descendent.
9. This word is displaced in the text.
10. Here a passage was left out by scribal error.
11. This term corresponds to the Arabic *maṭāliᶜ shuᶜāᶜiyya.*
12. The text means that the smaller arc is part of the greater arc.
13. Ibn Muᶜādh likes to work with points above the horizon.
14. To "equalize the houses" is a technical term for finding the houses.