AL-ṢAGHĀNĪ'S TREATISE ON THE DISTANCES, VOLUMES AND SURFACE AREAS OF THE PLANETS AND FIXED STARS

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1. Introduction and summary

Abū Ḥāmid al-Ṣaghānī was a mathematician and astronomer who worked in the late fourth/tenth century [33, V, p. 311; VI, pp. 217-218], [30, pp. 292-295], [32, pp. 88-89], see the bibliography at the end of this paper. Among his contemporaries he was famous as a maker of astrolabes and other astronomical instruments. Together with Abū Sahl al-Kūhī and Abu'l-Wafā' Muḥammad ibn Muḥammad al-Būzjānī, he participated in the astronomical observations which were held for the Buwayhid ruler Sharaf al-Dawla in Baghdad in 378/988 (all dates of Islamic scholars will be given in the Islamic and Julian calendars). The reports of these observations have been preserved in the *Ta'rīkh al-Ḥukamā'* of Ibn al-Qifṭī [13, pp. 351-353]. Al-Ṣaghānī died in Baghdad in 390/1000 [13, p. 79].

This paper contains an edition and translation of a short and hitherto unpublished work by al-Ṣaghānī on the sizes, distances, volumes and surface areas of the celestial bodies. The translation is in Section 2, the edition in the appendix, and Section 3 contains my commentaries.

The sizes and distances of the planets, the sun, the moon and the fixed stars of the first magnitude were numerically determined in the end of Book I of the *Planetary Hypotheses* by Ptolemy (ca. A.D. 150, [37]), which is extant (only) in a medieval Arabic translation [10]. The topic was very popular in medieval Islamic astronomical tradition, and a long list of authors who wrote on "sizes and distances" can be found in [3, pp. 78-79]. All these authors determined the sizes and distances according to the assumptions of the Ptolemaic universe. Excepting the moon and the sun, their results are unrelated to determinations of sizes and distances of celestial bodies by modern science.

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The following five authors will be relevant in this paper, in addition to al-Ṣaghānī himself. Aḥmad ibn Muḥammad ibn Kathīr al-Farghānī (ca. 240/850) devoted Chapters 22 and 23 of his *Summary of Astronomy* (Jawāmiʿ ʿilm al-nujūm [33, VI, p. 150 no. 1]) to the subject. The Arabic text can be consulted in the edition by Jacobus Golius (1596-1667) [11, pp. 80-83], now available online. Al-Farghānī also discussed the sizes of the fixed stars of the second through sixth magnitudes. Thābit ibn Qurra (221/836 - 288/901) gave numerical values for the distances and volumes of the planets and the stars of the first magnitude in his *Simplification of the Almagest* (Tashīl al-Majisṭī, [33, VI, p. 90 no. 5a]), which has been edited with French translation in [22, pp. 1-17], see also the commentary in [22, pp. 179-180]. Abū Jaʿfar Muḥammad ibn al-Ḥusayn al-Khāzin (d. ca. 360/970) wrote a treatise on sizes and distances [33, V, p. 299 no. 10] which has not come down to us, but we will see below that some traces survive in the treatise by al-Ṣaghānī published in this paper. The treatise on sizes and distances [33, V, p. 312 no. 2] by Abu'l-Ṣaqr al-Qabīṣī (ca. 350/960) is extant [29]. Finally, Abū Rayḥān al-Bīrūnī (362/973-440/1046) presented a list of sizes and distances [5, pp. 115-118] in the *Introduction to Astrology* (al-Tafhīm fī Ṣināʿat al-Tanjīm [33, VII, p. 189 no. 1]), and he discussed the subject in more depth in Chapter 6 of Book 10 [8, pp. 1301-1314] of the *Qānūn al-Masʿūdī* [33, VI, p. 265 no. 1].

We will now give a brief summary of al-Ṣaghānī's treatise, which is extant in incomplete form in a unique manuscript. The treatise does not contain original work on the part of al-Ṣaghānī, but reports on computations by Thābit ibn Qurra and Abū Jaʿfar al-Khāzin. These astronomers expressed the distances to the planets in miles, and al-Khāzin even computed the surface areas of the celestial bodies in square miles. The resulting numbers are huge and thus they must have fascinated the medieval readers; they are interesting for modern historians of mathematics as well because some of the underlying computations can be reconstructed in detail.

Al-Ṣaghānī 's treatise consists of three brief chapters which will now be summarized. Chapter 1 is about preliminaries of the Ptolemaic universe. The universe is conceived as a collection of nested spheres (for the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn) with the Earth at the center. Each planet moves on circles in its own sphere, and because useless space cannot exist, the spheres must be tangent so the

maximum distance of the moon is the minimum distance of Mercury, and so on. The fixed stars are in the outermost sphere and their distance to the earth is equal to the maximum distance of Saturn. Al-Ṣaghānī informs the reader that Ptolemy had only determined the distances of the sun and moon. It follows that al-Ṣaghānī did not know the *Planetary Hypotheses*, in which Ptolemy computed the distances of all planets and of the stars of the first magnitude.

Chapter 2 is on the distances of the planets and fixed stars. Al-Ṣaghānī first presents the distances of the planets and the fixed stars according to Thābit ibn Qurra, in earth radii and in miles as well. The fact that the distances are given in two different units of measurement makes it possible to correct scribal errors, as we will see in Section 3. There it will also be shown that al-Ṣaghānī took his information from a lost work by Thābit, in which it was assumed that one degree on the spherical earth corresponds to 56 miles, and which must have based on Ptolemy's *Planetary Hypotheses*.

In Chapter 3 al-Ṣaghānī discusses the sizes of the planets and fixed stars. He first renders the volumes of the planets (not the fixed stars) according to Thābit, in terms of the volume of the earth. Al-Ṣaghānī then presents a whole set of celestial distances in miles by "the shaykh Abū Jaʿfar al-Khāzin". These distances are analyzed in Section 3 and it turns out that al-Khāzin used Ptolemy's values from the *Planetary Hypotheses* and the assumption that 1 degree on the earth's surface is equal to $66\frac{2}{3}$ miles. Then al-Ṣaghānī renders what turns out to be al-Khāzin's values for the surface areas of the planets and stars in square miles. For example, the surface area of Saturn is stated to be 3,310,915,386 (square) miles. Because these numbers are huge but with many non-zero digits, it is possible to correct most of the scribal errors in the numbers and to reconstruct the computations in detail. As far as we know, no medieval Islamic author other than al-Khāzin dealt with surface areas of the stars and planets. We will also compare al-Khāzin's computations of the fixed stars of the first through sixth magnitudes to the assumptions on the diameters of these fixed stars that were made by al-Farghānī.

Al-Ṣaghānī's treatise on sizes and distances is not mentioned in medieval Islamic bibliographical works and one may wonder whether the manuscript we have represents an authentic treatise by him. In order to answer this question, we will finish Section 3 by comparing al-Ṣaghānī's treatise with references made by al-Bīrūnī to al-Ṣaghānī himself and to al-Khāzin. It will turn out that al-Ṣaghānī's treatise is probably authentic and that he preserved sections of al-Khāzin's computations of sizes and distances which have not come down otherwise.

Al-Ṣaghānī's treatise exists in a unique manuscript Damascus, Ẓāhiriyya 4871, ff. 78b-79b, written around A.H. 555 / 1160 CE. See for this manuscript [31], and for al-Ṣaghānī's treatise [31, p. 98]. As we will see below, the text in the manuscript is incomplete because a few passages are evidently missing.

In the English translation in Section 2, my own explanatory additions appear in square brackets. The numbers which are rendered in words in the Arabic manuscript (such as khamsīn, "fifty") have been rendered in the translation in Hindu-Arabic numerals in normal print (50). Fractions in words have also been rendered in numerals but the fractions have not been simplified. For example, the Arabic equivalent of "thirty-three and one-half and one-tenth" is rendered in the translation as $33\frac{1}{2} + \frac{1}{10}$ and not as $33\frac{3}{5}$. Some very large numbers appear in Hindu-Arabic numerals in the manuscript; these numbers have been rendered in boldface in the translation. In large numbers (ten thousand or more) I have put separation signs between the thousands, as in: "24,000", for sake of clarity. No such separation signs are found in the manuscript. Section 3 of this paper contains an analysis and recomputation of the numbers to the extent possible. In the translation in Section 2, the results of my analysis have been indicated as follows. In case the number in the manuscript is a clear scribal error, the reconstructed number is rendered in the translation, followed by the manuscript reading in square brackets, as in "107612 [MS. 107621]", where 107612 is the reconstruction and 107621 is the scribal error. A question mark after the number means that the number in the translation is exactly the manuscript reading, and that I have not been able to recompute the number in Section 3. This may be the result of a misunderstanding on my part or of an as yet unidentified scribal or computational error in the manuscript. If a number in the translation is not followed by a question mark or an alternative manuscript reading, the manuscript reading agrees with my recomputation in Section 3. In Section 3 and in the footnotes in Section 2 I have used standard sexagesimal notation such as $31;15^\circ$, meaning 31 degrees and 15 minutes, and $10;19^r$ to indicate $10 + \frac{19}{60}$ times the unit denoted as ^{r} (i.e., the earth-radius). A notation

such as 202;16,54 means $202 + \frac{16}{60} + \frac{54}{3600}$.

The appendix of this paper contains a critical edition of the Arabic text. For easy comprehension, the Arabic text has been divided into paragraphs, and some punctuation has been added. Words in pointed brackets $<\ >$ have been added by me to restore what I believe to be the original text. Words in square brackets [...] are in the Arabic manuscript but were crossed out by the scribe so they do not belong to the text. Numbers in square brackets refer to the critical apparatus at the end.

2. Translation

In the name of God, the Merciful, the Compassionate. In Him I take refuge.

Treatise by al-Ṣaghānī on the distances and the (volumes of the celestial) bodies.

The first chapter, on the presentation of the things which need to be established before the discussion of the distances and the bodies.

The second chapter, on the distances of the stars (and planets) from the center of the earth.

The third chapter, on the magnitude of the bodies.

The first chapter, on the presentation of things which need to be established before the discussion of the distances and the bodies. First we have to accept some things before we discuss the distances of the stars (and planets) and their bodies. In the first place, we assume that the order of the (celestial) spheres is as follows: The sphere of the moon is the celestial sphere nearest to us, then (that of) Mercury, then Venus, then the sun, then Mars, then Jupiter, then Saturn, then the fixed stars. The second notion is that we assume that the maximum distance of the moon is the minimum distance of Mercury, and that the maximum distance of Mercury is the minimum distance of Venus, and so on, until (we finally assume) that the maximum distance of Saturn is at (the distance of) the fixed stars. And the third notion is that the fixed stars are all on the same surface of the same (spherical) solid, so their distances (to the earth) are not different.

In the Book *Almagest*, Ptolemy only mentioned the distances and magnitudes of the bodies for the two luminaries (sun and moon). He explained the magnitude of their bodies and the magnitude of their distances from the center of the earth, and in this he made the comparison with the radius of the earth. The present book does not contain a long explanation of these subjects. I only say that Ptolemy first found, by means of two lunar eclipses, the magnitude of the radius of the moon compared to the radius of the parecliptic orb if it is assumed to be sixty parts; and with this comparison, he also found the diameter of the shadow where the moon passes around it, as compared to the magnitude of the radius of the earth. Then he derived from these two (data), the length of the (earth) shadow and the diameter of the sun.[1] The magnitude of the radius of the earth has already become available to us as we have mentioned previously.[2]

The second chapter, on the distances of the planets. As to how the distances of the planets can be known, we have already said that we have to assume that the maximum distance of the moon is the minimum distance of Mercury, and that the maximum distance of Mercury is the minimum distance of Venus, and so on until the fixed stars. As for the distance of the moon from the center of the earth, if the center of its epicycle is at the apogee (of the deferent) and the moon is at the apogee of the epicycle, then it is $64\frac{1}{6}$ times the radius of the earth, if the radius of the earth is one; and in what follows (what we mean) has to be understood in a similar way if we say: in comparison to the radius[3] of the earth. And if the epicycle is in the same situation but the moon is in the perigee of the epicycle, its distance from the earth is $53\frac{5}{6}$ times the radius of the earth. And if the center of the epicycle is in the perigee

[1] The ratio of the radius of the moon to the radius of the parecliptic orb (that is, a great circle parallel to the ecliptic with radius equal to the maximum lunar distance) can be interpreted as a measure of the apparent diameter of the moon when it is at maximum distance. In *Almagest* V, 14 [27, pp. 251-254], Ptolemy used two lunar eclipses to determined the apparent diameter of the moon at maximum distance, as well as the apparent diameter of the circle at which the shadow cone of the earth intersects the sphere with center the earth and radius the maximum lunar distance. In the subsequent chapter *Almagest* V,15 [27, pp. 255-257], Ptolemy found the length of the shadow cone of the earth as 268, and the distance of the sun as 1210, if the radius of the earth is 1. In *Almagest* V,16 [27, p. 257] he concluded that the diameter of the sun is 5.5 times the diameter of the earth.

[2] It seems that the present treatise by al-Ṣaghānī is an extract from a longer work, which has not come down to us in complete form.

[3] The manuscript has "diameter".

(of the deferent) and the moon is at the apogee (of the epicycle), then its distance to the earth is . . . (text missing) [4]

And first we say that if the situation of the magnitude of the sphere (i.e., the body) of the sun is compared to that of the sphere of earth, only (the following is possible): it can be greater than it, or equal to it, or less than it. If the sphere of the sun is less than the sphere of the earth, it is necessary that the shadow of the earth widens when it extends into the air, and increases indefinitely. As a consequence of this, the moon will be eclipsed in every month, and if the moon is further away of the earth, the duration of the total obscuration in the eclipse is longer. And if the sphere of the sun is equal to the sphere of the earth, it is a necessary consequence of this (case) and also in the first (case) that the stars (i.e., planets and fixed stars) are passing the shadow of the earth when they are in opposition to the sun, so eclipses would happen to them in an irregular way. But we do not find anything like this (i.e., eclipses of the planets), so the sphere of the sun is not less than (nor equal to) the sphere of the earth. Therefore it is greater than it.[5]

Thābit ibn Qurra mentioned that the diameter of the earth is 6415 miles, and that the minimum distance of the moon from the earth, compared to the radius of the earth, if it is one, is approximately $33\frac{1}{2} + \frac{1}{20}$ [MS. $33\frac{1}{2} + \frac{1}{10}$], that is 107,612 [MS. 107,621] miles. And the maximum distance of the moon, which is the minimum distance of Mercury, is $64\frac{1}{6}$ (times the radius of the earth), that is 205,815 miles. The maximum distance of Mercury, which is the minimum distance of Venus, is 166 times the radius of the earth, and it is 532,445 miles. The maximum distance of Venus, which is the minimum distance of the sun, is 1079 times (the radius of the earth), and it is 3,460,890 miles. The maximum

[4] In *Almagest* V, 17 [27, p.259], Ptolemy mentions the numbers 64;10 and 53;50 for maximum and minimum distance of the moon if the center of the epicycle is in the apogee of the deferent, as well as the corresponding distances for the center of the epicycle in the perigee: 43;53 (moon in the apogee of the epicycle) and 33;33 (moon in the perigee of the epicycle). All four numbers were probably found in al-Ṣaghānī's original and the abrupt ending of the passage shows that the extant manuscript is incomplete.

[5] This clumsy paragraph seems to be out of place in al-Ṣaghānī's technical account of Ptolemaic astronomy, and therefore it probably did not belong to the original. Perhaps it was a marginal addition by another more philosophically inclined author.

distance of the sun, which is the minimum distance of Mars, is 1260 times (the radius of the earth), and it is 4,041,450 miles. The maximum distance of Mars, which is the closest distance of Jupiter, is 8820 [MS. 8028] times the radius of the earth, and it is 28,290,150 miles. The maximum distance of Jupiter, which is the minimum distance of Saturn, is approximately $14188\frac{2}{3}$ times (the radius of the earth), and it is 45,510,241 miles. The maximum distance of Saturn, which is the minimum distance of the fixed stars, is approximately $19864\frac{1}{6}$ [MS. $9864\frac{1}{6}$] times (the radius of the earth), and it is 63,714,337 miles. The circumference of the greatest circle in which the fixed stars revolve is 403,200,000 miles, and the amount (in miles) of 1/360 part of this circle is 1,120,000 miles. So this is what Thābit ibn Qurra mentioned about the distances of these stars and planets from the center of the earth, if the radius of the earth is one.

And he said that he found by scrupulous consideration that the average speed of female camels in every time-degree of the revolution of the orb[6] is one thousand ells. So if the steps of the female camel are one thousand ells in 450 steps, that is, the step is close to two ells and a quarter, then the magnitude of the motion of the orb of the fixed stars is 2490 miles in the time interval of one step of a female camel.

The third chapter on the magnitudes of the bodies of the planets. Concerning the magnitudes of the bodies of these planets, Thābit mentioned that they are the following. The body of the sun is $166\frac{1}{4} + \frac{1}{8}$ times (the earth). The body of the moon is $1/39\frac{1}{4}$-th of the body of the earth. The body of Saturn is $79\frac{1}{2}$ times (the earth). The body of Jupiter is $81\frac{1}{2} + \frac{1}{4}$ times (the earth). The body of Mars is $1\frac{1}{2}$ times the earth. The body of Venus is $1/44$ of the body of the earth. The body of Mercury is $1/22,000$ (times the earth). Thābit did not mention anything about the bodies of the fixed stars.

The excellent master Abū Ja^{c}far Muḥammad ibn al-Ḥusayn, may God have mercy upon him, mentioned in a book by him, concerning

[6] A complete revolution of the orb is one complete (apparent) rotation of the heavens, which corresponds to one complete rotation of the earth. One time-degree is 1/360-th part of a complete revolution, i.e. almost 4 minutes of time in the modern view. Since the ell is close to 0,5 meters, Thābit's camels could walk 500 meters in 4 minutes, that is, 7.5 kilometers per hour.

what he computed himself, some things which I have to report here. I will mention what he said for all these planets and stars. He said: We have mentioned that the ancients found that a great circle on the sphere of the earth is 24,000 miles, where each mile is 3000 ells. If we divide by $3\frac{1}{7}$, the outcome is 7637 miles, and that is the (number of) miles of the diameter of the earth. So we take half of it, $3818\frac{1}{2}$, and that is the (number of) miles of the radius of the earth.

After this he derived by means of these things that the (distance from) the center of the earth to the outer limit of the sphere of fire, which is the place of the four elements: Earth, Water, Air and Fire, is 128,110 miles. This is the distance within which the rest of the existing perishable things are contained. He mentioned that the maximum distance of the moon, which is the minimum distance of Mercury, is 244,352 miles [MS. 244,052]. The maximum distance of Mercury is 633,788. The maximum distance of Venus, which is the minimum distance of the sun, is 4,119,564 (?) miles. The maximum distance of the sun is 4,811,310. The maximum distance of Mars is 33,675,760 [MS. 33,675,460]. The maximum distance of Jupiter is 54,165,960 miles. The maximum distance of Saturn, which is also the distance of the fixed stars, is 75,825,480 [MS. 75,825,430].

And because the center of the earth, which is the beginning (point) of these distances, is the center of the universe, the distance of each planet or star is the radius of its orb, which also has that (same) distance. The fixed stars are all in one orb, (namely) the sphere containing the ecliptical signs. So their distance, which is the maximal distance of Saturn, is the radius of the orb of the ecliptical signs. So we double the (distance in) miles of the fixed stars which we have obtained, and it amounts to 151,650,960, and this is the diameter of the orb of the zodiac in miles. So the length of the ecliptic in miles is 476,617,303 [MS. 476,617,003], and the measure of one degree (of the celestial sphere) is 1,323,937 miles. The surface area of the sphere of the circle of the ecliptic is 72,279,471,352,560,880 [MS. 72,279,471,552,560,880].

The diameter of the moon is 2246 miles.

The measurement of the outside of the[7] body of the moon is 15,854,514 (square) miles.

[7] The words "outside of the" are missing in the manuscript but have to be added for mathematical sense.

The outside of the body of Mercury is 217,238.
The outside of the body of Venus is 14,680,503 (?).
The outside of the body of the sun is 5,545,008,046
[MS. 5,112,145,008,586].
The outside of the body of Mars is 239,616,459.
The outside of the body of Jupiter is 3,468,554,535 (?).
The outside of the body of Saturn is 3,310,915,386.

The outside of the body of each of the fixed stars of the first magnitude is **3,794,759,584** [MS. **379,479,584**].
The outside of each of them of the second magnitude is **3,415,508,362**.
The outside of the (fixed stars) of the third magnitude is **2,932,852,384**
[MS. **293,285,384**],
and of the fourth magnitude **2,419,847,584** [MS. **2,419,848,284**],
and of the fifth magnitude **1,857,506,266**,
and of the sixth magnitude **1,161,003,320**.

This is the end of what we mention about the distances and the bodies. With God is our success.[8]

3. Commentary and reconstruction of the computations

In this section we will analyze and recompute the numbers in al-Ṣaghānī's treatise and relate them to other sources, to the extent possible. We first turn to the numbers which al-Ṣaghānī adopted from Thābit ibn Qurra.

For the diameter $2r$ of the earth, Thābit used $2r = 6415$. This corresponds to an earth-radius $r = 3207\frac{1}{2}$ miles and to a circumference of the earth of $\pi \cdot 6415 = 20160$ miles, using the Archimedean $\pi = 3\frac{1}{7}$. These values were based on a ninth-century determination of 1 degree on the surface of the earth as 56 miles, which was mentioned by al-Bīrūnī in the *Taḥdīd Nihāyāt al-Amākin*. Most Islamic astronomers seem to have used $2r \approx 6500$, corresponding to $1^o = 56\frac{2}{3}$ miles [7, p. 214], [6, p. 179], [11, pp. 30-31].

Because al-Ṣaghānī presents Thabit's planetary distances in earth radii as well as in miles, it is possible to identify scribal errors. Here is an example. According to the manuscript, the minimal lunar distance

[8] The manuscript ends with further religious formulas which may have been added by the scribe.

is $(33\frac{1}{2} + \frac{1}{10})r$ and 107621 miles. We note that $(33\frac{1}{2} + \frac{1}{10}) \times 3207\frac{1}{2} = 107,722$ which differs in only two of the six digits from the number 107,621 in the manuscript, so a scribal error is more likely than a computational error. Because the lunar distance $(33\frac{1}{2} + \frac{1}{10})r$ is not attested elsewhere in the literature, we will suppose that it is a scribal error of the value $(33\frac{1}{2} + \frac{1}{20})r$ which Ptolemy used in the *Almagest*. We have $33\frac{1}{2} + \frac{1}{20} \times 3207\frac{1}{2} = 107,611\frac{5}{8}$. Thus we will take the rounded value 107612 as Thābit's minimal distance of the moon, and we will explain 107,621 as an error made by a scribe who exchanged the last two digits.

For the maximum distance of the moon, Thābit also used the value $64\frac{1}{6}$ from Ptolemy's *Almagest*. For the maximum distances of Mercury, Venus, the sun, Mars, Jupiter and Saturn, Thābit, as quoted by al-Ṣaghānī, presented the values from Ptolemy's *Planetary Hypotheses*, namely $166r, 1079r, 1260r, 8820r, 14,188\frac{2}{3}r, 19,864\frac{1}{6}r$, respectively. Unlike Ptolemy, Thābit also used $1079r$ as the minimal distance of the sun.

We now recompute Thābit's maximum distances in miles.
Moon: $64\frac{1}{6} \times 3207\frac{1}{2} = 205,814\frac{7}{12}$ rounded to 205815 in the manuscript.
Mercury: $166 \times 3207\frac{1}{2} = 532,445$, as in the manuscript.
Venus: $1079 \times 3207\frac{1}{2} = 3,460,892\frac{1}{2}$, rounded to 3,460,890.
Sun: $1260 \times 3207\frac{1}{2} = 4,041,450$ as in the manuscript.
Mars: $8820 \times 3207\frac{1}{2} = 28,290,150$ as in the manuscript.

For Jupiter and Saturn, we have $14,188\frac{2}{3} \times 3207\frac{1}{2} = 45,510,148\frac{1}{3}$, and $19,864\frac{1}{6} \times 3,207\frac{1}{2} = 63,714,314\frac{7}{12}$, but the manuscript reads 45,510,241 and 63,714,339. Note that the distances of Jupiter and Saturn are stated to be "approximately" $14,188\frac{2}{3}r$ and $19,864\frac{1}{6}r$. Therefore the following explanation is plausible.

Ptolemy obtained the maximum distance of Jupiter $14,188\frac{2}{3}r$ as the approximate product of the maximum distance $8820r$ of Mars times the ratio (37:23) between maximum and minimum distances of Jupiter. The numbers in the manuscript show that Thābit used the same method for the distance in miles. For Jupiter we have $28,290,150$ miles $\times\ 37/23 = 45,510,241\frac{7}{23}$ miles, which can be rounded to the number 45,510,241 in the manuscript. For Saturn: if $45,510,241\frac{7}{23}$ is multiplied by the ratio (7:5) between the maximum and minimum distance of Saturn, the product is $63,714,337\frac{19}{23}$, rounded to $63,714,337$ in the manuscript.

Thābit (as quoted by al-Saghani) stated that the maximum distance

$19,864\frac{1}{6}$ of Saturn is equal to the minimum distance of the fixed stars, but nevertheless he used the minimum distance $20000r$ from the *Planetary Hypotheses* in his further computation. The circumference of the "great circle in which the stars revolve" (i.e., the celestial equator) is $2 \times 3\frac{1}{7} \times 20,000 \times 3207\frac{1}{2} = 403,200,000$ miles and one degree on the celestial equator is $\frac{1}{360} \times 403,200,000 = 1,120,000$ miles. The two numbers 403, 200, 000 (miswritten as 403, 020, 000) and 1, 120, 000 are also found in the *Treatise on Sizes and Distances* (*Kitāb fi'l-'abʿād wa'l-ajrām*) by al-Ṣaghānī's near-contemporary Abu'l-Ṣaqr al-Qabīṣī [29, fol. 94a:24-25].

We now turn to the question whether al-Ṣaghānī used Thābit's extant *Simplification of the Almagest* (*Tashīl al-Majisṭī*). In that introductory work, Thābit gave planetary distances in earth radii but he does not give the distance in miles, and he rounded the distances for Jupiter and Saturn to $14,187r$ and $19,865r$ [22, p. 15], corrupted to 17865 in the Latin tradition [9, p. 137]. In the *Simplification of the Almagest*, Thābit also stated that the volume of fixed stars of the first magnitude is 94 times the volume of the earth [22, p. 13], whereas al-Ṣaghānī says that "Thābit did not mention anything about the bodies of the fixed stars". I conclude that al-Ṣaghānī did not use Thābit's *Simplification of the Almagest* but one of his lost astronomical works listed in [22, p. xiv].

The volumes of the moon and planets which al-Ṣaghānī attributes to Thābit (Th-Ṣagh in the following table) are also close but not identical to the values in Thābits *Simplified Almagest* (Thabit-SA). Al-Ṣaghānī seems to have preserved more "accurate" values given by Thābit. The following table displays the volumes in Thābit's lost astronomical work preserved by al-Ṣaghānī (Th-Ṣagh), in the *Simplified Almagest* (Th-SA), and in Ptolemy's *Planetary Hypotheses* and al-Farghānī's *Summary of Astronomy*. The unit is always the volume of the earth.

Body	Th-Ṣagh	Th-SA	Plan. Hyp.	Al-Fargh.
Sun	$166\frac{1}{4} + \frac{1}{8}$	166	$166\frac{1}{3}$	$166\frac{1}{4} + \frac{1}{8}$
Moon	$1/39\frac{1}{4}$	$1/40$	$1/40$	$1/39$
Saturn	$79\frac{1}{2}$	79	$79\frac{1}{2}$	91
Jupiter	$81\frac{1}{2} + \frac{1}{4}$	82	$81\frac{1}{2} + \frac{1}{4} + \frac{1}{20}$	95
Mars	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2} + \frac{1}{8}$
Venus	$1/44$	$1/37$	$1/44$	$1/37$
Mercury	$1/22,000$	$1/19,683$	$1/19,683$	$1/22,000$

We now turn to the numbers which al-Ṣaghānī took from Abū Jaʿfar Muḥammad ibn al-Ḥusayn al-Khāzin.

Al-Khāzin took the circumference of the earth as 24,000 miles, on the basis of Ptolemy's value of 180,000 stades and the conversion factor 1 mile = 7.5 stades.

Using the Archimedean value $\pi = \frac{22}{7}$, the diameter of the earth is $2r = 24,000/\pi = 7636\frac{4}{11}$ miles, which al-Khāzin rounded to 7637 miles. Then $r = \frac{1}{2} \cdot 7637 = 3818\frac{1}{2}$ miles, but as we will see below, al-Khāzin often used the rounded value $r = 3818$.

Al-Khāzin presented the distance of the planets in miles only, but dividing by r it is easy to see that he used the following values for the outer limit of the sphere of the four elements (i.e., the minimum distance of the moon) and the maximum distances of the moon, Mercury, Venus, the sun, Mars, Jupiter, and Saturn: $(33 + \frac{1}{2} + \frac{1}{20})r$, $64r$, $166r$, $1079r$, $1260r$, $8820r$, $14,187r$, $19,860r$ respectively. The minimum value for the moon is from the *Almagest*, and the other values were taken from the *Planetary Hypotheses*, except the value for Saturn which was rounded from $19865r$ in the *Planetary Hypotheses*.

Most of al-Khāzin's computations of the distances in miles can be reconstructed in detail:

Minimum distance of the moon: $(33+\frac{1}{2}+\frac{1}{20})\times 3818\frac{1}{2} = 128,110\frac{27}{40}$, rounded to 128,110 in the manuscript.

Maximum distance of the moon: $64 \times 3818 = 244,352$; the manuscript has 244,052 by scribal error, which can be explained by the omission of one word if the number was written out in words.

Maximum distance of Mercury: $166 \times 3818 = 633,788$ as in the manuscript.

Maximum distance of Venus: $1079 \times 3818 = 4,119,622$ but the manuscript has 4, 119, 564. I am unable to explain the difference.

Maximum distance of the sun: $1260 \times 3818\frac{1}{2} = 4,811,310$ as in the manuscript.

Maximum distance of Mars: $8820 \times 3818 = 33,674,760$. The manuscript has 33,675,460, probably by scribal error.

Maximum distance of Jupiter: $14187 \times 3818 = 54,165,966$. The manuscript has the rounded value 54,165,960.

Maximum distance of Saturn, which is also the distance of the fixed stars and the radius of the ecliptic: $19,860 \times 3818 = 75,825,480$; the manuscript has the scribal error 75,825,430.

The diameter of the ecliptic is $2 \times 75,825,480 = 151,650,960$, which appears correctly in in the manuscript.

Based on the Archimedean value $\pi = 3\frac{1}{7}$, the circumference of the ecliptic is $\pi \times 151,650,960 = 476,617,302\frac{6}{7}$ rounded to 476,617,303. The manuscript has 476,617,003 by scribal error.

The length of an arc of one degree of the ecliptic is derived from the reconstructed number: $476,617,303/360 = 1,323,936\frac{343}{360}$ rounded to 1,323,937 in the manuscript.

The surface area of the sphere of the ecliptic is the product of the (rounded) circumference times the diameter ($4\pi r^2 = 2\pi r \cdot 2r$), that is $476,617,303 \times 151,650,960 = 72,279,471,352,560,880$. The number in the manuscript is 72,279,471,552,560,880, which differs by only one digit in the middle and must therefore be a scribal (not a computational) error.

In the following attempt to reconstruct al-Khāzin's computation of the surface areas of the celestial bodies, I begin with the fixed stars, which he seems to have treated by a uniform method. The surface areas of the fixed stars are huge numbers with many different digits, so the reconstruction is reasonably certain, as we will see below.

In the *Planetary Hypotheses*, Ptolemy only mentioned stars of the first magnitude, and for the stars of the second through sixth magnitude, al-Khāzin was probably inspired by al-Farghānī's *Summary of Astronomy*. Al-Farghānī argued, in a way which does not concern us here, that the volume of each star of the first magnitude is approximately 107 times the volume of the earth. He also supposed that the volumes of the stars of first through sixth magnitude are in the ratio 6 : 5 : 4 : 3 : 2 : 1 respectively, so the volume of a star of the second magnitude is $\frac{5}{6}$ times the volume of a star of the first magnitude, and so on. He rounded the volume 107 to the closest multiple of 6, that is 108, and found the volumes of the stars of second through sixth magnitudes as integer multiples (90, 72, 54, 36, 18) of the volume of the earth [11, pp. 84-85].

Here is my reconstruction of al-Khāzin's computation. From the *Planetary Hypothesis* he must have taken the diameter of stars of the first magnitude as $4 + \frac{1}{2} + \frac{1}{20}$ (that is, 4;33) times the diameter of the earth, and the volume of stars of the first magnitude as ($4;33^3 \approx$) 97 times the volume of the earth. In the same vein as al-Farghānī, he rounded 97 to 96, which is an integer multiple of 6, and he then found the volumes of the stars of the second through sixth magnitude as 80, 64, 48, 32, and 16 times the volume of the earth. Hence the diameters of these stars are, in terms of the diameter of the earth, $\sqrt[3]{80} = 4;18,32,\ldots$, $\sqrt[3]{4 \cdot 16} = 4$, $\sqrt[3]{48} = 3;38,3,\ldots$, $\sqrt[3]{32} = 3;10,29,\ldots$, $\sqrt[3]{16} = 2;31,11,\ldots$. He must have rounded these five values to $p = 4;19$, $p = 4$, $p = 3;38$, $p = 3;11$, $p = 2;31$ respectively.

The steps in al-Khāzin's reconstructed computation are presented in the two tables below. The rounded ratios p between diameters of the fixed stars and the diameter of the earth are displayed in the second column of the first table. The third column contains the diameter of the star in miles, computed as $2pr = 7637p$, where $2r = 7637$ is al-Khāzin's value for the diameter of the earth in miles, and p the number in the second column. In the fourth column, this product is rounded to an integer number d of miles. In the fifth column, the circumference of the star is computed as πd with $\pi = \frac{22}{7}$. In the first column of the second table, the number πd is rounded to an integer number of c miles. The surface area s is obtained in the third column of the second table as the product $d \times c$. The fourth colum in the second table displays the number in the manuscript.

star	factor		diameter	
	p	$2pr$	d	πd
first	4;33	34,748;21	34,748	109,208
second	4;19	32,966;23	32,966	$103,607\frac{3}{7}$
third	4	30,548	30,548	96,008
fourth	3;38	27,747;46	27,748	87,208
fifth	3;11	24,311;07	24,311	76,406
sixth	2;31	19,219;47	19,220	$60,405\frac{5}{7}$

star	circumference	surface area	number in ms.
	c	s	
first	109,208	3,794,759,584	379,479,584
second	103,607	3,415,508,362	3,415,508,362
third	96,008	2,932,852,384	293,285,384
fourth	87,208	2,419,847,584	2,419,848,284
fifth	76,406	1,857,506,266	1,857,506,266
sixth	60,406	1,161,003,320	1,161,003,320

For the stars of the second, fifth and sixth magnitude, the recomputed surface areas in square miles agree exactly with the numbers in the manuscript. The numbers 379,479,584 for stars of the first magnitude and 293,285,384 for stars of the third magnitude can be explained as scribal errors made by a scribe who left out one digit of the correct numbers. The number 2,419,848,284 for stars of the fourth magnitude differs only in the third and the fourth digit, perhaps as the result of a small error made in the multiplication.

Now the question arises to what extent we can be sure that al-Khāzin really computed this way. Take the stars of the second magnitude as an example and let $s = 3,415,508,632$ be the number in the manuscript. Then we have $s = \pi d^2$ for some diameter d of the star and $\pi = 3\frac{1}{7}$. Solving for d we obtain $d = \sqrt{\frac{s}{\pi}} = 32,965.93\ldots$. Since it is likely that an integer number was used, we can suppose $d = 32,966$. In the *Planetary Hypotheses*, Ptolemy computes $d = f \cdot 2r$ where f is a "nice" factor and $2r$ the diameter of the earth, which al-Khāzin took as 7637. Solving for f we obtain $f = 4.316616\ldots$ or in sexagesimals, $f = 4\frac{18.997\ldots}{60}$, so we can be sure that al-Khāzin used $f = 4;19$. From these data, the computation can be reconstructed. The factor $f \approx \sqrt[3]{80}$ can be explained as above.

By means of this method I have also tried to reconstruct al-Khāzin's computations of the surface areas of the sun, moon and planets. The results are displayed in the two tables below. The notations are the same as in the tables for the computations for the fixed stars, with the difference that the numbers are not always rounded.

body	factor		diameter	
	p	pr	d	πd
Moon	5/17	$2246\frac{3}{17}$	2246	$7058\frac{6}{7}$
Mercury	1/29	$263\frac{10}{29}$	263	$826\frac{4}{7}$
Venus	0;17	2163;49	2163;49	6800; 34
Sun	5;30	$42,003\frac{1}{2}$	$42,003\frac{1}{2}$	132, 011
Mars	8/7	8728	8728	$27,430\frac{6}{7}$
Jupiter	4;21	33,220;57	33,220;57	104, 408; 42
Saturn	4;15	$32,457\frac{1}{4}$	$32,457\frac{1}{4}$	$102,008\frac{1}{2}$

body	circumference	surface area	number in ms.
	c	s	
Moon	7059	15,854,514	15,854,514
Mercury	826	217,238	217,238
Venus	6800;34	14,715,179;29,46	14,680,503
Sun	132,011	$5,545,008,045\frac{1}{2}$	5,112,145,008,586
Mars	27,430;51	239,416,458;48	239,616,459
Jupiter	104,408;42	3,468,556,202;16,54	3,468,554,535
Saturn	$102,008\frac{1}{2}$	$3,310,915,386\frac{5}{8}$	3,310,915,386

The value $p = 5/17$ for the moon is my reconstruction. The other values for p are also attested in Ptolemy's *Planetary Hypotheses*. For the moon and Mercury, the recomputed surface areas agree with the manuscript.

The surface area of the sun appears in the manuscript as the number 5,112,145,008,586, which is unreasonably large and must therefore be corrupted. I note that its successive erroneous digits 1,1,2,1 add up to the second digit 5 in the recomputed number, so the number in the manuscript may have been caused by al-Khāzin who miscopied it from his own computation. I am unable to explain the difference between the final digits 046 in the recomputed (and correctly rounded) number and 586 in the manuscript. For Mars, my recomputation produces the number in the manuscript (with one trivial scribal error) if we assume that al-Khāzin rounded πd not to an integer but to the first sexagesimal digit.

For Saturn, al-Khāzin seems not to have rounded pr and πd at all.[9]

For Venus and Jupiter, al-Khāzin must have assumed $p = (\frac{1}{4} + \frac{1}{30})$

[9] To put these errors in perspective, one may compare them to al-Qabīṣī's erroneous computation of the surface area of the sphere of the fixed stars. Al-Qabīṣī computed the circumference of the sphere of fixed stars in miles, just like Thābit (as quoted by al-Ṣaghānī), but instead of the correct result 403, 200, 000 miles, al-Qabīṣī wrote 403, 020, 000 miles. He continued the computation with this wrong number and found the surface area as the product of circumference times diameter: $403,020,000 \times 128,291,000 = 5,190,303,882,000,000$ square miles [29, fol. 94a:29]. The same number is written as 5, 170, 303, 882, 000, 000 in [29, fol. 88b:10]. A correct multiplication with the erroneous number would have produced $403,020,000 \times 128,291,000 = 51,703,838,820,000,000$ and the correct computation with the correct number would have resulted in $403,200,000 \times 128,291,000 = 51,726,931,200,000,000$. The example of al-Qabīṣī shows that scribal and computational errors in large numbers are likely.

and $p = 4\frac{1}{4} + \frac{1}{10} = 4;21$ as in Ptolemy's *Planetary Hypotheses*. In the table I have presented the recomputations as if al-Khāzin did not round his intermediate results. The results are somewhat different from the numbers in the manuscript. Thus it is an open question how exactly al-Khāzin rounded the intermediate steps in his computations for Venus and Jupiter.

We note that al-Khāzin could have made his computations in an easier way. Starting from the value 24000 for the circumference of the earth in miles, the circumference of the planets can be computed as $24,000p$ miles. For the sun and Saturn, this method would have produced 'nice' circumferences of 132,000 and 102,000 miles, and corresponding nice surface areas ending in zeroes. The surface areas in the manuscript cannot have been computed by this easy method.

My reconstructed computation of al-Khāzin agrees to a large extent (but not completely) with a passage in the Qānūn al-Masʿūdī, where al-Bīrūnī says:

"As for the fixed stars, Ptolemy only mentioned stars of the first magnitude, and made them equal to Mars by putting their diameters equal to one-twentieth of the diameter of the sun. Abū Jaʿfar al-Khāzin mentioned in his book on the sizes and distances that their diameters are as follows: for the first magnitude, one part of seventeen (i.e. 1/17) of the diameter of the sun, for the second magnitude, one part of twenty and one-quarter; for the third magnitude, one part of twenty-one and four-fifths; for the fourth magnitude, one part of twenty-four; for the fifth magnitude, one part of twenty-seven and half, and for the sixth magnitude, one part of thirty-six. Then he did not attribute this to himself nor to anyone else, and he did not indicate the way he had derived it or discovered it" (My translation from [8, p. 1312]). Al-Bīrūnī then used these apparent diameters in the tables of sizes of the fixed stars in his *Introduction to Astrology* [5, pp. 115-118].

On the basis of Ptolemy's *Planetary Hypotheses*, we can reconstruct four of these apparent diameters from the p in my computation, which is the diameter of the star divided by the diameter of the earth. Assume with Ptolemy that the diameter of the sun is $5\frac{1}{2}$ times the diameter of the earth, then the real diameter of the star is $\frac{p}{5.5}$ times the real diameter of the sun. Now take for the distance of the stars and of the sun the values $20,000r$ and $1260r$, then the apparent diameter a of the star is $\frac{p}{5.5} \cdot \frac{1260}{20,000}$ times the apparent diameter of the sun. For stars of the second through fifth magnitudes, we have $p = 4;19$, $p = 4$, $p = 3;38$, $p = 3;11$ respectively so we obtain $a = 1/20.22\ldots$, $a = 1/21.82\ldots$, $a = 1/24.03\ldots$, $a = 1/27.42\ldots$,, which can be rounded to the quantities $1/20\frac{1}{4}, 1/21\frac{4}{5}, 1/24$, and $1/27\frac{1}{2}$ in the passage in the Qānūn al-Masʿūdī. For the first and sixth magnitude we obtain $a = 1/19.18\ldots$ and $a = 1/34.68\ldots$ so the agreement with $1/17$ and $1/36$ is not so good.

Finally we turn to the question whether al-Ṣaghānī's treatise which we have published in this paper is an authentic work, and whether his information on Thābit and al-Khāzin is correct. We have seen above that al-Ṣaghānī's reports about Thābit ibn Qurra are based on the assumption that one degree on the surface of the earth corresponds to 56 miles. This value is not very common among Islamic astronomers but al-Bīrūnī says that "the same was reported by Abū Ḥāmid al-Ṣaghānī on the authority of Thābit ibn Qurra" [7, p. 214] [6, p. 179]. The agreement is a strong argument for the authenticity of al-Ṣaghānī's text, and it is likely that al-Bīrūnī had seen the complete version of it. Al-Bīrūnī's quotation of al-Khāzin shows that al-Khāzin had his own theory about fixed stars of the second through sixth magnitude, and the quotation is in reasonable agreement with the data about al-Khāzin as reported by al-Ṣaghānī. Thus al-Ṣaghānī's treatise is an authentic work, and his information about Thābit and al-Khāzin must be reasonably accurate.

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Appendix: Arabic text

بسم الله الرحمن الرحيم وبه استعين
مقالة للصغاني في الأبعاد والأجرام
الباب الأول في تقديم الأشياء يجب الإقرار بها قبل الكلام في الأبعاد والأجرام
الباب الثاني في أبعاد الكواكب من مركز الأرض
الباب الثالث في كميّة الأجرام

الباب الأول في تقديم أشياء يجب الإقرار بها قبل الكلام في الأبعاد والأجرام. أمّا أوّلًا فيجب أن نتسلم أشياء قبل أن نتكلّم في أبعاد الكواكب وأجرامها. أمّا أوّلًا فإن نفرض أن مراتب الأكر هي أن فلك القمر أقرب الأفلاك إلينا ثم عطارد ثم الزهرة ثم الشمس ثم المريخ ثم المشتري ثم زحل ثم الكواكب الثابتة. والمعنى الثاني أن نفرض أن أبعد بعد القمر هو أقرب قرب عطارد وأن أبعد بعد عطارد هو أقرب قرب الزهرة ثم كذلك حتى أن أبعد بعد زحل هو عند الكواكب الثابتة. والمعنى الثالث أن الكواكب الثابتة كلّها في سطح واحد من جسم واحد حتى لا يكون أبعادها مختلفة.

أمّا بطلميوس فلم يذكر في كتاب المجسطى من أبعاد الكواكب ومقادير أجرامها إلّا في النيّرين فقط فإنّه بيّن مقدار جرم كل واحد منهما ومقدار أبعادهما [١] من مركز الأرض وجعل القياس في ذلك نصف قطر الأرض. فأمّا هذا الكتاب فلا يحتمل الشرح الطويل في هذه الأحوال إلّا أنّي أقول إن بطلميوس وجد أوّلًا مقدار نصف قطر القمر بكسوفين قمريين بالقياس إلى نصف قطر [الأرض] الفلك المثّل إذا كان ستين جزءًا ووجد بهذا القياس أيضًا قطر الظلّ يمرّ حولها إلى مقدار نصف قطر الأرض ثم استخرج بهما طول الظلّ وقطر الشمس وقد حصل لنا مقدار

نصف قطر الأرض كما ذكرنا قبل.

الباب الثاني في أبعاد الكواكب. أمّا كيف يعلم أبعاد الكواكب فقد قلنا أنه يجب أن نتسلم أن أبعد بعد القمر هو أدنى بعد عطارد وأبعد بعد عطارد أدنى بعد الزهرة وكذلك إلى الكواكب الثابتة. أمّا بعد القمر من مركز الأرض إذا كان مركز فلك تدويره في [٢] البعد الأبعد والقمر في بعده الأبعد من فلك التدوير يكون أربعة وستين وسدس مثل نصف قطر الأرض إذا كان < نصف > قطر الأرض واحدًا وكذلك يجب أن يفهم بعد ذلك إذا قلنا بالقياس إلى < نصف > قطر الأرض. وإذا كان فلك التدوير في هذه الحال والقمر في بعده الأقرب من فلك التدوير يكون [٣] بعده من الأرض ثلاثة وخمسين مرة مثل < نصف > قطر الأرض وخمسة أسداسه. وإذا كان مركز فلك التدوير في البعد الأقرب والقمر في بعده الأبعد من فلك التدوير يكون بعده من مركز الأرض ....

ونقول أوّلًا أن حال كرة الشمس عند كرة الأرض في العظم لا يخلوا إمّا أن يكون أكبر منها أو مثلها أو أصغر منها.

فإن كانت كرة الشمس أصغر من كرة الأرض لزم أن يكون ظلّ الأرض كلما ارتفع في الهواء اتّسع [٤] ومرّ بلا نهاية فيلزم من ذلك أن يكون القمر ينكسف في كل شهر ويكون كسوفه كلما كان أبعد من الأرض زمان مكثه أكثر. وإن كانت كرة الشمس مثل كرة الأرض لزم من ذلك ومن الأوّل أن يكون الكواكب إذا كانت في مقابلة الشمس أن يمرّ في ظلّ الأرض ويعرض لها كسوفات على غير نظام. ولسنا نجد شيئًا من ذلك فليس كرة الشمس بأصغر من كرة الأرض فهي أعظم منها.

وأمّا ثابت بن قرة فقد ذكر أن قطر الأرض ستة ألف وأربعمائة وخمسة عشر ميلًا وأن أدنى بعد القمر من الأرض بالقياس إلى نصف قطر الأرض إذا كان واحدًا ثلاثة وثلاثون مرة ونصف و < نصف > عشر

بالتقريب يكون بالأميال مائة ألف وسبعة ألف ميل وستمائة واثنى عشر [٥] ميلًا. وأبعد بعد القمر الذي هو أدنى بعد عطارد أربعة وستين مرة وسدس يكون بالأميال مائتى ألف وخمسة ألف ميل وثمان مائة وخمسة عشر ميلًا. وأبعد بعد عطارد الذي هو أدنى بعد الزهرة مائة وستة وستون مرة مثل < نصف > قطر الأرض ومن الأميال خمسة مائة ألف واثنين وثلاثين ألفًا وأربعمائة وخمسة وأربعين ميلًا. وأبعد بعد الزهرة الذي هو أدنى بعد الشمس ألف وتسعة وسبعين مرة يكون بالأميال ثلاثة ألف ألف وأربعمائة ألف وستين ألفًا وثمان مائة وتسعين ميلًا. وأبعد بعد الشمس الذي هو أدنى بعد المرّيخ ألف ومائتى وستين مرة يكون بالأميال أربع ألف ألف [ص ٧٩ ا] وأحد وأربعين ألفًا وأربعمائة وخمسين ميلًا. وأبعد بعد المرّيخ الذي هو أدنى بعد المشتري ثمانية ألف وثمان < مائة > وعشرين مرة يكون بالأميال ثمانية وعشرين ألف ألف ومائتى وتسعين ألفًا ومائة وخمسين ميلًا. وأبعد بعد المشتري الذي هو أدنى بعد زحل أربعة عشر ألفًا ومائة وثمان وثمانين مرة وثلثي مرة بالتقريب يكون بالأميال خمسة وأربعين ألف ألف [مرة وثلثى] وخمسمائة ألف وعشرة ألف ومائتى وأحد وأربعين ميلًا. وأبعد بعد زحل الذي هو أدنى بعد الكواكب الثابتة تسعة < عشر > ألفًا [٦] وثمان مائة وأربعة وستون مرة وسدس بالتقريب يكون بالأميال ثلاثة وستون ألف ألف وسبعة مائة ألف وأربعة عشر ألفًا وثلاثمائة وسبعة وثلاثين ميلًا.

ودور أعظم دائرة يدور فيها الكواكب الثابتة أربعمائة ألف ألف وثلاثة ألف ألف ومائتى ألف ميل وحصة جزء من ثلاثمائة وستين جزءًا من هذه الدائرة ألف ألف ومائة ألف وعشرين ألف ميل فهذا ما ذكره ثابت بن قرة في ابعاد هذه الكواكب من مركز الأرض إذا كان < نصف > قطر الأرض واحدًا.

وذكر أنّه وجد بالمحنة أن أوسط سير الجمّازات يكون في كل جزء

من مدار الفلك ألف ذراع فإذا كان خطوات [٧] الجمازة ألف ذراع بأربع مائة وخمسين خطوة على أن الخطوة قريب من ذراعين وربع كان مقدار ما يتحرك فلك الكواكب الثابتة في وقت خطوة واحدة من خطى الجمّازة ألفي وأربعمائة وتسعين ميلًا.

الباب الثالث في كميّة أجرام الكواكب. فأمّا مقادير أجرام هذه الكواكب فقد ذكر ثابت على هذا المقدار. إن جرم الشمس مائة وست وستين مرة وربع وثمن مرة وإن جرم القمر من جرم الأرض جزءٌ من تسعة وثلاثين جزًا وربع. جرم زحل مثل الأرض تسعًا وسبعين مرة ونصف. جرم المشتري أحد وثمانين مرة ونصف وربع مرة. جرم المريخ مثل الأرض مرة ونصف. جرم الزهرة جزء من أربعة وأربعين جزءًا من جرم الأرض. جرم عطارد جزء من اثني وعشرين ألف جزء ولم يذكر ثابت من أجرام الكواكب الثابتة شيئًا.

فأمّا الشيخ الفاضل أبو جعفر محمّد بن الحسين رحمة الله عليه فقد ذكر في كتابٍ له بحسب ما حسب هو ما يجب أن أحكيه هاهنا وأذكر ما قاله في كلّ واحد من هذه الكواكب قال

وقد كنّا ذكرنا أن القدماء وجدوا أوسع دائرة يقع على كرة الأرض أربعة وعشرين ألف ميل كل ميل ثلاثة ألف ذراع فإذا قسمنا على ثلاثة وسبع خرج سبعة ألف وستمائة وسبعة وثلاثين وهي أميال قطر الأرض فنأخذ نصفها وهو ثلاثة ألف وثمان مائة وثمانية عشر ونصف فهي أميال نصف قطر الأرض.

فأخرج بعد ذلك بهذه الأشياء أن مركز الأرض إلى أقصى كرة النار التي هي مكان الأربعة الأركان الأرض والماء والهواء والنار مائة ألف ميل وثمانية وعشرين ألف ميل ومائة وعشرة أميال وهي المسافة التي يكون فيها سائر الأشياء الكائنة الفاسدة. وذكر أن أبعد بعد القمر الذي هو أدنى

بعد عطارد مائتا ألف وأربعة وأربعون ألفًا < وثلاثمائة > واثنان وخمسون ميلًا. وأبعد بعد عطارد ستمائة ألف وثلاثة وثلاثين ألفًا وسبعمائة وثمانية وثمنون ميلًا. وأبعد بعد الزهرة وهو أدنى بعد الشمس أربعة ألف ألف ومائة وتسعة عشر ألف وخمسمائة وأربعة وستون ميلًا. وأبعد بعد الشمس أربعة ألف ألف وثمان مائة ألف وأحد عشر ألفًا وثلاثمائة وعشرة. وأبعد بعد المرّيخ ثلاثة وثلاثون ألف ألف وستمائة وخمسة وسبعون ألفًا وسبعمائة [٨] وستون. وأبعد بعد المشتري أربع وخمسون ألف ألف ومائة وخمسة وستون ألفًا وتسعمائة وستون ميلًا وأبعد بعد زحل الذي هو أيضًا بعد الكواكب الثابتة خمسة وسبعون ألف ألف وثمان مائة ألف وخمسة وعشرين ألفًا وأربعمائة وثمانون [٩].

ولأن مركز الأرض الذي منه ابتداء هذه الأبعاد هو مركز الكل فبعد كل كوكب من الكواكب هو نصف قطر فلكه الذي له ذلك البعد. فأمّا الكواكب الثابتة فإنّها كلّها في فلك واحد الكرة التي فيها البروج فبعدها الذي هو أبعد بعد زحل هو نصف قطر فلك البروج فنضعف ما خرج لنا من أميال بعد الكواكب الثابتة فيبلغ مائة ألف ألف وأحد وخمسين ألف ألف وستمائة وخمسين ألفًا وتسعمائة وستين وهي أميال قطر فلك البروج يكون مسافة منطقة البروج بالأميال أربعمائة ألف ألف وستة وسبعين ألف ألف وستمائة وسبعة عشر ألفًا < وثلاثمائة > وثلاثة يكون مساحة درجة واحدة ألف ألف وثلاثمائة ألف [١٠] وثلاثة وعشرين ألفًا وتسعمائة وسبعة وثلاثين ميلًا. ويكون مساحة بسيط كرة دائرة البروج اثنين وسبعين ألف ألف ألف ألف وثلاثمائة ألف ألف ألف ألف ألف وتسعة وسبعين ألف ألف ألف ألف وأربعمائة ألف ألف ألف وأحد وسبعين ألف ألف ألف وثلاثمائة [١١] ألف ألف واثنين وخمسين ألف ألف وخمسمائة ألف وستين ألفًا وثمان مائة وثمانين

أميال قطر القمر ألفان ومائتان وستة وأربعون. مساحة < ظاهر > جرم القمر خمسة عشر ألف ألف وثمان مائة وخمسون ألفًا وأربعة ألف

وخمسمائة وأربعة عشر ميلًا. ظاهر جرم عطارد مائتى ألف وسبعة عشر [ص ٧٩ ب] ألفًا ومائتى وثمانية وثلاثين . ظاهر جرم الزهرة أربع عشر ألف ألف وستمائة وثمنون ألفًا وخمسمائة وثلاثة. ظاهر جرم الشمس خمسة ألف ألف ألف ومائة ألف ألف ألف واثنا عشر ألف ألف ألف ومائة وخمسة وأربعين ألف ألف وثمانية ألف وخمسمائة وستة وثمانون. ظاهر جرم المرّيخ مائتى ألف ألف وتسعة وثلاثين ألف ألف وستمائة ألف وستة عشر ألفًا وأربعمائة وتسعة وخمسون. ظاهر جرم المشتري ثلاثة ألف ألف ألف وأربعمائة ألف ألف وثمانية وستين ألف ألف وخمسمائة ألف وأربعة وخمسون ألفًا وخمسمائة وخمسة وثلاثون ظاهر جرم زحل ثلاثة ألف ألف ألف وثلاثمائة ألف ألف وعشرة ألف ألف وتسعمائة ألف وخمسة عشر ألفًا وثلاثمائة وستة وثمانين

وظاهر جرم كل واحد من الكواكب الثابتة التي في العظم الأوّل ٣٧٩٤٧٥٩٥٨٤ [١٢]

وظاهر كل واحد ممّا في العظم الثاني ٣٤١٥٥٠٨٣٦٢

وظاهر ما في العظم الثالث ٢٩٣٢٨٥٢٣٨٤ [١٣]

وما في العظم الرابع ٢٤١٩٨٤٧٥٨٤ [١٤]

وما في العظم الخامس ١٨٥٧٥٠٦٢٦٦

وما في العظم السادس ١١٦١٠٠٣٣٢٠

وهذا آخر ما نذكره في الأبعاد والأجرام. وبالله التوفيق.

[١] أبعادها [٢] من [٣] تكون [٤] تسع [٥] واحد وعشرون [٦] الف [٧] خطوط [٨] وأربعمائة [٩] وبلثون. [١٠] الف الف [١١] وخمسمائة [١٢] ٣٧٩٤٧٩٥٨٤ [١٣] ٢٩٣٢٨٥٣٨٤ [١٤] ٢٤١٩٨٤٨٢٨٤