# Al-Nayrīzī's Influence in the West

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## 1 Introduction

Abū'l-ᶜAbbās al-Faḍl ibn Ḥātim al-Nayrīzī was a mathematician and astronomer who originated from the city of Nayrīz in Iran and who worked in Baghdād.[1] Since he wrote a treatise for Caliph al-Muᶜtaḍid, who reigned from 279 H./A.D. 892 to 289 H./ A.D. 902, he must have lived around 287 H./A.D. 900. Following Suter [15, p. 45], many authors have stated that al-Nayrīzī died in or around the year 310 H./A.D. 922/3. I have not found evidence for this date in medieval sources, and I suspect that the date 310/922-3 is an invention of Suter.[2]

Al-Nayrīzī had an influence on the development of mathematics in medieval Europe because his commentary to the *Elements* of Euclid was translated into Latin in the 12th century A.D. In this paper I will sketch the influence of Al-Nayrīzī in medieval Europe and on the modern historiography of Greek mathematics.

## 2 Al-Nayrīzī's Commentary on Euclid's Elements

The most important Greek mathematical work is the *Elements* of Euclid, written around 300 B.C. and consisting of 13 "books" or large chapters [9]. In this work Euclid presents elementary plane geometry, proportions, numbers,

[1] On al-Nayrīzī see [8, vol. 5, pp. 283-285; vol. 6, pp. 191-192; vol. 7, pp. 156, 268], [7, vol. 10, pp. 5-7], [14, pp. 513-518]

[2] In his German translation of a chapter of the *Fihrist* [16, p. 67], Suter says: "Weitere Angaben über sein Leben habe ich nicht gefunden, da er aber für al-Muᶜtaḍid ein Werk verfasst hat, so muss er ums Jahr 900 zur Zeit Ṯābits gelebt haben." [I have not found further indications on his life, but since he wrote a work for al-Muᶜtaḍid, he must have lived around the year 900, at the time of Thābit.]

irrational magnitudes, and stereometry on a systematic basis. He begins with definitions, axioms, and postulates, and he then presents theorems and constructions in logical order. His work had a tremendous influence on the history of mathematics until the ninteenth century A.D., even though Euclid wrote the *Elements* as a textbook for his mathematical colleagues, not as a work for students who have to learn geometry. Some of the topics in the *Elements* are difficult or obscure.

Already in antiquity, commentaries on the *Elements* appeared. The *Elements* were translated into Arabic in the ninth century A.D./ second and third centuries of the Hijra, and al-Nayrīzī was one of the first medieval Islamic mathematicians who wrote a commentary on (at least) the first ten books of the work. Most of al-Nayrīzī's commentary on the first six books of the *Elements* survives in an Arabic manuscript in Leiden (Netherlands) together with an Arabic translation to the *Elements* attributed to al-Ḥajjāj ibn Maṭar [2]. The manuscript gives each definition, axiom, postulate and proposition from the *Elements* followed by the commentary of al-Nayrīzī. The Arabic manuscript is incomplete and breaks off at the beginning of Book VII, and an important section of the commentary on Book I is missing.

Al-Nayrīzī's commentary was translated into Latin by Gerard of Cremona (ca. 1114-1187 A.D.).[3] This translation is extant in three manuscripts and makes it possible to fill the gaps in the Arabic version [6], [1].

Considerable confusion surrounds the commentary by al-Nayrīzī to Book X of the *Elements* on the theory of irrational magnitudes. The authorship of the second part of his commentary to Book X [6, p. 252-386] has been doubted by historians of mathematics for the following reasons. In 1902 Bjørnbo discovered that another Latin manuscript contained the same text as this second part of the commentary [3, p. 71]. No author was mentioned but this Latin manuscript begins with "Abbacus." On the basis of this beginning, Suter conjectured that the author was not al-Nayrīzī but a certain Abū Bakr Muḥammad ibn ᶜAbd al-Bāqī from Baghdād who was born in 442 H. / 1050 A.D. and who died in 535 H. /1141 A.D. [17, p. 22-23]. Suter then published a study of this second part of the commentary under the misleading title "On the Commentary by Muḥammad ibn ᶜAbd al-Bāqī on the Tenth Book of the *Elements*" [18].[4] In [8, vol. 5, p.388-389] Sezgin has objected, in my opinion correctly, that Muḥammad ibn ᶜAbd al-Bāqī lived so late that his work could hardly have become available in Spain in time to

[3]The translator's name is not mentioned in the Latin manuscripts but Al-Nayrīzī's commentary is included in a list of works translated by Gerard of Cremona, see [6, p. viii], [3, p. 71].

[4]Suter's article is important because it contains many corrections to the edition by Curtze.

be translated by Gerard of Cremona. (For it is unlikely that an unimportant mathematical text such as the second part of the commentary to Book X was transmitted in the 6th/12th century from the Eastern Islamic world via the Islamic part of Spain to the part that had been conquered by the Christians.) Recently, Busard has published a new edition of the Latin text of the second part of the commentary [5], and he has suggested that this part was written by Yuḥannā al-Qāss [8, vol. 5, p. 298] who lived in the early fourth/tenth century. He authored a short Arabic text on rational and irrational magnitudes which, however, is different from the commentary edited by Busard. Thus the authorship of the second part of the commentary to Book X remains unsettled.[5]

I do not know whether al-Nayrīzī also composed a commentary to Books XI-XIII of the *Elements*, dealing with stereometry.

# 3 The Historical Value of al-Nayrīzī's Commentary

In his commentary al-Nayrīzī wanted to make the *Elements* more accessible to the reader, and to include a collection of interesting material which he had found in other commentaries to the *Elements*. This extra material consists of alternatives to the definitions given by Euclid, discussions of the structure of geometric propositions, "proofs" of some of the Euclidean postulates, notes and alternative proofs to propositions in the *Elements*, etc. His most important sources were the commentaries on the *Elements* by Heron of Alexandria (first century A.D.)[6] and Simplicius (sixth century A.D.).[7] These commentaries are lost in Greek.[8]

Al-Nayrīzī took from the commentary of Simplicius references to the fol-

[5]There is one reference to a mathematician other than al-Nayrīzī, but this reference occurs in the part of the commentary which Suter considered as genuine: "Diachasimus asks: I wonder why Anaritius has added this..." [6, p. 232]. Thus the version which was translated into Latin may have contained some small additions by other Islamic mathematicians. The identity of "Diachasimus" has not been established [19].

[6]On Heron see [7, vol. 6, pp. 310-315].

[7]On Simplicius see [7, vol. 12, pp. 440-443].

[8]Al-Nayrīzī mentioned the additions of Heron to the following propositions of Euclid: I:1, 11, 35, 38, I:46 (=I:47 Greek), I:47 (=I:48 Greek); II:1,2,3,4,5,6, 8, 9, 10, 11, 12, 13; III definitions, III:7, 8, 10, 11, 12, 13, 16, 19, 20, 21, 24, 31; IV:definitions, IV:1, 3, 15, 16; V: definitions; VI:18 (=19 in Greek), VII:3, VIII:25. Al-Nayrīzī mentions Simplicius only in connection with Book I of the *Elements*, namely in all definitions, postulates, common notions, the introduction before the first proposition ([2, part 1, p. 41]), and in the "proof" of Euclid's parallel postulate before Book I, Proposition 29.

lowing Greek mathematicians:

- Abthiniatus: I, postulate 5, preliminaries to I:29;
- Aghānis: I: def. 23 (parallel lines), preliminaries to I:29;
- Archimedes (died 212 BC):[9] I: def. 4, 20bis;
- Diodorus (first century A.D.):[10] I, postulate 5; preliminaries to I:29;
- Hermides: I: def. 1, 2, 5, 7;
- Others: I: def. 1, 2, 4, 7, 8;
- Pappus (ca. A.D. 300):[11] I: common notions;
- Plato (427 - ca. 347 B.C.):[12] I: def. 4;
- Posidonius (died ca. 50 B.C.):[13] I: def. 1;
- Ptolemy (ca. 150 A.D.):[14] preliminaries to I:29.

The geometers Abthiniatus, Aghānis,[15] and Hermides are only known through these quotations in al-Nayrīzī's commentary, and the identification of their Greek names has been problematic.

Only two other commentaries by Greek authors have come down to us: the commentary by Proclus on Book I of the *Elements* still exists in the Greek original [13],[16] and the commentary by Pappus of Alexandria to Book X of the *Elements* survives in an Arabic translation [12]. Because al-Nayrīzī preserved large parts of the commentaries by Simplicius and Heron, his commentary is very important for the history of Greek mathematics. For this reason al-Nayrīzī's commentary attracted the attention of European historians of mathematics in the end of the 19th century. This led to the publication of the Arabic text [2] and the first edition of the medieval Latin translation [6].

Al-Nayrīzī also presents an interesting alternative proof of the theorem of Pythagoras (*Elements* I:46 in the Arabic manuscripts, I:47 in the Greek)

[9] On Archimedes see [7, vol. 1, pp. 213-231].
[10] On Diodorus see [11, vol. 2, pp. 840-841].
[11] On Pappus see [7, vol. 10, pp. 293-304].
[12] On Plato see [7, vol. 11, pp. 22-31].
[13] On Posidonius see [7, vol. 11, pp. 103-106].
[14] On Ptolemy see [7, vol. 11, pp. 186-206].
[15] Aghānis was probably a contemporary of Simplicius. He is not identical to Geminus (ca. 70 B.C.), see [7, vol. 5, pp. 344-347].
[16] Proclus' commentary was not translated into Arabic and was unknown to al-Nayrīzī.

by Thābit ibn Qurra. In comparison to the material which al-Nayrīzī quoted from other sources, his own contributions to the commentary are modest. They amount to the following:

- He added numerical examples to some theorems in order to make these theorems easier to understand for the reader. Example: In theorem 4 of Book II Euclid proves that if a line (segment) is divided into two parts, the square of the whole line is equal to the squares of the two parts plus two times the rectangle contained by the two parts. Al-Nayrīzī illustrates this with the example where the whole line is 10, the parts 7 and 3, and $10 \times 10 = 7 \times 7 + 3 \times 3 + 2 \times 7 \times 3$ )[2, part 2, fasc. 1, p. 21]. Other examples are in Book II:1,2,3,5, Book V, definitions 9, 10, and Book VI:26 (=25 in the Greek), 28, 29. In the commentary to Book X there are also many numerical examples, which may or may not be due to al-Nayrīzī.

- He made remarks on *Elements* III:15,19; IV:5,6,10; V:definitions, propositions 1,2,21; VI: definitions, propositions 4,7,18,25,32 in the numbering of the manuscript (corresponding to VI:4,7,19,23,31 in the numbering of the Greek edition, see [9, vol. 2]). Some of these remarks are simple, but al-Nayrīzī's commentary to Book V is more profound. He discusses a definition of a:b = c:d which is unlike Euclid's definition, and which is related to the modern theory of continued fractions [2, part 3, fasc. 2, pp. 10-13]. It is likely that this is a subject which was actively researched in the time of al-Nayrīzī and which was only partially understood by him (but fully understood by al-Māhānī).

- There are parts in the commentary whose origin is uncertain, for example the direct proofs of I: 8, 9, 14; the division of the proof of III:34 into six cases, etc. These parts may be due to al-Nayrīzī, they may have been taken by al-Nayrīzī from an earlier source, or they may even be due to a later redactor of the text. Sometimes al-Nayrīzī presents a proof but says that the author is not known to him (I: 25, 26). Further research will be necessary in order to identify the exact contribution of al-Nayrīzī.

# 4 An Example

I now discuss an interesting theorem from the commentary of Heron of Alexandria, which has been preserved thanks to al-Nayrīzī. The theorem is related to Euclid's proof of the theorem of Pythagoras, which will first be summarized [9, vol. 1, pp. 349-350].

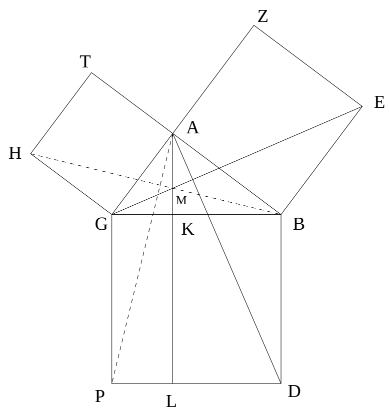


Figure 1

In order to prove the theorem of Pythagoras, Euclid considers a right-angled triangle $ABG$ and he constructs the squares $ABEZ$, $AGHT$ and $BGPD$ as in Figure 1 (notations adapted to Figure 2). He notes that $BA$ and $AT$ are on the same straight line, and that $GA$ is on the same straight line as $AZ$. He drops perpendicular $AKL$ to $BG$ and he draws the straight lines $EG$ and $AD$. Now he proves that the square $ABEZ$ is equal in area to parallelogram $KLDB$ in the following way: Since $EB = BA$, $GB = BD$ and $\angle EBG = \angle ABD$, triangles $EBG$ and $ABD$ are congruent and hence have the same area. Since $ZAG$ is parallel to $EB$, the square $ABEZ$ is two times the triangle $EBG$. Since $AL$ is parallel to $BD$, parallelogram $KLDB$ is two times triangle $ABD$. Therefore the square $ABEZ$ is equal in area to parallelogram $KLDB$.

He then draws lines $AP$ and $BH$ and he says that one can now prove in a similar way that the square $AGHT$ is equal to parallelogram $KLPG$. Thus the sum of the squares $ABEZ$ and $AGHT$ is equal to the square $BGPD$. This is Pythagoras' theorem.

Figure 1 suggests that lines $BH$, $GE$ and $AK$ intersect in the same point. Heron proves in his commentary that this is indeed the case. In the following translation of Heron's theorem as quoted by al-Nayrīzī [2, part 1, pp. 182-185], "line" always means straight line, and my explanatory additions are in parentheses.

> (Figure 2) Since we have now treated these preliminary theorems,[17] let us suppose that angle $A$ of triangle $ABG$ is (a) right (angle). On $BG$ the square $GD$ has been constructed, on $AB$

[17] Heron adds three preliminary theorems which I do not translate here

the square $ABEZ$, and on $AG$ the square $AGHT$. From point $A$ line $AKL$ has been drawn parallel to line $BD$. Line $EG$ has been drawn, it intersects line $AL$ at point $M$. Line $HM$ has been drawn. Then point $M$ was joined with point $B$. I say that line $BM$ is at the rectilinear extension of line $HM$.

(Proof): Let lines $EB$ and $HG$ be extended rectilinearly to meet < at point $N$, and let lines $EZ$, $HT$ be extended to meet >[18] at point $S$. Through point $M$ let line $OMF$ be drawn parallel to line $SE$ and line $CMQ$ parallel to line $ZG$ as has been shown in the proof of (proposition) 31. Let lines $SA, TZ$ be drawn. Then line $TA$ is equal to line $AG$ and line $ZA$ is equal to line $AB$. Thus the two lines $BA, AG$ are equal to the two lines $ZA, AT$ and angle $BAG$ is equal to line $ZAT$. Thus the base $BG$ is equal to the base $TZ$ because of the proof of (proposition) 4,[19] and the other angles are equal to the other angles, so angle $ABG$ is equal to angle $TZA$. But angle $ABG$ is equal to angle $GAK$ since $AK$ is a perpendicular in the right-angled triangle $ABG$. Thus angle $TZA$ is equal to angle $GAK$. Angle $TZA$ is equal to angle $SAZ$ because in the parallelogram $SA$ the two diagonals $SA, TZ$ have been drawn to intersect at point $X$, so $ZX$ is equal to line $AX$, so angle $SAZ$ is equal to angle $GAK$. We add angle $SAG$, then the sum of the two angles $SAZ, SAG$ is equal to the sum of the two angles $MAG, GAS$. But according to the proof of (proposition) 13, the sum of the two angles $SAG, SAZ$ is equal to the sum of two right angles. Therefore the sum of the two angles $SAG, GAM$ is equal to the sum of two right angles. Thus, according to the proof of (proposition) <14>, line $SAM$ is straight, and it is a diagonal in parallelogram $SM$. Thus, according to the proof of (proposition) 43, the complement[20] $AC$ is equal to the complement $AO$. We add area $AM$, then area $MT$ is equal to area $MZ$. Again, area $ZN$ is a parallelogram, its diagonal is $EMG$, and on its two sides are the parallelograms $ZM, MN$, and they are complements. Thus the complement $ZM$

[18] Besthorn and Heiberg added the passages in angular brackets to make mathematical sense.

[19] The references are always to Book I of the *Elements*.

[20] "Complement" (Greek: paraplèrōma, Arabic: mutammim) is a technical term introduced in *Elements* I:43. In this proposition Euclid proves that if in a parallelogram $MOSC$ one draws the diagonal $MS$ and through a point $A$ on this diagonal one constructs two new parallelograms $AM$ and $ATSZ$, the "complements" or remaining parallelograms $AO$ and $AC$ are equal, that is to say, equal in area [9, vol. 1, pp. 340-341].

is equal to the complement $MN$. Area $MN$ is therefore equal to area $MT$. Then, according to the proof in the third preliminary lemma for this proposition,[21] $BMH$ is a straight line, and that is what we wanted to demonstrate.

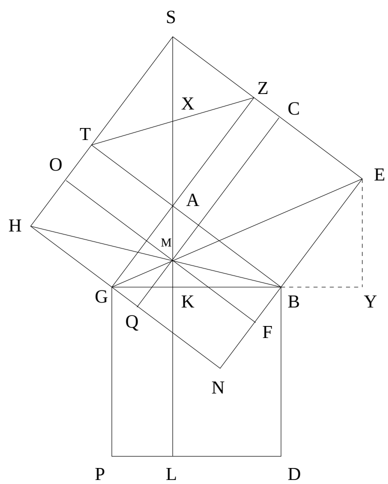


Figure 2

The proof is remarkable because it does not use the theory of proportions and similar triangles from Books V and VI of the *Elements*.[22] Thus Heron wrote his commentary not for accomplished mathematicians but for students.

# 5 Influence on Albertus Magnus

The Latin version of al-Nayrīzī's commentary to the *Elements* does not seem to have had a very wide distribution in the Middle Ages. However, the

[21] This "third preliminary lemma" is a converse of *Elements* I:43 to the effect that if in a parallelogram $BNHT$ two smaller parallelograms $MT$, $MN$ are drawn which meet at $M$, and if these parallelograms are equal in area, then line $BMH$ must be a straight line (and the diagonal of the parallelogram $BNHT$). Heron proves this theorem using the methods of Book I of the *Elements*, without the theory of proportion and similarity, see [2, part 1, p. 180-182].

[22] Using proportions and similarity, the proof can be much shortened. For example, define $M$ again as the intersection of lines $GE$ and $AK$. Through $E$ draw a perpendicular to $GB$ to meet $GB$ extended at point $Y$ (broken lines in Figure 2), then by similar triangles $EY : YG = MK : KG$. Triangles $EYB$ and $BKA$ are congruent so $EY = BK$ and $YB = KA$, hence $YG = AL$. Therefore $BK : AL = MK : KG$, hence $AL.MK = BK.KG (= AK^2)$. If we define $M'$ as the point of intersection of $BH$ and $AK$, we have similarly $AL.M'K = AK^2$. Therefore $M' = M$. Note that $AD$, $BG$ and $MF$ intersect at one point, and that similarly $AP, BG$ and $MQ$ intersect at one point.

commentary was studied by two authors from the middle of the 13th century: Albertus Magnus and Roger Bacon. We first turn to Albertus Magnus.

Albertus Magnus ("the great Albertus") was born around 1200 in Germany. He studied in Italy and became a member of the Dominican order. He spent the period 1241-1247 in Paris and travelled extensively in Germany. Between 1250 and 1270, he wrote a very large number of works in which he discussed Aristotelian philosophy and Greek and Arabic science. He was not a slavish follower of Aristotle, and his works include a list of Aristotle's errors. As a result of Albertus' activity, Christian scholars became less hostile toward Greek and Arabic science. On December 16, 1931, Pope Pius XI made Albertus a saint, and in 1941 the Roman Catholic church declared him the patron of all students of natural science. [7, vol. 1, pp. 99-103].

Albertus wrote a work entitled *Geometry*, which is essentially a paraphrase of Books I-IV of the *Elements* with philosophical introductions and a mathematical commentary. I have used the edition and analysis of Book 1 of the *Geometry* by Paul Tummers of Nijmegen (Netherlands) [21]. Tummers is currently preparing an edition of the whole *Geometry* of Albertus.

The *Geometry* contains Albertus' own philosphical discussions about the nature of mathematics and its place among the other sciences. In this sense the *Geometry* is an original work. Albertus depended on al-Nayrīzī for many technical mathematical matters.[23]

The following example will make this clear. Below I have translated Albertus' paraphrase [21, vol. 2, pp. 98-99] of the theorem of Heron that has been translated above. I have adapted the letters in the geometrical figure to the notation of Figure 2. By comparing the two translations,[24] the reader can see how Albertus modified al-Nayrīzī's text. See also the analysis in [21, vol. 1, pp. 315-319]. In the following proof, reference is made to preliminarty lemmas, which are easy and which I have therefore not included.

> (Figure 2) Now the fifth (proposition) of Heron is presented, and this is what Euclid assumed in the proof of proposition 46, that is to say that if squares are described on the sides of a right-angled triangle, the two lines which contain one side of the square (with endpoint) at the right angle, will be in a straight line with the sides of the triangle containing the same (i.e. right) angle.[25]

[23] Albertus called him "Alfarabius" in the beginning of his work (although it is clear that al-Fārābī is not involved), and only later gave the correct name "Anarizius" [21, p. 63]. For another instance of the influence of al-Nayrīzī on Albertus Magnus see [**?**].

[24] The Latin version of the commentary of al-Nayrīzī is similar to the Arabic, see [1, pp. 69-70].

[25] This is to say that line $BAT$ and $GAZ$ are straight lines. This statement has nothing to do with the theorem of Heron in the commentary of al-Nayrīzī.

For let $ABG$ be a right-angled triangle with right angle $BAG$. Let squares be described on all sides, let the square on side $BG$ which subtends the right angle ($A$) be $BGPD$, let the square on the side $AG$ be $AGTH$ and let the square of side $AB$ be $ABZE$. Thus I say that line $AT$ of square $AH$ is in a straight line with line $AB$ of triangle $ABG$, and in the same way, line $AZ$ of square $AE$ is in the same straight line as line $AG$ of triangle $ABG$.

For I draw from point $A$ to line $PD$ of the square $GD$ a parallel to the side $BD$ by (prop.) 31. This will also be a parallel to side $GP$ by (prop.) 32,[26] let it be line $AL$, and this line will intersect the base of triangle $ABG$ perpendicularly at point $K$. Then I draw lines $BH$ and $GE$, and I extend lines $HT$ and $EZ$ to meet at point $S$. In the same way I extend lines $HG$ and $EB$ to meet at point $N$. I extend line $LKA$ on the side of $A$ until point $S$,[27] which (line) will intersect lines $GE$, $HB$ at point $M$[28] which is in the area of triangle $ABG$ (i.e. inside $ABG$). I join points $T$ and $Z$ by line $TZ$, and I draw two lines, one (line) parallel to $TAB$, let it be line $OMF$, and the other (line) parallel to $ZAG$, let it be line $CMQ$.

I say therefore: because the lines $TA$ and $AG$ are the sides of one square, they are equal by (prop.) 45, and for the same reason lines $ZA$ and $AB$ are equal; thus two lines $AG$ and $AB$ are equal to two lines $AT$ and $AZ$. Angle $BAG$ is equal to angle $TAZ$ by (prop.) 15.[29] Thus by (prop.) 4 the two triangles $ABG$ and $TAZ$ are equal (in area) and angle $ABG$ is equal to angle $TZA$ and angle $AGB$ is equal to angle $ZTA$. But by the immediately (preceding) lemma,[30] angle $ABG$ is equal to angle $GAK$, so angle $TZA$ is also equal to angle $GAK$. I continue by saying that parallelogram $SA$ is divided by the two diagonals at point $X$, namely(diagonals) $AXS$ and $TXZ$, which are bisected as is proved by the second lemma, therefore line $XZ$ is equal to line $XA$. Thus by the first part of the fifth (proposition) the angles at the base are

[26] The reference should have been to proposition 30 of Book I of the *Geometry*, which is the same as proposition 30 of Book I of the *Elements*. The proposition numbers in the *Geometry* are the same as the numbers in the *Elements*.

[27] It would be correct to say: draw a straight line joining $A$ and $S$. It will be proved below that segment $AS$ is the rectilinear extension of segment $LKA$.

[28] It has not yet been proven that the three lines $GE$, $HB$ and $AK$ intersect at one point $M$. This is the purpose of Heron's proof.

[29] Here it is assumed that lines $BAT$ and $GAZ$ are straight.

[30] This easy lemma is not discussed by al-Nayrīzī.

equal, that is to say, angle $XZA$ and angle $XAZ$. But angle $XZA$ was (shown to be) equal to angle $GAK$, so angle $XAZ$ is equal to angle $GAK$. By (prop.) 13 it is established that angles $XAZ$ and $SAG$ are equal to two right (angles). But angles $GAK$ and $SAG$ are equal to those (two right angles) because (to) angle $SAG$ on both sides two equal angles are applied, so they themselves are equal to two right (angles). Thus by (proposition) 14 line $SAMKL$ is one straight line.[31] But this is a diagonal of parallelogram $SM$, whose complements are $CA$ and $OA$, so by (prop.) 43 they are equal. If area $AM$ is added to both, they (the sums) will still be equal, so $ZM$ is equal to $TM$. But consider again parallelogram $ZN$, whose diagonal is $GE$ and the complement about the diameter is $ZM$ on one side and $MN$ on another side, so by (prop.) 43 they are equal.[32] But $ZM$ was (shown to be) equal to $TM$, so $MN$ is also equal to $TM$. But then consider parallelogram $HB$ whose diagonal is $HMB$,[33] divided in two equal parallelograms, namely $TM$ and $MN$, and the diagonal divides that. Therefore this diagonal is a straight line by the preceding lemma,[34] so line $BMH$ is one (straight) line.[35] In the same way it can be shown that line $GME$ is one straight line.[36] Also angle $BAG$ of triangle $ABG$ is a right (angle). In the same way, angle $TAG$ it a right angle since it is the angle of a square by (prop.) 45, so by (prop.) 14 lines $TA$ and $AB$ which issue on different sides from point $A$ are one (straight) line. And similarly it is proved that $ZA$ and $AG$ are one (straight) line, and in this way it can be proved what is required in the demonstration of Euclid.[37] And this is what we wanted to prove.

Know, however, that none of these theorems are necessary for the proof of Euclid, they have only been added for the pleasure of study ...[38]

[31] In other words, it now follows that segment $AS$ is the rectilinear extension of segment $LKA$.

[32] Here it is assumed that point $M$ is on line $GE$ and on line $AK$, as in Heron's theorem.

[33] Here it has not yet been proved that line $HB$ passes through $M$.

[34] This is Heron's third lemma immediately preceding the theorem translated above.

[35] Here ends the proof of Heron's theorem.

[36] This remark shows that Albertus Magnus understood very little of the reasoning. Point $M$ is defined as the point of intersection of $AK$ and $EG$, so there is nothing to be proved here.

[37] The last lines are sufficient as a proof of the fact that lines $TAB$ and $GAZ$ are straight. One does not need point $M$ or Heron's theorem for this purpose.

[38] Note that this statement is in contradiction with Albertus' earlier statement to the

Albertus reworked al-Nayrīzī's text but the remarks in my footnotes show that he missed the point of Heron's theorem completely. Tummers also mentions other instances where Albertus misunderstood al-Nayrīzī's commentary [21, pp. 191, 246, 248, 286-7, 319, 320]. Clearly Albertus had not received sufficient training in mathematics, probably because such training would be very difficult to find or unavailable in his time. Albertus lived in the beginning of the period of assimilation of mathematics in Western Europe when the level of mathematical knowledge at the universities was still low.[39]

# 6 Influence on Roger Bacon

Roger Bacon was born around 1215 in England. He studied in Oxford and lectured in Paris between 1241 and 1246, so he could have known Albertus Magnus personally. Around 1257 he became a member of the Franciscan order. He disliked the fact that people regarded Albertus Magnus as an authority on the same level as as Aristotle, Avicenna (=Ibn Sīnā) and Averroes (=Ibn Rushd). Roger Bacon died in 1292 [7, vol. 1, pp. 377-385].

Like Albertus Magnus, Roger Bacon wrote many works which facilitated the spread of Greek and Arabic Science in Western Europe. Roger Bacon was enthousiastic about mathematics but even less familiar with it than Albertus Magnus. He wrote a summary of geometry and appended it to the end of his work *Communia Mathematica*. This summary has been edited and translated into English by Molland [10].

In this summary, Roger Bacon mentions Al-Nayrīzī five times. He quotes a few simple proofs from the commentary of al-Nayrīzī and also a discussion of theoretical and practical geometry which al-Nayrīzī had adopted from Simplicius.

Thus al-Nayrīzī's commentary had an influence on Roger Bacon and also Albertus Magnus, in the sense that it was read and studied by them, and reworked by Albertus Magnus. However, neither Roger Bacon nor Albertus Magnus were able to fully appreciate its contents.

# 7 Conclusion

As we have seen, al-Nayrīzī's commentary has been studied by European historians in the late 19th and early 20th century because of its interest for

effect that the fifth proposition of Heron proves "what Euclid assumed in the proof of proposition 46" (that is, the theorem of Pythagoras).

[39] The real experts in mathematics were not connected to the universities.

the history of Greek mathematics. Recently, the historian Paul Tummers has studied the Latin translation of the commentary to Books 1 to 4 in connection with Albertus Magnus. Busard has compared the Latin translation by Gerard of Cremona with the Latin translations of Euclid's *Elements* and with the. modern Latin translation of the Arabic text of al-Nayrīzī's commentary [4]. These studies all focus on the Greek or the Latin tradition.

One would also like to know how al-Nayrīzī modified or selected his sources, which passages he added himself, whether the last part of al-Nayrīzī's commentary on Book X in the Latin version was really by him. Perhaps these questions can be answered by a historical analysis of the Arabic text. Al-Nayrīzī's commentary on Euclid's *Elements* is important, but what exactly his own contribution was is still unclear at present.

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