

113. Kaif darb al-shai'. Shai' when multiplied by a number n gives n shai', when multiplied by minus (illā, kam<sup>p</sup>) shai' gives a minus square,  $x^2$  māl nāqis mustathnā; when minus shai' is multiplied by a number the result is minus n shai', and when minus shai' is multiplied by itself a positive square, māl zā'id results, because the minus sign, istithnā', is not abolished except by multiplication by itself.

114. Mā hisāb al-dirham wa'l dīnār. This is a method of reckoning derived from Algebra. As there are sometimes more unknown quantities than one, it becomes necessary to have names for them. Some people call them dinars, dirhams and fulus, while others adopt the Hindu method of naming the ashya' black, yellow and gray.<sup>1</sup>

115. Mā hisāb al-khaṭā'ain. A number is assumed which appears to be suitable for solving the problem; if a test indicates its accuracy, it is unnecessary to proceed further, but if  
 RULE OF TWO ERRORS it has led to an error, the amount of that must be noted and the process repeated after the unsuccessful guess, when either you hit upon the correct answer or else you have a second error. Then the answer can be deduced from the two errors by a method which is known [to Arithmeticians].<sup>2</sup>

116. Kaif ithbāt al-a'dād bi ḥurūf al-'arab. The notation of numbers by Arabic letters is a matter of agreement and convention; for it  
 ARABIC LETTERS FOR NUMERALS would have been possible to use the letters in the ordinary order of the alphabet, a, b, t, th, etc., because there are nine units, nine tens, nine hundreds, which, with a sign for a thousand, are provided for by the twenty-eight letters.

<sup>1</sup> v. Colebrook, l.c.p.139. For akhab read ashhab.

<sup>2</sup> Carra de Vaux quotes an example from Al-khwārizmī, l.c.II, 117.  $x - \frac{1}{3}x - \frac{1}{4}x = 8$ . Try 12, result 5, (-3); try 24, result 10 (+2);  

$$\begin{array}{r} 12 \times -3 \\ 24 \times +2 \end{array}$$
 multiply diagonally, result 24, and -72; add, because signs contrary, result  $96/5 = 19 \frac{1}{5}$ .

**كيف ضرب الاشياء الشئ اذا ضرب في الشئ اجمع منه مالا**  
 واذا ضرب في عدد اجمع اشياء بمثل تلك العدة فاذا ضرب الشئ في الاشئ اجمع  
 مال باخر مستثنى قليل الاما واذا ضرب الاشئ في عدد اجمع اشياء باقصه بمثل  
 تلك العدة قليل الاكدا شيئا واذا ضرب الاشئ في عدد اجمع مال زايد فان  
 الاستثنا لا يطل الا بمثله ما حساب الدنيا وهو حسب مستخرج  
 من الجزو والمقابل واما كانت الاشياء المجهولة اكثر من واحد واجتمع اليها  
 تلقبها فمهم من تلقب بدنيا ودرهم وفسر منهم من يلون الاشياء كالهند فانهم  
 يقولون شئ اسود واصفر واكعب ما حساب الخطاين  
 يفيض في المطلوب ما انفق من الاعداد فان اداء امتحانه الى الصواب نقد وجد  
 وان اداءه الى الخطا حفظ مقداره واعاد ذلك بعد اخر كيف ما انفق فلما  
 ان يعبر على المطلوب واما ان تحط ثانيا ثم يستخرج المطلوب من ثوب  
 هذين الخطاين بطريق مشهور كيف اثبات الاعداد كجوف العرب  
 هذا مواضع واصطلاح فقد كان يمكن ان يجعل ترتيب العدد في حروف  
 المعجم المنضود على ا ب ت ث فانها تستوفى عن الاعداد التسعة  
 والپشرات التسع والمائتين التسع والالف ميعالان عدتها ثمانية وعشرون

However, people selected the order of the hurūf al-jummal because this was widely diffused among people of the Book before the time of the Arabs. The mode in which the letters are apportioned to the several numbers is shown in the accompanying table.

Units	ا	ب	ج	د	هـ	و	ز	ح	ط
	a	b	j <sup>a</sup>	d	h	w <sup>a</sup>	z	ḥ <sup>u</sup>	ṭ
	1	2	3	4	5	6	7	8	9
Tens	ع	ك	ل	م	ن	س	ر	ف	ص
	y	k <sup>a</sup>	l	m	n	s	r	f <sup>a</sup>	ṣ
	10	20	30	40	50	60	70	80	90
Hun- dreds	ق	ر	ش	ت	ث	خ	ذ	ض	ظ
	q	r	sh <sup>a</sup>	t	th <sup>a</sup>	kh <sup>a</sup>	dh <sup>a</sup>	ḍ <sup>a</sup>	ẓ <sup>h</sup>
	100	200	300	400	500	600	700	800	900
									1000

117. Hal ikhtilāf fiḥā. The object in using these letters is economy of space and ease of writing numbers especially in astronomical tables. Among astronomers there is no disagreement as to their use, but there are perverse people outside the profession who put sa'faḍ for sa'faṣ, thus making s 60, and d 90, and qarasaṭ for qarashat, basing their objections, some on linguistic, others on religious grounds; but this is all nonsense. Were it not that a general agreement has

<sup>1</sup> A memoria technica is made by the insertion of vowels as indicated in the transliteration above. Pococke, Spec. Hist. Arab., p. 308, refers to the use of the first six words for the days of the week from Saturday to Thursday, Friday being al-'arūbah.

<sup>2</sup> The Muslims of Morocco make the last four words, sa'faḍ, qarasaṭ, thakhadh, ṣaḡhash.

وَلِكِنَّ الْقَوْمَ جَعَلُوا تَرْبِيبَ هَذِهِ الْأَعْرَادِ فِي حُرُوفِ الْجُمْلَةِ مِنْ هَذَا التَّرْبِيبِ  
 هُوَ الْفَاعِلُ فَمَا بَيْنَ مَنْ تَقْدِمُ الْعَرَبُ مِنْ أَهْلِ الْكِتَابِ وَهُوَ أَجْزَعُ مِنْهُ هُوَ  
 حُطِّيٌّ كَلْنٌ سَعَفَضٌ قُرْشَتٌ خَذَّاعٌ صَنْطَغٌ مَجْمَعٌ  
 مِنَ الْعُودِ مُشْتَبِهٌ فِي هَذَا الْحَدُولِ

ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع
ط	ح	ز	و	ه	د	ج	ب	ل	ع

## هَلْ خُتِلَفَ فِيهَا

العرض في هذه الحروف هو اختصاص في أبحاث الأعداد في الحول ومن أمثلة الجداول  
 هم المجرز ولم تختلفوا فيما بينهم وإنما خالفهم من ليس هذه صناعته فجعل مكان سَعَفَضٍ  
 صَعَفَضٍ وجعل الصاد غير المعجمة سَيْنٌ والمعجمة سَعِينٌ وجعل كَلْنٌ قُرْشَتٌ  
 قُرْشَتٌ غير المعجمة سَيْنٌ وإنما حارب بعضهم على ذلك أي أنه للكلام لغوي وبعضهم

سن

been arrived at as to this order by its users, their objection might be allowed, but it would constitute a departure from an established custom.

118. Kaif yakūn tarkībā. Certain rules have to be observed in combining these letters. Of the units, tens and hundreds, the hundreds come first, the tens next and the units last; thus  
 COMBINATION OF LETTERS 115 is written qyh, and [when written in black]<sup>P</sup> must have a line over it to show that it is not a word. So 105 is written qh, and 42 mb, and 1002 ghb. On the other hand 2000 is written bgh, the smaller number coming first to distinguish it from ghb, and to show what is intended.

The customary way of writing these letters is that jīm 3, is written without a tail to distinguish it from hā' 8; little attention need be given to khā' 600, as astronomical calculations are concerned with numbers less than 360; [it is pointed without a tail]. Yā' must have its tail drawn backwards, in case it should be mistaken for a nūn with its point. Kāf must be written horizontally, so as not to resemble lām, and nūn must be written larger than zā' and pointed, for zā' also is pointed, and rā' unpointed. Shīn 300, must be pointed to distinguish it from sīn 60, and attention must be paid to other pointed letters.

When nūn or yā' are combined with units, nā 51, yā 11, nūn must be pointed, and indeed the yā' also should have its points.

When zero, sifr, has to be written in places lacking a number, its circle must have a line over it, ʿ touching, to distinguish it from hā', but in the Indian notation this line is unnecessary, for there, there is no resemblance to hā'.

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Generally the right end of the line in question is continued downwards to the left to join a very small circle ʿ, or the circle has two horns, ʿ; compare ʿ and hā' in future tables.

ما وبلا لا يعارض في اعتقاد وهو كالهذيان ولو اطباق المستعملين على هذا  
 الترتيب لجوزنا ظاهراً ولصحة خروج عن العباد كيف يكون تركيبها  
 اذا جمع من العدد مراتب كالاجاد والعشرات والمائين فابداً لا اعظم  
 اعني المائين ثم بالعشرات ثم بالاجاد مثال ذلك مائة وخمسة عشر فكتبه  
 فقه وخط فوقه خطاً يدل على انه حساب لأكمله فاذا كان العدد مائة وخمسة  
 فكتبه فقه وان كان اثنين فارتفع فكتبه مائة فان كان القواش  
 فكتبه غيب فان اجمعنا الى كتبه الفين فكتبه بع لانه قد تم الاصغر  
 على الاعظم تفصله عن غيب ويدل على انه لا من ما والعباد جاريه في هذه الجوف  
 ان لا يعطف الجيم فها بينهما وبين الكا وفل ما يحتاج الى الخ لانه جبايات النجوم تقصر  
 على ثمانية وستين وان يعطف اليها الي الورد لان لا يشابه النون مع النقطة ويحذف  
 الكاف مبسوطة غير مشابه للام وتكون النون كغير من الزا ومنقوطة  
 للفرق بينهما فان الراء مشقوطة والراء غير منقوطة والسين والسين  
 ان السين منقوطة وسائر ما بعدها كذلك مجعده ومتى تركب النون والياء مع الجاد  
 ثباتها تكون نقطة النون فاصلاً بينهما والاختياط واجب باعجام اليا ايضاً  
 وان اُجيب الى كتبه صفر ليملا الموضع الفارع عملت دائرة وفوقها خط



119. Hal tusta'mal hadhihi 'l-hurūf fī ghayr al-a'dād. The same letters are used for another purpose, namely, for designating the signs of the zodiac as in the following table:-

FURTHER USE OF LETTERS

## Names of Signs. Symbols

Aries	♈ 0
Taurus	♉ 1
Gemin	♊ 2
Cancer	♋ 3
Leo	♌ 4
Virgo	♍ 5
Libra	♎ 6
Scorpius	♏ 7
Sagittarius	♐ 8
Capricornus	♑ 9
Aquarius	♒ 10
Pisces	♓ 11

The foregoing amount of arithmetic will suffice for anyone who desires an introduction to it.

Now, however, we must proceed to the description of the form of the heavens.

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 ASTRONOMY
 

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120. Mā al-falak. The celestial sphere is a body like a ball revolving in its own place; it contains within its interior objects whose movements are different from those of the sphere itself, and we are in the centre of it. It is called falak on account of its circular movement like that of the whirl of a spindle, and its name, athīr, (ether) is current among philosophers.

THE CELESTIAL SPHERE

121. A huwa wāhid au akthar. There are eight such spheres enclosed the one within the other, like the skins of an onion; the smallest sphere is that which is nearest to us, within which the moon is always travelling alone, rising and setting, within its limits. To each sphere there is a certain amount of space between the outer and inner boundaries so that the planet to which it belongs has two distances, the one further, the other nearer. The second sphere above that of the moon belongs to Mercury, the third to Venus,

MORE SPHERES THAN ONE ?

