New insights in the oldest textbook on algebra and geometry in the Dutch language (1583)

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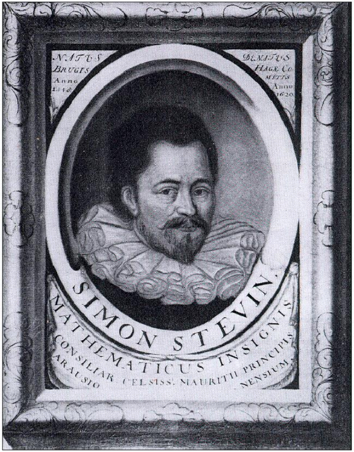
There are many Dutch words for mathematical concepts

Wiskunde [instead of mathematics]
Evenwijdig [instead of parallel]
Loodlijn [instead of perpendicular]
Middellijn [instead of diameter]
Middelpunt [instead of center]
Evenaar [instead of equator]
Meetkunde [instead of geometry]
etc. etc.

thanks to, among others, Simon Stevin van Brugge, Belgium (1548-1620)
who emigrated to Leiden, Netherlands in 1581, student in Leiden University in 1583.



The mathematical community in 17th-century Netherlands included many people who did not know Latin.

A Dutch success story in mathematics:

1637: René Descartes published his philosophical work *Discours de la Méthode* with an appendix on *Géométrie*, in Leiden, Netherlands.

A radical new way to do mathematics (first idea of the equation of a curve, coordinates in x and y), which was understood in the Dutch community of mathematicians, and publicized by them.

New notation, for example $ax^3 + bx + c = 0$, $x^3 = 9x + 28$.

Descartes' mathematics was used by Newton and Leibniz in the 1660s and 1670s in the development of differential and integral calculus.

Arithmetics books in the Dutch language start in the Roman Catholic southern Netherlands, now Belgium.

First arithmetic book printed in Brussels (1508). Later arithmetic books were printed in Gent and Antwerpen, the economic center of the Netherlands.

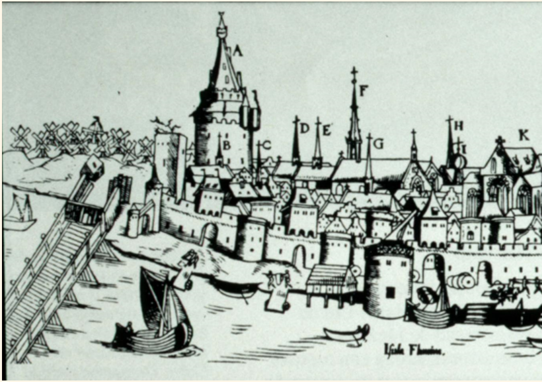
Antwerpen in 1572

The first Dutch mathematics books printed in the protestant Northern Netherlands

this was a simple arithmetics book by Martinus Carolus Creszfelt (German), Deventer 1555, reprints Kampen 1557, Rees 1577.

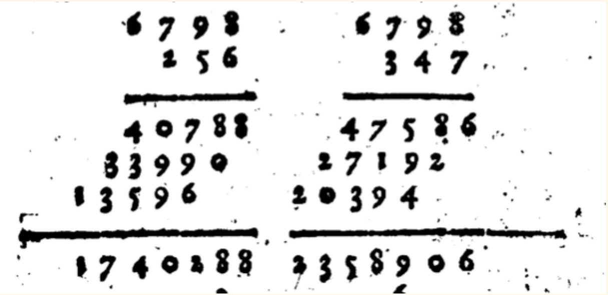
then the books written by Claes Pieterszoon of Deventer, the hero of this story.
Born around 1540? moved in 1567 to Amsterdam

Deventer in the sixteenth century



What was arithmetics like in 1557? from Creszfelt's book

Addition, subtraction, multiplication and division using the symbols, 1,2,3,4,5,6,7,8,9,0 also with fractions

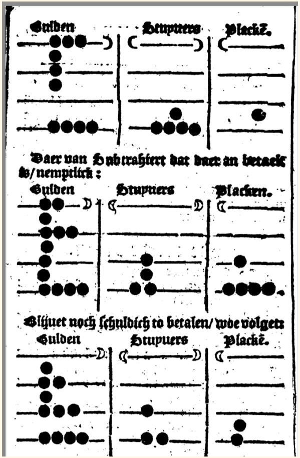


Also reckoning “on the lines”

Origin of this system: Roman computation with pebbles = calculi (whence our word calculus)

In the 16th century this was done by little coins (on a computorium, Frans comptoir)

Example: 3654 guilders 9 shilling 5 pence minus 2869 guilders 17 shilling 14 pence is 784 guilders 12 shilling 6 pence



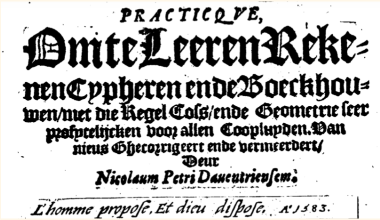
Creszfelt wrote in a funny mixture of Dutch and German: see the poem at the end on how to do mathematics (Konst).

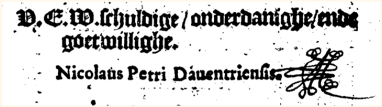
Eyn Phylosophus in Greken Landt
Voer olden tyden Plato ghenant /
Tekent an / dat oeveninge
sonder Const,
Verloren is / unde gaer omme
sonst:
Der gelycken Konst sonder
oeveninge twaer
Eygentlick nichtes is / verswindet
gaer.

A Philosopher in Greece
called Plato in the old days
notes that practice without art
(of mathematics)
is lost and in vain
Similarly, art (mathematics)
without practice
is nothing and vanishes

Claes Pietersz van Deventer (ca. 1540- 1602) and his alter ego Nicolaus Petri Daventriensis

Nicolaus Petri: author


PRACTICQVE,
Omte Leeren Reke-
nen Cypheren ende Boeckhou-
wen/met die Regel Coſs/ende Geometrie seer
profijtelijcken voor allen Coopluyden. Van
nieus Ghecorrigeert ende vermeerdert/
Deur
Nicolaum Petri Daventrienſem.
L'homme propoſe, Et dieu diſpoſe. A°1583.


V. E. W. ſchuldige / onderdanighe/ende
goetwillighe.
Nicolaus Petri Daventrienſis.

Claes Pietersz: bookseller and school master


T'AMSTELREDAM,
By Barendt Adriaenſz. Wo-
nende inde Warmoeſtraet/Int Gulden
Schrijff-boeck. A° M. D. XCI.
Ende men vintſe te coop by Claes Pieterſz.
Schoolmeeſter/wonende op de Oude-zijdts
Burch-wal/Int Gulden Claver-blat.

Claes Pietersz of Deventer (ca, 1540-1602) had engravings of himself made by Hendrik Goltzius, Haarlem

1583

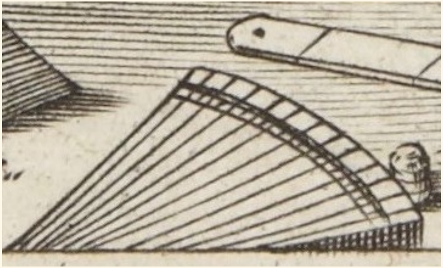
1595

Claes Pietersz of Deventer, detail 1

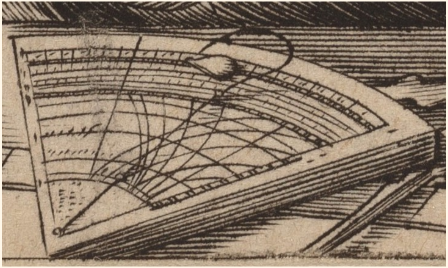
1583

1595

Claes Pietersz van Deventer, detail 2



quadrant, 1583

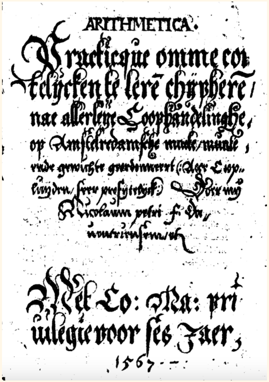


quadrant, 1595

Claes Pietersz van Deventer's first math book: an arithmetics book published in 1567

This appeared after his emigration from Deventer to Amsterdam

Standard contents, weights and measures of Amsterdam


ARITHMETICA.

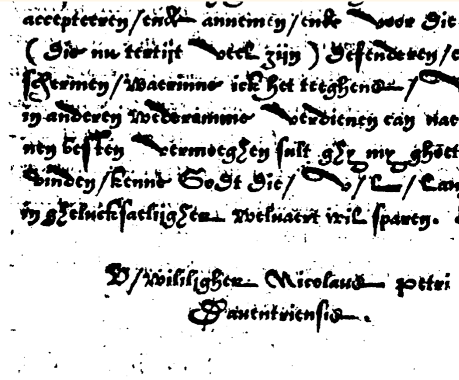
Met Co: Ma: pri uilegie voor ses Jaer,

1567

Claes Pietersz van Deventer, Arithmetic book 1567

Printed in civilité-font.
150 pp.

Signed by his alter ego Nicolaus Petri Daventriensis.



Claes Pietersz van Deventer, Main work 1583: Practique

Four parts:
1. Arithmetic
2. Algebra (first book on algebra in the Dutch language)
3. Geometrical computations (first book on the subject in the Dutch language)
4. Accounting

Total more than 650 pp.
Much material taken over from foreign sources (German, French) - he says so.

Claes Pietersz van Deventer, Practique

First edition 1583
Second edition 1591. Print run 525 copies
Edition 1598 [there is one copy in Göttingen, digitized for me, 2024]
Reprint 1605 (posthumous)
Reprint 1635 (posthumous)
Enormously influential in the Netherlands.

1605 with the new engraving

Claes Pietersz van Deventer, Practique, The Disaster of 1596!

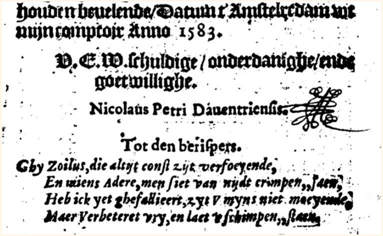
Pirated edition! (so it seems that algebra and geometry were considered good business)

Without his permission printed in Alkmaar, sold in Haarlem by Barent Barentz.
(worker in the press where the 1591 editon was printed)

note the fake engraving

Claes Pietersz van Deventer, Practique

1591 with signature



houden beuelende/Datum t'Amstelredam uit
mijn comptoir Anno 1583.

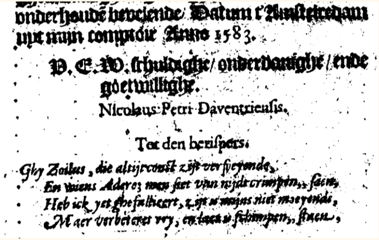
V.E.W. schuldige/onderdanighe/ende
goetwillighe.

Nicolaus Petri Dauentriensis.

Tot den berispers.

Ghy Zoilus, die altyt const zyt verfoeyende,
En wiens Adere, men siet van nydt crimpen,, saen,
Heb ick yet ghefallieert, zyt V myns niet maeyende,
Maer Verbeteret vry, en laet v schimpen,, staen.

1596 pirated edition without signature



onderhoude bevelende Datum t'Amstelredam
uyt mijn comptoir Anno 1583.

V.E.W. schuldighe/onderdanighe/ende
goetwillighe.

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Claes Pietersz van Deventer, Practique: the pirated edition was unfortunately very good

1591: real

117. Item daer is ee Icosahedron besloten in een sphere wiens diameter is V ✓. 90+2 ✓1620/ Die vrage is hoe vele dat een syde vanden Icosahedron maken sal? Fa.6.

Spreket deur die 115e van desen 4 den Diameter sphere gheeft my V ✓.8—✓12⅘ eene syde vanden Icosahedron/ wat

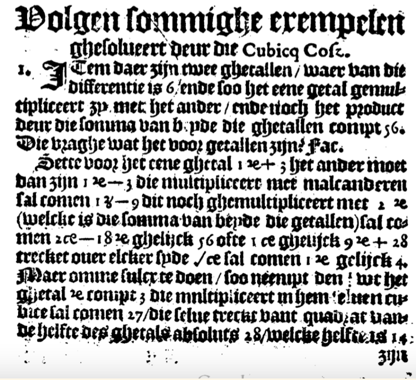
1596: pirated

117. Item daer is een Icosahedron besloten in een sphere/ wiens Diameter is V ✓.90+2 ✓1620. Die vraghe is/ hoe vele dat een zijde van den Icosahedron maken sal? Fa.6.

Spreket deur die 115e van desen/ 4 den Diameter sphere gheeft my V ✓.8—✓12⅘ eene zijde van den Icosahedron/ wat

Claes Pietersz van Deventer, Pratique, 1591

Algebra: more than in our highschools, but other concepts than the modern ones (basicallly old Arabic algebra).
His examples are very good:
1 cube is equal to 9 thing + 28
Modern (Descartes):
$x^3 = 9x + 28$


Volgen sommighe exempelen
gheſolueert deur die Cubicq Coſz.
1. ITem daer zijn twee ghetallen / waer van die
differentie is 6 / ende ſoo het eene getal gemul-
tipliceert zy met het ander / ende noch het product
deur die ſomma van beyde die ghetallen comt 56.
Die vraghe wat het voor getallen zijn? Fac.
Sette voor het eene ghetal 1 ꝛe + 3 het ander moet
dan zijn 1 ꝛe — 3 die multipliceert met malcanderen
ſal comen 1 ȝ — 9 dit noch ghemultipliceert met 2 ꝛe
(welcke is die ſomma van beyde die getallen) ſal co-
men 2 ce — 18 ꝛe ghelijck 56 ofte 1 ce ghelijck 9 ꝛe + 28
trecket ouer elcker ſyde √ce ſal comen 1 ꝛe gelijck 4.
Maer omme ſulcx te doen / ſoo neempt den ⅓ vut het
ghetal ꝛe compt 3 die multipliceert in hem ſelven cu-
bice ſal comen 27 / die ſelue treckt vant quadraet van-
de helfte des ghetals abſoluts 28 / welcke helfte is 14
zijn

Claes Pietersz van Deventer, Pratique, 1591

$x^3 = 9x + 28$.

Recipe: take half the number which stands on its own: $\frac{1}{2}$ times 28 is 14. Remember this.

Take $\frac{1}{3}$ times the number of things (x) $\frac{1}{3}$ times 9 is 3. Take the cube of this (27) Subtract from the square of 14, that is, 196. We obtain 196-27=169. Also remember this.

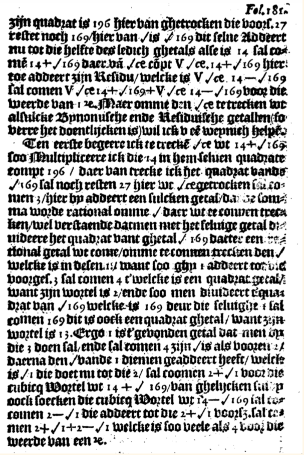
Now we have the two numbers 14 and 169.

Now take

$\sqrt[3]{14+\sqrt{169}} + \sqrt[3]{14-\sqrt{169}}$

=3+1=4)

This is the solution.



Fol. 181.

zijn quadraat is 196 hier van ghetrocken die voorf. 27
restet noch 169/hier van √is √169 dit selue Addeert
nu tot die helfte des ledich ghetals alse is 14 sal co-
mē 14+√169 daer vā √ce cōpt V √ce.14+√169 hier-
toe addeert zijn Residu/welcke is V √ce. 14—√169
sal comen V √ce 14+√169+V √ce 14—√169 voor die
weerde van 1 2e. Maer omme den √ce te trecken wt
alsulcke Bynomische ende Residuische getallen(so
verre het doentlijcken is)wil ick v eē weynich helpē.
Ten eerste begeere ick te treckē √ce wt 14+√169
soo Multipliceere ick die 14 in hem seluen quadrate
compt 196 / daer van trecke ick het quadraat vande
√169 sal noch resten 27 hier wt √ce getrocken sal co-
men 3/hier by addeert een sulcken getal/da... com-
ma worde rational omme √ daer wt te connen trec-
ken/wel verstaende daermen met het seluige getal di-
uideere het quadraat vant ghetal √169 datter een ra-
tional getal wt come/omme te connen trecken den √
welcke is in desen 1 / want soo ghy 1 addeert tot die
voorgef. 3 sal comen 4 t'welcke is een quadraat getal/
want zijn wortel is 2/ende soo men diuideert equa-
draat van √169 welcke is 169 deur die seluighe 1 sal
comen 169 dit is oock een quadraat ghetal / want zijn
wortel is 13. Ergo 1 is t'gevonden getal dat men by
die 3 doen sal/ende sal comen 4 zijn √is als vooren 2/
daerna den √vande 1 diemen geaddeert heeft/ welcke
is √1 die doet nu tot die 2/ sal coomen 2+√1 voor die
cubicq Wortel wt 14 + √ 169/van ghelijcken sal ...
oock soecken die cubicq Wortel wt 14—√169 sal co-
comen 2—√1 die addeert tot die 2+√1 voorsz. sal co-
men 2+√1+2—√1 welcke is soo veele als 4 voor die
weerde van een 2e.

Claes Pietersz van Deventer, Pratique, 1591

Why is $x^3 = 9x + 28$ a "nice" problem?

If $x^3 = ax + b$ we have

$x = \sqrt[3]{(\frac{b}{2}) + \sqrt{(\frac{b}{2})^2 - (\frac{a}{3})^3}} + \sqrt[3]{(\frac{b}{2}) - \sqrt{(\frac{b}{2})^2 - (\frac{a}{3})^3}}$

If $a = 9, b = 28$

$\sqrt[3]{14 + \sqrt{169}} + \sqrt[3]{14 - \sqrt{169}}$

$=3+1=4$

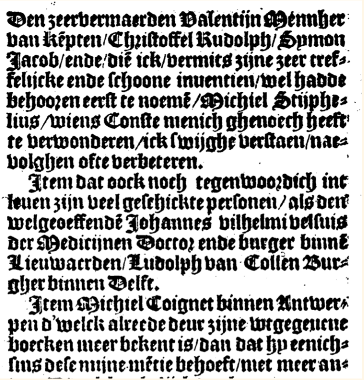
$(\frac{b}{2})^2 - (\frac{a}{3})^3$ is a rational square n^2 (here $n = 13$)

$\frac{b}{2} \pm n$ are cubes of rational numbers.

Claes Pietersz van Deventer, Pratique

the parts on algebra and geometry were a collation ("collatie") of material by French and German, by authors from Germany and Antwerpen.

Valentin Mennher, Coignet, Symon Jacob, etc. etc.


Den zeerbermaerden Valentijn Mennher van Kēpten/Christoffel Rudolph/Symon Jacob/ende/diē ick/vermits zijne zeer treffelijcke ende schoone inuentien/wel hadde behooren eerst te noemē/Michiel Stijphelius/wiens Conste menich ghenoech heeft te verwonderen/ick swijghe verstaen/naevolghen ofte verbeteren.

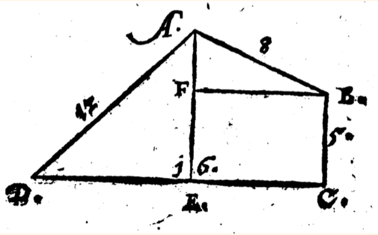
Item dat oock noch tegenwoordich int leuen zijn veel geschickte personen/als den welgeoeffendē Johannes vilhelmi velsuis der Medicijnen Doctor ende burger binnē Lieuwaerden/Ludolph van Collen Burgher binnen Delft.

Item Michiel Coignet binnen Antwerpen d'welck alreede deur zijne wtgegeuene boecken meer bekent is/dan dat hy eenichsins dese mijne mētie behoeft/niet meer an-

Did he understand what he took over? Yes! You can see this in some controversies.

Willem Goudaen, arithmetics teacher in Haarlem, attached in 1581 this problem to the door of a church
Prize for a correct solution: 1 jug of wine.

Consider a field $ABCD$ with a right angle at C.
Given: $AB = 8, BC = 5, CD = 16, DA = 12$.
To compute the perpendicular AE and the surface area.

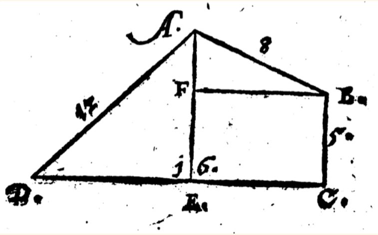


First act. The answers by Claes Pietersz and Goudaen.

Goudaen:
$AE = 3\frac{119}{562} + \sqrt{25\frac{44215}{78961}}$

Claes Pietersz: $AE =$
$\sqrt{35\frac{276445}{315844} + \sqrt{1054\frac{395520866}{6234839521}}}$

Claes did not receive his jug of wine, and Goudaen did not even let him in in his office.



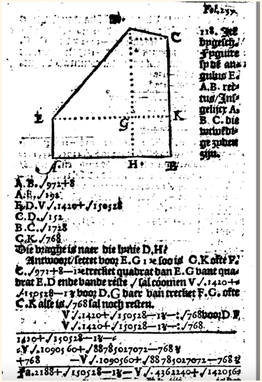
Act 2: Claes proposed a new and much more difficult problem to Goudaen

We have a field *ABCDE* with two right angles at *A* and *B*.
Given $AB = 8 + \sqrt{972}$,
$BC = \sqrt{1728}$, $CD = \sqrt{152}$,
$DE = \sqrt{1420 + \sqrt{150528}}$,
$EA = \sqrt{192}$.
To compute the perpendicular *DH* (and some other things)

Act 3: Goudaen attached this problem in 1583 to the door of a church and claimed that he had invented it himself!

Act 4: Problem solved by Claes Pietersz in the Practique, 1591.

We have a field *ABCDE* with two right angles at *A* and *B*. Given $AB = 8 + \sqrt{972}$, $BC = \sqrt{1728}$, $CD = \sqrt{152}$, $DE = \sqrt{1420 + \sqrt{150528}}$, $EA = \sqrt{192}$.
To compute the perpendicular *DH*.


Fol. 257.

118. Jck

A.B. √972+8
A.E. √192
E.D. V √.1420+√150528
C.D. √152
B.C. √1728
C.K. √768
Die vraghe is naer die lynie D.H?
Antwoort/settet voor E.G 1 ꝛe soo is C.K ofte F. E. √972+8—1ꝛe trecket quadraet van E.G vant quadraet E.D ende vande reste / sal coomen V √.1420+√150528—1ꝫ voor D.G daer van trecket F.G. ofte C.K alse is √768 sal noch resten.
V √.1420+√150528—1ꝫ—:√768 voor D.F
V √.1420+√150528—1ꝫ—:√768.
1420+√150528—1ꝫ—
V √.1090560+√88785027072—768ꝫ
+768 —V √.1090560+√88785027072—768ꝫ
Fa.2188+√150528—1ꝫ—V √.4362240+√1420564

After a few pages of computation, Claes finds $DH = 8 + \sqrt{1472}$.

We have a field $ABCDE$ with two right angles at A and B.
Given $AB = 8 + \sqrt{972}$,
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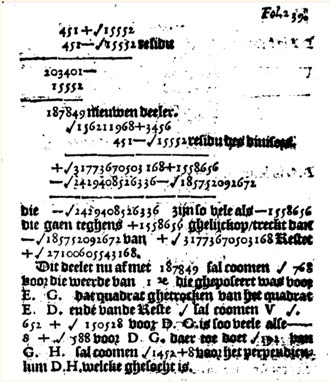

Fol. 39

451 + √15552
451 — √15552 residu

203401 —
15552

187849 nieuwen deeler.
√156211968 + 3456
451 — √15552 residu des divisors.

+ √31773670503168 + 1558656
— √24294085263 36 — √185752092672

die — √24294085263 36 zijn so vele als — 1558656 die gaen teghens + 1558656 ghelijckop/treckt dan — √185752092672 van + √31773670503168 Restet + √27100605543168.

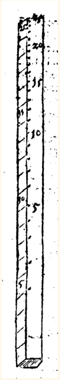
Dit deelet nu af met 187849 sal coomen √ 768 voor die weerde van 1 ℞ die gheposeert was voor E. G. dat quadrat ghetrocken van het quadrat E. D. ende vande Reste √ sal coomen V √. 652 + √ 150528 voor D. G. is soo veele alse — 8 + √ 588 voor D. G. daer toe doet √ 192 van G. H. sal coomen √ 1452 + 8 voor het perpendiculum D.H. welcke ghesocht is.

Pracital applications in the Practique: 1 Wine gauging

Given a wine barrel which is partly full. How to correctly determine the quantity of wine in it?

Answer: make a gauging rod
How to make such a rod (which can be used with many types of barrrels)

Important for taxation



Valentin Mennher, 1565

Practical applications in the Practique: 2 Sundials

Important because mechanical clocks in towers had to be adjusted every few days.
Horizontal sundial is relatively simple.
Vertical sundial is also relatively simple, but only if it is oriented precisely towards the south.

Photo: Sundial oriented towards the south, made by Hendrik Hollander
www.zonnewijzer.nl

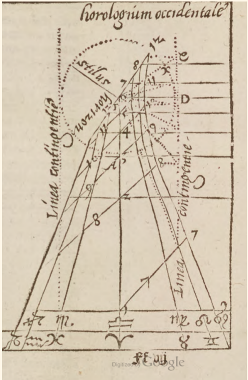
NB the angles are not exactly equal . . .
Table of sines necessary for the computation.

Claes Pietersz, part of his table of sines (1589) , R=100000

Graden M.	Sinus	1. Sinus	2. Sinus	3. Sinus	4. Sinus
0	00	1745	3489	5233	6975
1	29	1774	3519	5262	7004
2	58	1803	3548	5291	7033
3	87	1832	3577	5320	7062
4	116	1861	3606	5349	7091
5	145	1890	3635	5378	7120
6	174	1919	3664	5407	7149
7	208	1948	3693	5436	7178
8	232	1977	3722	5465	7207
9	261	2007	3751	5495	7236
10	290	2036	3780	5524	7265
11	319	2065	3809	5553	7294
12	349	2094	3838	5582	7323
13	378	2123	3867	5611	7352
14	407	2152	3896	5640	7481
15	436	2181	3925	5669	7310
16	465	2210	3955	5698	7439
17	494	2239	3984	5727	7468
18	523	2268	4013	5756	7497
19	552	2297	4042	5785	7526
20	581	2326	4071	5814	7555
21	610	2355	4100	5843	7584
22	639	2385	4129	5872	7613
23	669	2414	4158	5901	7642
24	698	2443	4187	5930	7671
25	727	2472	4216	[illegible]	[illegible]

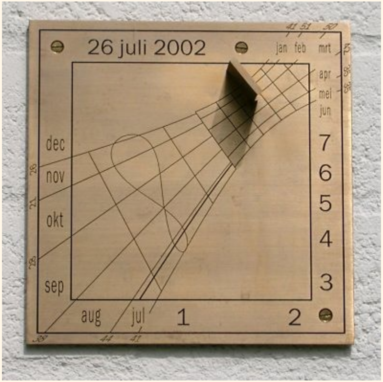
Practical applications in the Practique: 2 Sundials

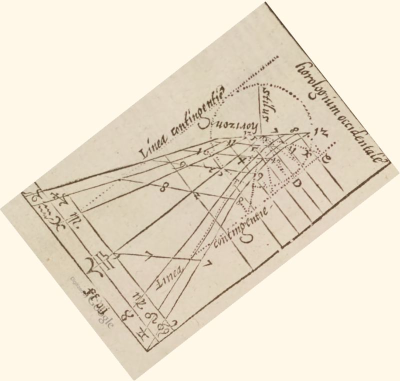
Sundial on vertical wall oriented towards the East or the West: also doable needed: table of tangents.



Compare Claes Pietersz with modern

Sundial made by Hendrik Hollander

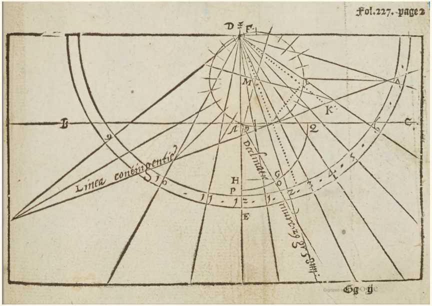




Vertical sundial on wall with arbitrary orientation is more difficult.

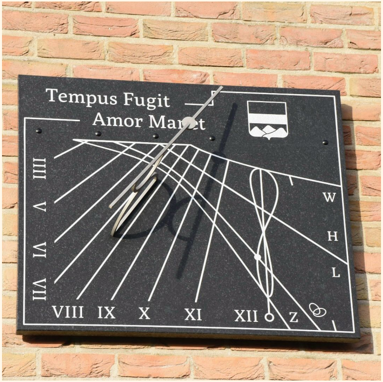
Geographical latitude $52^o 40'$ (Amsterdam)
Vertical wall declining $29^o 58'$ towards the West (from South) - how come?

Pages of computation of Claes Pietersz: if the sun is in Leo 7, and a rod perpendicular to the wall throws a vertical shadow if the altitude of the sun is $53^o 30'$, then the wall declines $29^o 58'$ towards the West.

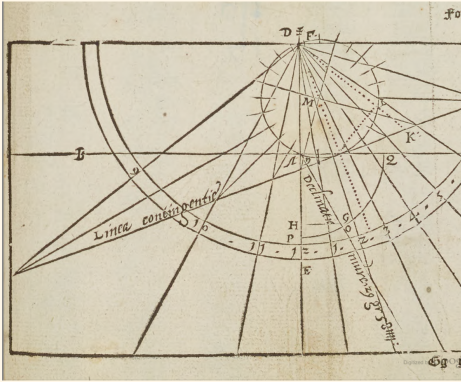


Vertical sundial with arbitrary orientation: similar patterns

Hendrik Hollander, vertical sundial declining towards the East



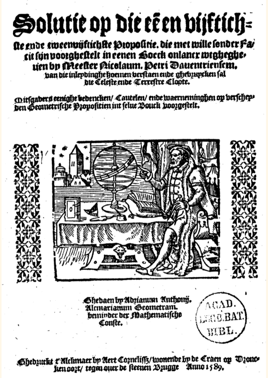
Geographical latitude 52°40′ (Amsterdam)
Vertical wall declining towards the West 29°58′.



Was this original work by Claes Pieterszoon?

The theory was not new (known in what is now Belgium)
But Claes Pietersz had to redo all the computations for the latitude of Amsterdam $52^o 40'$.

(He was criticized by the famous Adriaan Anthoniszoon, fortress designer, 1541-1620, who computed $\pi \approx 355/113$)



Solutie op die eeen vijftich-
ste ende tweenvijftichste Propositie. die met wille sonder Fa-
cit sijn voorghestelt in eenen Boeck onlancx uitghegheve-
ven by Meester Nicolaum. Petri Daventriensem.
van die inleydinghe hoemen verstaen ende ghebruycken sal
die Celeste ende Terrestre Cloote.

Mitsgaders eenighe bedencken/ Cautelen/ ende waerneminghen op verschey-
den Geometrische Propositien int selve Bouck voorgestelt.

Ghedaen by Adrianum Anthonij.
Alcmariamum Geometram.
beminder der Mathematische
Conste.

Ghedruckt t'Alckmaer by Aert Cornelisz/ wonende by de Craen op Vrouc-
ken poort/ tegen over de steenen Brugge Anno 1589.

Why is Claes Pietersz important?

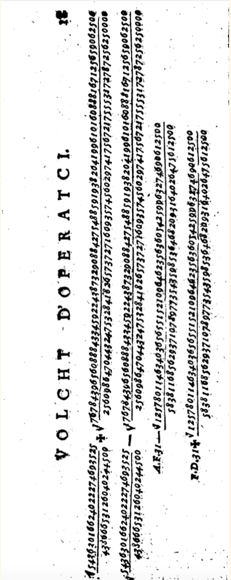
First extensive book in Dutch language on algebra and geometry

Used for generations

Was the beginnng of a mathematical literature in the Netherlands - so people who did not know Latin could learn mathematics and become part of the Dutch mathematical community.

Beginning in 1600 there were mathematics curricula in Dutch at two universities (Leiden, Franeker).

Jan Beerntsen, sailor and a good mathematician (1612).



Thanks for your attention!

See http://www.jphogendijk.nl
for a webpage on 16-th and 17th century Dutch mathematics and separate webpages for arithmetics and Claes Pietersz.