Miniworkshop on the abjad number system

0. Aim. This miniworkshop is about the Arabic abjad notation for numbers in the sexagesimal system. This system was used by astronomers and mathematicians who wrote in Arabic (and Persian) between 800 and 1900 AD. We will read a value of π in a medieval manuscript. You don't need to be familiar with Arabic writing.

1. We first explain the principle of the abjad-system in the familiar (Latin) alphabet in a system which we have invented for this workshop, purely for didactic reasons. We assign a numerical value to some letters (including a few upper-case letters) as follows.

a=1	**b**=2	**j**=3	**d**=4	**h**=5	**w**=6	**z**=7	**H**=8	**T**=9
i=10	**k**=20	$\boldsymbol{\ell}$=30	**m**=40	**n**=50				

We use these letters to write all integer numbers less than 60: first write the tens, then the units, not the other way around. Thus 11 is **ia**, not **ai**

Exercise 1: Read the abjad-numbers: **nw,** $\boldsymbol{\ell}$**d, kH.** Write your answers: ...

Exercise 2: Write in abjad: 13, 31, 49. Write your answers: ...

In modern life we use a sexagesimal (base-60) notation for hours, minutes and seconds. In antiquity and the middle ages, the system was also used to write integers and fractions as in the folowing example in abjad:

a kd na i means $$1 + \frac{24}{60} + \frac{51}{3600} + \frac{10}{60^3}.$$

(an apprroximation of $\sqrt{2}$ used by al-Khwārizmī (ca. 830 AD).)

Exercise 3: Write in abjad:
$$3 + \frac{8}{60}$$

Write your answer: ...

Preliminaries about Arabic writing: Arabic is written from right to left. Every Arabic letter has several forms: (1) an isolated form, (2) an initial form at the beginning (right side) of a word, (3) a final form at the end (left side) of a word, and (4) an intermediate form (which we will not need in the miniworkshop). The Arabic does not have upper-case letters.

There are a normal letter h and a sharp guttural H. Also a normal t and a 'thick' palatal T.

The Arabic letters that we will read are in a shorthand notation used by medieval mathematicians and astronomers; they deviate in some details from the modern shapes.

2. Table of the numbers 1-59 in Arabic abjad notation.
The number 34 is in the column for 30 and the row for 4.

		(0)	10	20	30	40	50
	letter		**i**	**k**	**ℓ**	**m**	**n**
			ى	ك	ل	م	ن
+1	**a**	ا	يا	كا	لا	ما	نا
+2	**b**	ب	يب	كب	لب	مب	نب
+3	**j**	ج	يج	كج	لج	مج	نج
+4	**d**	د	يد	كد	لد	مد	ند
+5	**h**	ه	يه	كه	له	مه	نه
+6	**w**	و	يو	كو	لو	مو	نو
+7	**z**	ز	يز	كز	لز	مز	نز
+8	**H**	ح	يح	كح	لح	مح	نح
+9	**T**	ط	يط	كط	لط	مط	نط

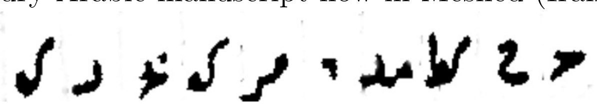
Exercise 4. Find in the table, in the third column to the left, the isolated forms of the letters **a**=1, **b**=2, **j**=3, **d**=4, **h**=5, **w**=6, **z**=7, **H**=8, **T**=9. Find in the third row the isolated forms of **i**=10, **k**=20, **ℓ**=30, **m**=40, **n**=50.

Exercise 5. Find the numbers 51 and 52 in the table. Identify the initial form of **n**=50 and the final forms of 1= **a** and 2= **b**. Write your answers:

What are the initial forms of **k** and **m**, and the final forms of **d** and **T**? Write your answers: ...

Exercise 6. Write in Arabic abjad 3, 8. Write your answer: ...
Again, but now from right to left: ...

Written from right to left, this means the number $3 + \frac{8}{60}$.
This number can be seen on the right side of the figure, taken from a 15th century Arabic manuscript now in Meshed (Iran):



Exercise 7. The number in the manuscript continues. Read the third abjad-number from the right, and add this number times $\frac{1}{60^2}$ to the fraction $3 + \frac{8}{60}$ (do not simplify the result). Then read the fourth abjad-number from the riigt, multiply by $\frac{1}{60^3}$, and add to the fraction $3 + \frac{8}{60} + \frac{\cdots}{60^2}$. The fifth number to the right is a zero (originally a little circle with an overbar), meaning $\frac{0}{60^4}$. Then read all the remaining abjad-numbers, multiply with the appropriate powers of $\frac{1}{60}$ and add them to the previous ones. The result can be interpreted as an approximation of π.

Find out how accurate this approximation is, either by computing the approximation with a calculator, or by entering the two words “pi sexagesimal” (without the quotation marks) in the website http://www.wolframalpha.com

End of the miniworkshop. See also: www.jphogendijk.nl/talks/abjad.html