Dead texts versus living teachers: remarks on the transmission of Greek mathematics into Arabic.

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In memoriam John Murdoch, 1927-2010

Questions: can we draw conclusions, on the basis of a text, about the mathematical competence of the author?
Murdoch said: Yes!

3. Examples from Murdoch's article Euclid

Transmission of the Elements DSB IV, pp. 437-459.

" the **particularly astute** commentary on book V written by the Andalusian mathematician Ibn Mu^{c}adh al-Jayyānī. It contains . . . the first known **comprehension** of the brilliant definition of the equality of ratios formulated by Eudoxus. In fact . . . this definition was seldom **properly understood** in the West before Isaac Barrow in the seventeenth century... [p. 441]

4. More examples from Murdoch

"a north German scholar with apparently **more mathematical wit** than, for example, the author of the two-book "Boethius" discussed above..." [p. 443]
"the translator **knew little** of what he was doing..." [p. 444]
"when one turns to the translation itself, it is immediately evident that its author was **extremely acute**, both as an editor and **as a mathematician**"

My questions (for the medieval Islamic tradition)

1. Is mathematical ability or competence a modern notion, which we superimpose on history, or was it somehow known in the medieval Islamic tradition?
2. What can we say, on the basis of a translation of a mathematical text, on the mathematical ability (or knowledge) of its translator?
3. Was there an oral transmission of mathematics from Greek into Arabic at the beginning of the Islamic scientific tradition?

Mathematical ability / competence

Two examples:
a. Banū Mūsā (9th c):
"there are bad mathematicians who prove false theorems, and there are only a few good mathematicians"

b. Sijzī (10th c.) explains how to become a good mathematician. People who have an "inborn natural talent" and who work hard will be outstanding, but even people without such a talent have a chance. "Archimedes, the culmination of the Greeks" could " visualize at one moment that many figures are constructed"

Mathematical ability of translators?

Example: Euclid, *Elements*, Book V, definitions 4, 5 - what Murdoch called the brilliant definition of equuality of ratios (by Eudoxos).

In modern terms:
Def. 4. A and B can have a ratio if there are integers m, n such that $mA > B$ and $nB > A$.

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In modern terms:
Def. 4. A and B can have a ratio if there are integers m, n such that $mA > B$ and $nB > A$.

Def. 5 $A : B = C : D$ if for any integers m, n
$mA > nB \rightarrow mC > nD$
$mA = nB \rightarrow mC = nD$
and
$mA < nB \rightarrow mC < nD$

Euclid's Greek text (tr. Heath)

Def. 4 "Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another."

Def. 5 "Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if equimultiples be taken of the first and third, and equimultiples of the second and fourth, whatever equimultiples they are, then the former equimultiples alike exceed, are alike equal to, or alike fall short of the latter equimultiples respectively taken in corresponding order."

Arabic translation (al-Ḥajjāj. 9th c.) is correct!

Def. 4 "Euclid said: magnitudes of which it is said that between them is a ratio, are (those magnitudes) which can, if they are multiplied (ḍa^{c}fa) , exceed one another."

Def. 5 "It is said of magnitudes that they are in the same ratio, the first to the second and the third to the fourth, if multiples of the first and third, which (multiples) are equal in number, either alike exceed multiples of the second and the fourth, which (multiples) are equal in number, whatever multiples they are, or they are alike equal, or they alike fall short, if they are compared in order."

Arabic translation (al-Ḥajjāj. 9th c., Codex Leidensis 399,1)

قال اقليدس المقادير التي يقال انّ بين بعضها وبعض نسبة هي التي قد يمكن اذا ضوعفت ان يفضل بعضها على بعضٍ

قال اقليدس يقال في المقادير انها في نسبة واحدة الأول الى الثاني والثالث الى الرابع متى كانت اضعاف الأول والثالث المتساوية المرات إما ان تفضل معًا على أضعاف الثاني والرابع المتساوية المرات اي الأضعاف كانت وإما ان تساويها معًا وإما ان تنقص عنها معًا اذا قست على الولاء بعضها ببعضٍ

Euclid's Greek text

Def. 4 *Logon echein pros allèla megethè legetai ha dunantai pollaplasiazomena allèlōn huperechein.*

Def. 5 *En tōi autōi logōi megethè legetai einai prōton pros deuteron kai triton pros tetarton hotan ta tou prōtou kai tritou isakis pollaplasia tōn tou deuterou kai tetartou isakis pollaplasiōn kath' hopoionoun pollaplasiasmon hekateron hekateron é hama huperechèi é hama isa é hama elleipèi lèfthenta katallèla*

Did al-Ḥajjāj understand what he was translating?

The text(s) we have may have been revised (by Thābit ibn Qurra).

Someone in the 9th century must have understood the definition. Mathematical meaning is a higher-level meaning encoded in the text. A translation can be grammatically correct but mathematically meaningless.

Available information on translation process

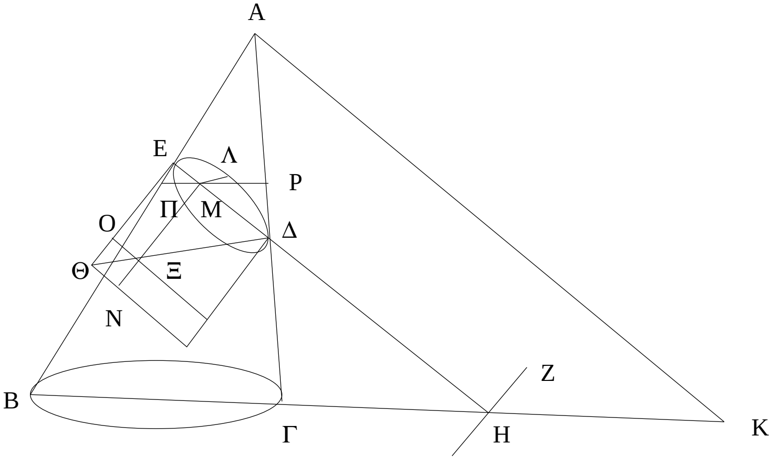
Conics of Apollonius, ca. 200 BC, Books 1-7.

translated under the supervision of the Banū Mūsā (late 9th c.) They say they first reconstructed the deductive structure of the (Greek) text by themselves.

Then the translation was commissioned (Bk. 1-4 to Hilāl ibn Abī Hilāl al-Ḥimṣī, Bk. 5-7 to Thābit ibn Qurra)

The Banū Mūsā understood the mathematics!

Example of text and translation: Conics I:13, Greek



Example of text and translation: Conics I:13, Greek

If a cone is cut by a plane through the axis, and by another plane meeting each side of the axial triangle, being neither parallel to the base nor subcontrary, and let the plane containing the base of the cone meet the cutting plane in a straight line perpendicular either to the base of the axial triangle or to its rectilinear extension; then any parallel line drawn from the section of the cone (parallel) to the common section of the planes as far as the diameter of the section will be equal in square to a certain area applied to a certain straight line, such that the diameter of the section will bear to it the same ratio as the square on the line drawn from the vertex of the cone parallel to the diameter of the section as far as the base of the triangle bears to the rectangle contained by the intercepts made by it on the sides of the triangle; the (area) having (as its) breadth the intercept made by it on the diameter in the direction of the vertex of the section, and being deficient (Greek: *elleipon*) by a figure similar and similarly situated to the rectangle contained by the diameter and the line along which they are equal in square, and let such a section be called an ellipse.

Example of text and translation

كل مخروط يعطع بسطحٍ يمرّ على سهمه ويعطع أيضًا بسطحٍ آخر يمرّ على كلي
ضلعي المثلث المخرج على سهم المخروط ولا يكون موازيًا لقاعدة المخروط ولا
مخالفًا لها ويكون السطح الذي فيه قاعدة المخروط والسطح الذي يقطع المخروط
إذا أخرجا يلتقيان على خط يكون على زوايا قائمة إمّا على قاعدة المثلث المخرج
على سهم المخروط وإما بالخط المتصل بالقاعدة على استقامة فإنّ الخط الذي يخرج
من أي موضع كان من خط القطع إلى قطره إذا كان موازيًا للخط الذي يتقاطع
عليه السطحان يقوى على سطحٍ مضافٍ إلى خطٍ يكون نسبة قطر القطع إليه
كنسبة مربع الخط الذي يخرج من رأس المخروط موازيًا لقطر القطع ويلقى قاعدة
المثلث الذي يمرّ على سهم المخروط إذا أُخرجت القاعدة على استقامة

Example of text and translation

إلى الذي يكون من ضرب وتر الزاوية التي عند رأس المخروط ويحيط بها الخط
الذي يخرج من رأس المخروط موازيًا لقطر القطع وأحد ضلعي المثلث الذي يمرّ
على سهم المخروط في وتر الزاوية التي عند رأس المخروط أيضًا ويحيط بها الخط
الذي خرج من رأس المخروط موازيًا لقطر القطع والضلع الآخر من المثلث الذي
يقطع المخروط على سهمه ويكون عرض ذلك السطح الخط الذي يكون من موضع
مسقط الخط المخرج من القطع على قطر القطع إلى رأس القطع ويكون ناقصًا
سطحًا شبيهًا بالسطح الذي يكون من ضرب قطر القطع في الخط الذي يقوى
عليه الخطوط المخرجة إلى قطر القطع على الترتيب وليُسَمّ هذا القطع الناقص

Other example of 'reconstruction'

Apollonius, *Determinate Section* (now lost, but mentioned by Pappus), Books I-II.
The *Fihrist* (10th c. Arabic bibliographical work) says: Book I was revised by Thabit ibn Qurra. Book II translated into Arabic but not understood.

This means that the meaning of Book 1 was reconstructed.

Dead texts or living teachers?

Apollonius; *Conics* and *Determinate Section* entered the Islamic tradition as dead texts without oral transmission.

What about Euclid's *Elements*, the work in which the Euclidean way of doing mathematics was explained, and which was the basis of everything else in Greek geometry [and in astronomy, of the *Almagest* of Ptolemy]

Another example

Example of a meaningless translation (this time from the Latin tradition). Geometrical figure with labels $A, B, \Delta, etc.$
Greek text: [from Elements book 2 prop. 8, cf Heath p. 391 line 5-6] Four times the [rectangle] under $AB, B\Delta$ is equal to the gnomon ΣTY
"Translation" [Latin palimpsest Munchen, 10th c.]: "That forty times what is under first second, second and fourth is equal to what is two hundred three hundred four hundred know" [etc.]

My hypothesis

Before people like al-Ḥajjāj could start to translate the *Elements*, someone had to explain the Euclidean way of doing mathematics to them.
(Byzantine or Christian Syrian scholars?)
This oral transmission must have been crucial.

An afterthought

The same question about oral transmission can be asked for the transmission of mathematics from Arabic into Latin. This question is relevant in contemporary Dutch politics

[i.e., can Muslims be mathematics teachers of Christians??? directly or possibly indirectly through Jews]

This presentation can be downloaded at

http://www.jphogendijk.nl/talks/montrealpres.pdf