Inspired science: methods for finding the qibla in the medieval Islamic tradition.

Jan P. Hogendijk

Department of Mathematics, Utrecht, Netherlands / KFUPM

April 17, 2012

Universiteit Utrecht

Introduction

1. Non-mathematical ways to find the qibla

2. Simple methods using geographical coordinates

3. Distance along a great circle; spherical trigonometry

4. Two examples of my own research

Universiteit Utrecht

Prayer facing the Kaaba

Since the second year of the Hijra (624 CE), the qibla (direction of prayer) in Islam has been towards the Kaaba in Makkah.

We know a lot about Islamic methods for finding the qibla thanks to two Western scholars:

E.S. Kennedy (1912-2009)

David A. King (Frankfurt, Germany; born 1941)

Three types of Islamic methods for finding the qibla.

1. Traditional non-mathematical methods (from early Islam until recently)

2. Simple approximation methods using geographical coordinates (since the time of Caliph al-Maʾmūn, ca. 200 H./ 810 CE until recently)

3. Exact mathematical computation along great circle on the spherical earth (from the early 4th century H./900 CE onwards)

Universiteit Utrecht

Traditional non-mathematical methods

Method A: Use the same qibla as the Prophet Mohammad (saws) and his Companions in Al-Medina: due south.

Traditional non-mathematical methods

Method A: Use the same qibla as the Prophet Mohammad (saws) and his Companions in Al-Medina: due south.

Method B: Use the direction of the road leaving your city towards Mecca.

Traditional non-mathematical methods

Method A: Use the same qibla as the Prophet Mohammad (saws) and his Companions in Al-Medina: due south.

Method B: Use the direction of the road leaving your city towards Mecca.

Method C: Use a symbolic world map with the Kaaba als its center (second century H. and later).
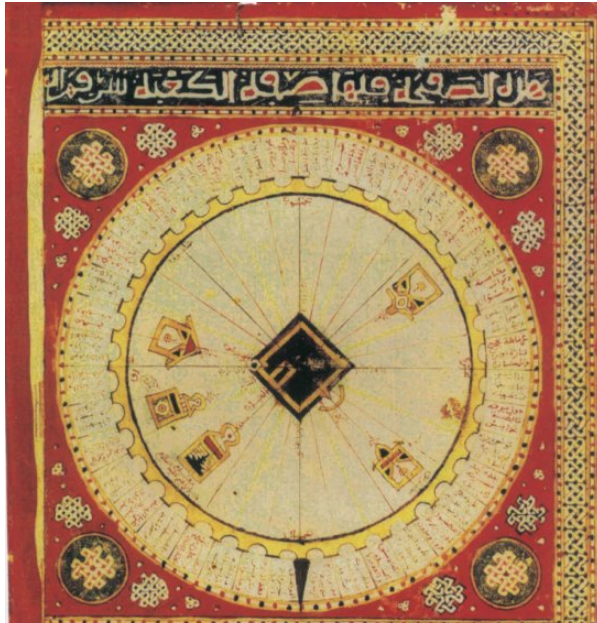
A symbolic world map (second century H.)

The qibla can be a direction on the horizon, for example where certain stars rise or set. No numbers are involved.
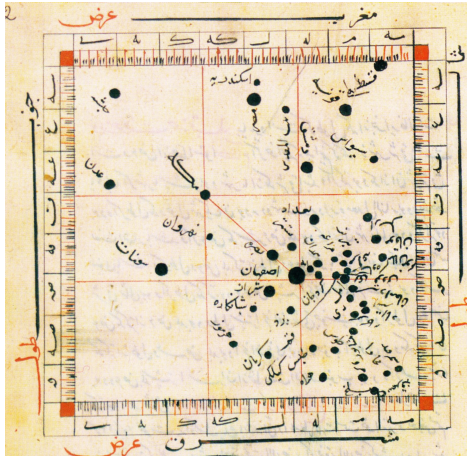

Universiteit Utrecht
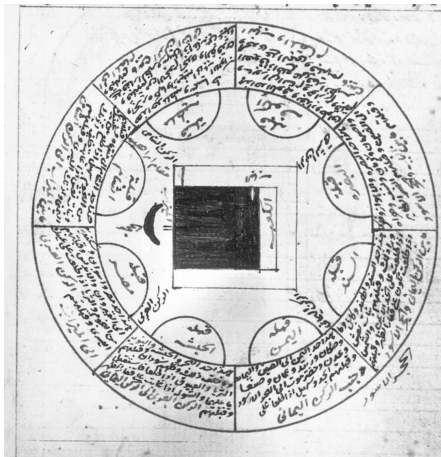
Another symbolic world map (10th c. Hijra / 16th c. CE)

2. Geographical coordinates: the general idea

Obtain geographical coordinates of your locality and of Makkah, and draw them on a map with a rectangular grid.

Geographical coordinates: the general idea

Then join your locality and Makkah by a straight line. This will indicate the direction of the qibla

Output: an angle, usually expressed in degrees, either the *samt al-qibla* (azimuth of the qibla, deviation from east or west) or the *inhirāf al-qibla*: its deviation from the south.

Universiteit Utrecht

Geographical coordinates were known to Caliph al-Maʾmūn (ca. 200 H./820 CE)

Zero meridian: usually Canary Islands (west of inhabited world)

1 degree on the sphere is $56\frac{2}{3}$ miles where 1 mile = 4000 ancient Babylonian cubits (49,3 cm).

so the circumference of the earth is ca. 40200 km

Globe owned by Maʾmūn (reconstruction by Fuat Sezgin)

Finding the qibla of Baghdad by the astronomers of Caliph al-Ma'mūn (as reconstructed from defective sources)

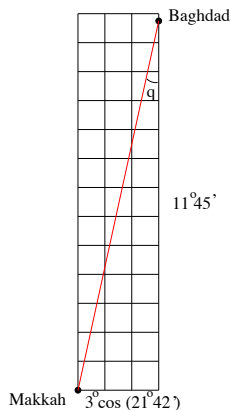Geographical latitude of Makkah $\phi_M = 21^o40'$, Baghdad $\phi = 33^o25'$, difference $\Delta\phi = 11^o45'$. Difference between geographical longitudes $\Delta\lambda = 3^o$ (found from lunar eclipse observed at Baghdad and Makkah) On the grid, longitudes are shortened by $\cos 21^o42' \approx \frac{56}{60}$.

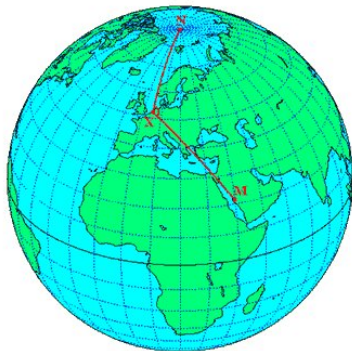The *inḥirāf* of the qibla was fonud as 13° West of South. Formula: $\tan q = \Delta\lambda \cos\phi_M/\Delta\phi$.

Baghdad

q

$11^o45'$

Makkah $3^o\cos(21^o42')$

According to the mathematicians from the 4th c. H/10th c. CE onwards

The qibla is along an arc of a great circle on earth (shortest distance to Makkah as a bird would fly)

The center of any great circle coincides with the center of the earth.

Modern formula

Notations: $\Delta\lambda$ difference between the geographical longitudes of one's city and Makkah, ϕ geographical latitude of one's city, ϕ_M geographical latitude of Makkah, q inḥirāf of the qibla (angle with due South), Then:

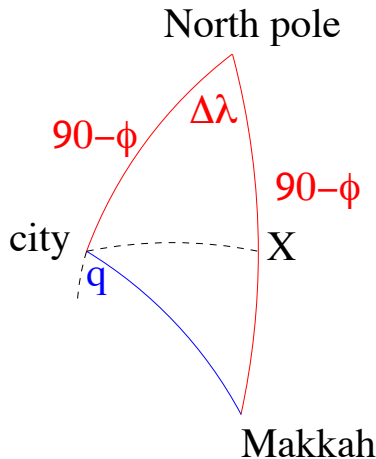$$\tan q = \frac{\sin\Delta\lambda}{\cos\Delta\lambda\sin\phi - \cos\phi\tan\phi_M}.$$

Muslim scientists used equivalent methods such as the followiing

Spherical triangle with three great circle arcs North pole - one's city - Makkah.
Red quantities are given. We want to compute the blue quantity.
From your city draw a great circle arc perpendicular to the arc North Pole - Makkah.
Then we obtain two right-angled spherical triangles with a right angle X.

North pole

$\Delta\lambda$

$90-\phi$

$90-\phi$

city

X

q

Makkah

Muslim scientists used equivalent methods such as the following (by Abū Rayḥān al-Bīrūnī, ca. 410 H. / 1020 CE)

We know $\Delta\lambda, \phi, \phi_M$.
Then compute:
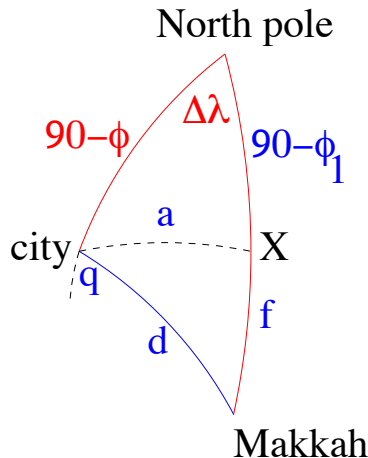(1) $\sin a = \cos\phi \sin\Delta\lambda$
(2) $\sin\phi_1 = \sin\phi / \cos a$
(3) $f = \phi_1 - \phi_M$
(4) $\cos d = \cos f \cos a$
and finally(5):
$\sin q = \cos\phi_M \sin\Delta\lambda / \sin d$
(sine rule in spherical triangle).

North pole

$\Delta\lambda$

$90-\phi$

$90-\phi_1$

a

city

X

q

f

d

Makkah

Al-Bīrūnī expressed this as follows (his jayb is modern 60 times the modern sine, al-jayb kulluhu = 60)

إذا أردنا سمت القبلة ، ضربنا جيب تمام عرض بلدنا فى جيب ما بينه وبين مكّة فى الطول ، وقسمنا المبلغ على الجيب كلّه ، فيخرج جيب // العمود ، نقوّسه ونأخذ جيب تمامها ، ونقسم عليه مضروب جيب عرض بلدنا فى الجيب كلّه ، فيخرج جيب نقوّسه ، ونأخذ الفضل بينه وبين عرض مكّة ، ونضرب جيب تمام هذا الفضل فى جيب تمام العمود ، ونقسم المبلغ على الجيب كلّه ، فيخرج جيب نقوّسه ، ونأخذ جيب تمامها ، ونقسم عليه مضروب جيب تمام عرض مكّة فى جيب ما بين الطولين ، فيخرج جيب بُعد السمت عن خطّ نصف النهار ببلدنا ، وعلى

Universiteit Utrecht

Al-Bīrūnī's numerical example: Ghazni, Afghanistan.

We know $\Delta\lambda, \phi, \phi_M$.
Then compute:
(1) $\sin a = \cos\phi \sin\Delta\lambda$
(2) $\sin\phi_1 = \sin\phi / \cos a$
(3) $f = \phi_1 - \phi_M$
(4) $\cos d = \cos f \cos a$
and finally(5):
$\sin q = \cos\phi_M \sin\Delta\lambda / \sin d$
(sine rule in spherical triangle).

$\Delta\lambda = 27°22'24''$,
$\phi = 33°35'$, $\phi_M = 21°40'$.
(1) $a = 22°31'19''$
(2) $\phi_1 = 36°46'48''$
(3) $f = 15°6'48''$
(4) $d = 26°54'6''$
(5) $q = 70°47'6''$

Reflections by medieval Islamic scientists on the qibla problem

Most Muslim scientists give only the mathematics, nothing else. Al-Bīrūnī is an exception.

He says that the qibla can only be found by "astronomers" if the distance to Makkah is large, but the computation is difficult and takes time. In order to find the correct qibla, correct geographical coordinates are necessary.

Universiteit Utrecht

Reflections by medieval Islamic scientists on the qibla problem

Most Muslim scientists give only the mathematics, nothing else. Al-Bīrūnī is an exception.

He says that the qibla can only be found by "astronomers" if the distance to Makkah is large, but the computation is difficult and takes time. In order to find the correct qibla, correct geographical coordinates are necessary.

He makes fun of the traditional non-mathematical solutions, and about one person who tried to prove that the qibla is always due south.

Universiteit Utrecht

Reflections by medieval Islamic scientists on the qibla problem

Most Muslim scientists give only the mathematics, nothing else. Al-Bīrūnī is an exception.

He says that the qibla can only be found by "astronomers" if the distance to Makkah is large, but the computation is difficult and takes time. In order to find the correct qibla, correct geographical coordinates are necessary.

He makes fun of the traditional non-mathematical solutions, and about one person who tried to prove that the qibla is always due south.

He says that for the qibla, "exactness can only exist in thought; in reality approximations are unavoidable" and relates this to Qur'an 2:144, 155.

Universiteit Utrecht

4. Two examples of historical research on the qibla problem

1. Islamic table for the inḥirāf of the qibla computed in degrees and minutes of arc for longitude and latitude differences $\Delta\lambda = 1\ldots30$ and $\Delta phi = 1\ldots30$. Source: Ashrafī Zīj (astronomical handbook, ca. 700 H/ 1300 CE).

Ashrafi Zīj: part of the table

It is necessary to identify and correct scribal errors in the manuscript.

Ashrafi Zīj: all values for multiples of 3 degrees were computed by an exact method, rest by linear interpolation

| $\Delta\phi$	24	25	26	27	28
$\Delta\lambda$					
1	2;17	2;13	2;08	2;03	2;00
2	4;34	4;25	4;15	4;06	3;58*
3	**6;51**	6;37	6;23	**6;09**	5;58
4	9;06	8;48	8;29	8;11	7;56
5	11;21	10;58	10;35	10;13	9;55*
6	**13;37**	13;09	12;42	**12;15**	11;53
7	15;47	15;17	14;42	14;13	13;47
8	17;55	?	16;46	16;11	15;40
9	**20;08**	19;28	18;49	**18;09**	17;37*
10	22;12	21;28	20;40	20;01	19;24*
11	24;16	23;30	22;44*	21;57	21;11

Second example. Instrument for qibla finding, constructed in the 11th/17th c. in Isfahan, discovered in 1990 by David A. King.

Universiteit Utrecht
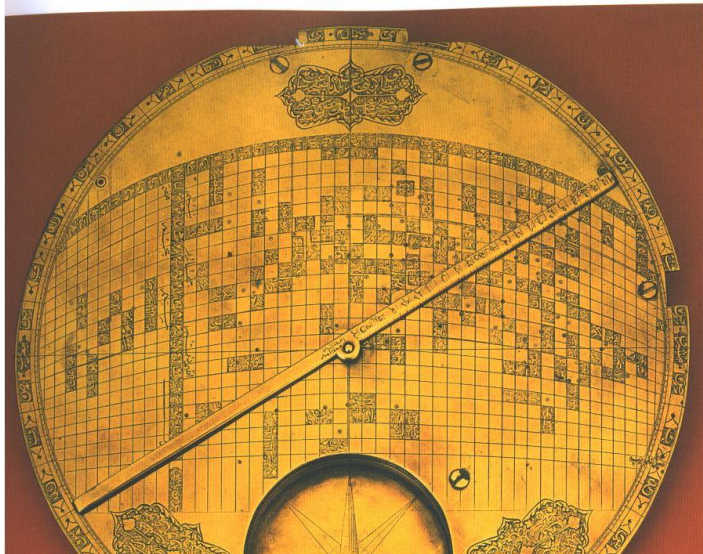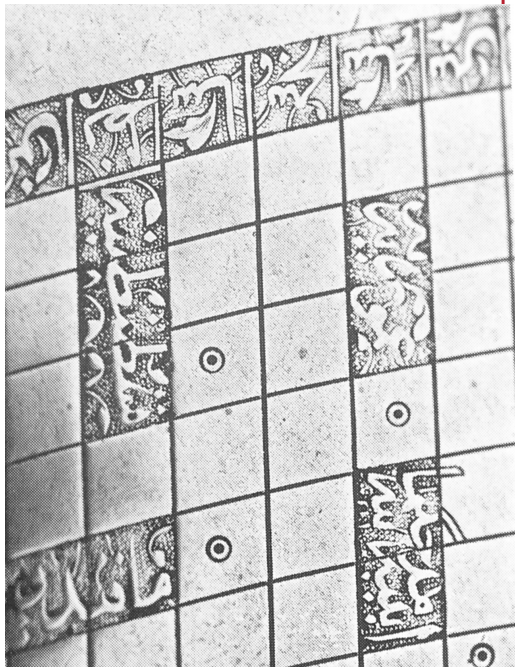
Finding the qibla in Istanbul
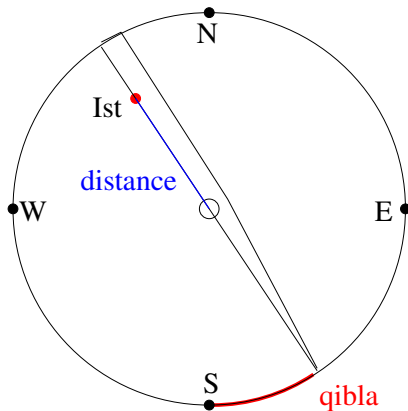
Questions

1. Is this instrument mathematically correct?

2. When and by whom was it invented?

A Dutch historian of science Elly Dekker answered in 2000;
1. yes, the curves on the instrument are ellipses.
2. Probably in Europe (France), and presented to the Shah of Esfahan in the 17th century.

My own research: this instrument is Islamic (not French) and related to an Islamic geometric construction of the qibla by al-Bīrūnī.

Only preserved in an Arabic manuscript in Leiden, Holland.

# Essentially the same construction, also by al-Bīrūnī

Starting with a circle with center C and the four directions, and inputting the geographical coordinates of a city (for example Istanbul) and Makkah, the construction produces a point M in such a way that CM is the direction of the qibla. See figure; note $LM = KJ$.
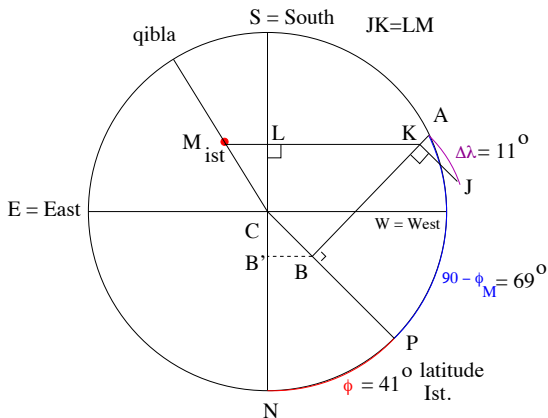
Point *M* depends on the geographical coordinates. For different cities, we obtain different points *M*.
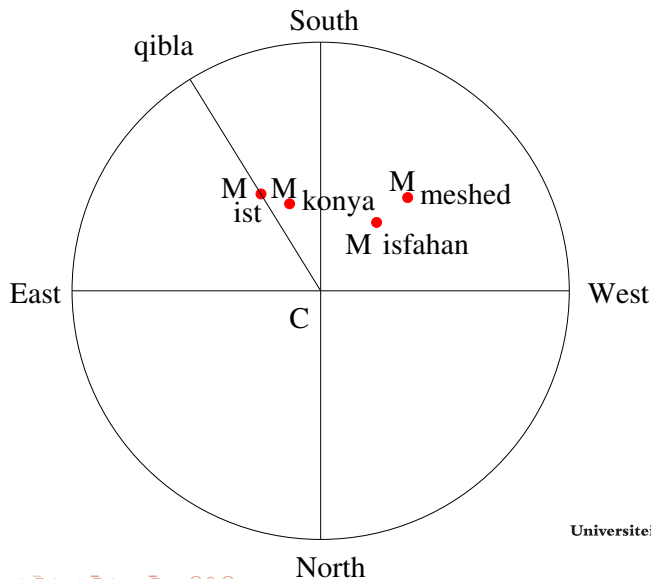
It turns out that the mathematics is exactly the same as in the instrument (if North/South and East/West are interchanged)
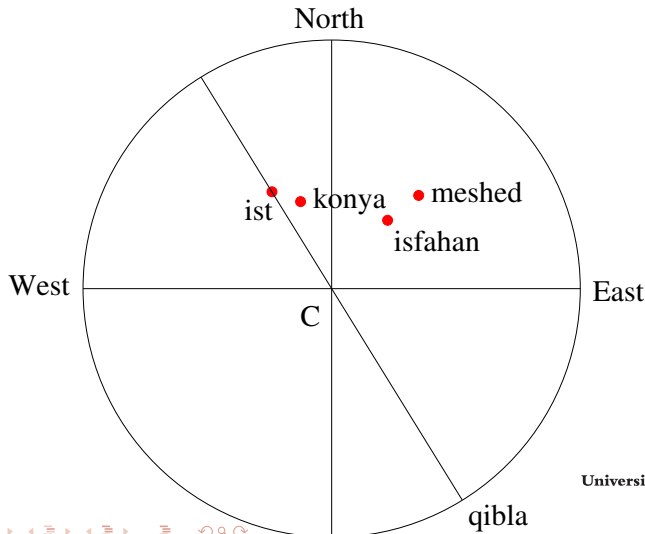
Modern proof (I will skip this in the lecture but you can download the presentation from the internet):

Referring to the previous figure, put $x = -LM$ and $y = LC$, and drop perpendicular BB' to CN. Put the radius of the circle equal to 1. Then we have $\angle NCP = \phi$, $\angle PCA = 90^o - \phi_M$ so $AB = \cos\phi_M$, $\angle ABJ = \Delta\lambda$ so $x = -LM = -KJ = -\sin\Delta\lambda\cos\phi_M$ (1). Also $CB = \sin\phi_M$ so $CB' = \sin\phi_M\cos\phi$ and since $\angle LKB = \angle BCN = \phi$, $LB' = KB\sin\phi = \cos\Delta\lambda\cos\phi_M\sin\phi$. Therefore $y = LB' - CB' = \cos\Delta\lambda\cos\phi_M\sin\phi - \sin\phi_M\cos\phi$, and $(\frac{x}{\cos\phi_M})^2 + (\frac{y + \sin\phi_M\cos\phi}{\cos\phi_M\sin\phi})^2 = 1$. (2).

By (1), the meridians (ϕ variable, $\Delta\lambda$ constant) are represented by lines $x = -\sin\Delta\lambda\cos\phi_M$ parallel to the y-axis, and by (2), the parallels ($\Delta\lambda$ variable, ϕ constant) by ellipses $(\frac{x}{\cos\phi_M})^2 + (\frac{y + \sin\phi_M\cos\phi}{\cos\phi_M\sin\phi})^2 = 1$. Note that the modern formula for the *inhirāf* $q = \angle MCL$ can be found from $\tan q = |x/y|$, and that by Al-Blruni's formula (5), $MC = |x|\sin q = \sin d$.

Universiteit Utrecht

Modern version of the instrument (programmed by my student Eelco Nederkoorn, joint project of Utrecht university and KFUPM)
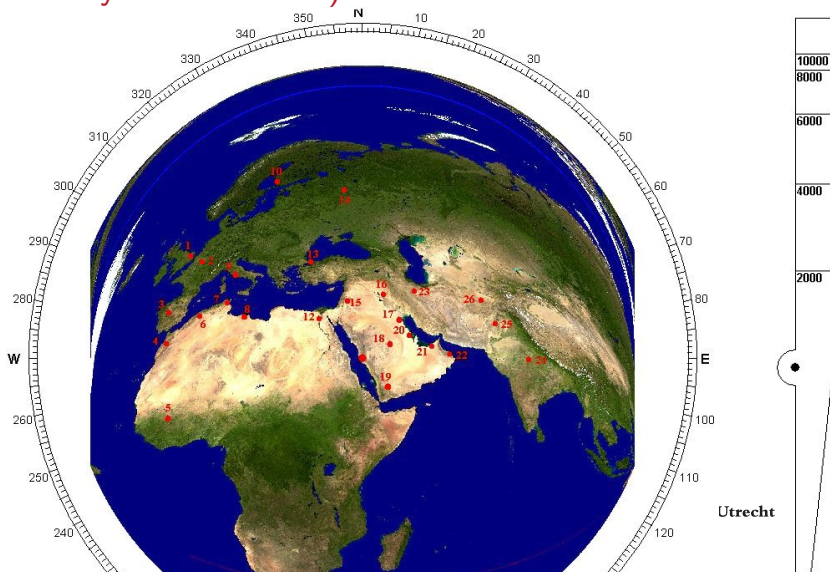
Instructions for use

Makkah is the center of the instrument.
Turn the ruler in such a way that the side which passes through the center (Makkah) also passes through the city where you want to know the qibla.
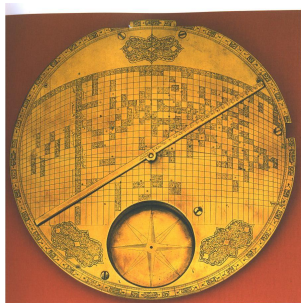
Then the tip of the ruler indicates the qibla on the circular scale. On the ruler itself you can read the (shortest) distance to Makkah along the great circle arc.

Universiteit Utrecht

What happens to the rest of the world?

Utrecht
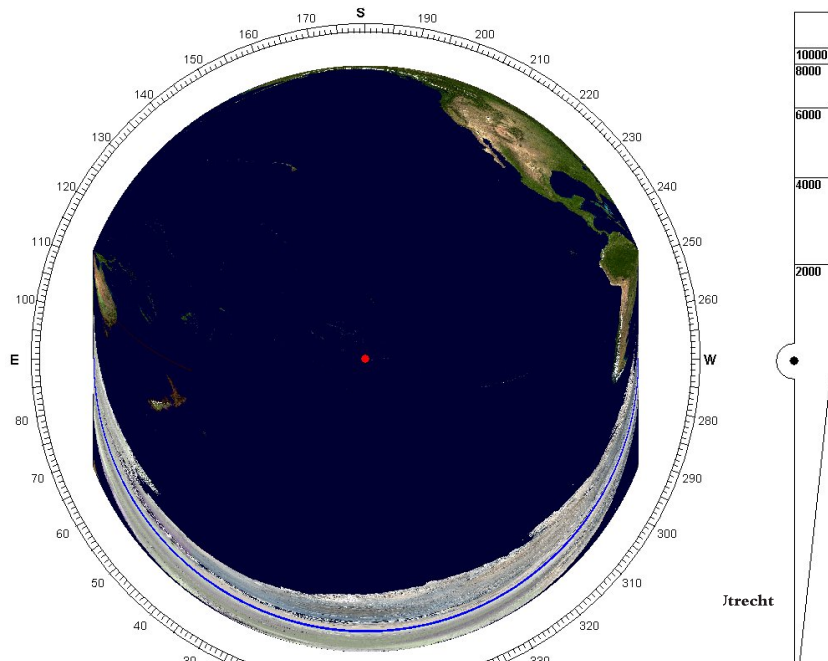
A proposal

Perhaps one can make, in the city of Makkah, a large copy of the instrument (ca. 3 meters of diameter), with the grid but with modern cities inscribed on it, and mount it horizontally so that people can walk around it and turn the ruler.

This will be a splendid example of the achievements of Islamic science, and will illustrate the central position of Makkah in Islam.