TWO TREATISES BY IBN AL-HAYTHAM ON THE DETERMINATION OF THE MERIDIAN LINE, TRANSLATED FROM THE EDITION BY FUAT SEZGIN

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1 Introduction

In 1986 Fuat Sezgin published an Arabic edition [8] of the two extant treatises by Ibn al-Haytham on the determination of the meridian line. Sezgin based his edition on a medieval Arabic manuscript which is now in Berlin.

The first treatise is entitled "Determination of the meridian line by means of one shadow" (*istikhrāj khaṭṭ niṣf al-nahār bi-ẓill wāḥid*) [6, p. 368 no. 21]. The treatise was probably written for craftsmen and contains instructions on how to determine the meridian line during the day from one shadow, if some extra information is available (declination of the sun and geographical latitude). Ibn al-Haytham presents one geometrical figure but he does not give proofs. In 1989, E.S. Kennedy published a brief summary and analysis of the first treatise in [3].

The second treatise bears the title "On the determination of the meridian line with utmost exactness" (*fī istikhrāj khaṭṭ niṣf al-nahār ʿalā ghāyat al-taḥqīq*) [7, p. 260 no. 23]. In it, Ibn al-Haytham deals with the theory and practical aspects of an instrument which he designed for the determination of the meridian line by the observation of an arbitrary fixed star before and after culmination. The instrument can be used for any geographical latitude. Fuat Sezgin also had Ibn al-Haytham's meridian instrument reconstructed for the Museum in the Institute for the History of Arabic-Islamic Science in Frankfurt [9, vol. 2, p. 146].

The purpose of this paper is to make the first and second treatise available to the reader in an English translation based of the Arabic edition by Fuat Sezgin. The Arabic manuscript has very few diacritical marks and readings are often ambiguous in trivial ways which

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I have not noticed. I have added a few notes to the Arabic edition, two explanatory figures, an explanatory note to the first text, and some thoughts on the reconstructed instrument. This paper does not contain a complete commentary, nor a comparison with other works by Ibn al-Haytham and other medieval Islamic astronomers.

Here are some notes to my translation. The first treatise is not exactly easy to follow, and the second treatise is written in an even heavier style, just like book 4 of Ibn al-Haytham's *Optics* [5] [10]. In the second treatise, Ibn al-Haytham presents complicated geometrical configurations and reasonings in long sentences without diagrams. I have divided the second treatise into numbered chapters and I have added chapter titles in square brackets. I have subdivided the chapters of the second treatise, as well as the first treatise as a whole, into numbered paragraphs. Square brackets contain the paragraph numbers and chapter numbers and all other explanatory additions by me, including page numbers in the manuscript and in Fuat Sezgin's edition. In the translation of the second text, I have used notations such as line[1] and line[2] to distinguish between different lines, and plane[1] and plane[2] to distinguish between different planes. The reader should remember that the numbers [1] and [2] are additions by me, which are found neither in Sezgin's edition nor in the Berlin manuscript. Words in pointed brackets < > are translations of Arabic words which have been added to the text in the (defective) manuscript in order to restore Ibn al-Haytham's original. The following notations have been used:

E = Fuat Sezgin's edition in [8]. The editions of Ibn al-Haytham's first and second text are in [8, pp. 16-19] and [8, pp. 20-43]. A notation such as E16:5 means line 5 of page 16 of E.

B = Manuscript Berlin, Staatsbibliothek, Or. 2970, ff. 44a-46a, 46b-59a, the manuscript on which Sezgin's edition is based. The whole codex Or. 2970 is available online on the website of the Staatsbibliothek.[1] The first treatise by Ibn al-Haytham is also extant in a manuscript in Istanbul (Atıf Efendi 1714). Sezgin did not use the Istanbul manuscript because it is a direct copy of the Berlin manuscript [8, p. 13]. A notation such as B44b:2 means line 2 of folio 44b (verso) of manuscript B.

The fact that Fuat Sezgin was able to produce high-quality editions of the two very technical treatises from a defective manuscript is a witness of his considerable skill in mathematics. He was a student of mathematics and engineering before he switched to oriental studies [2].

[1] http://resolver.staatsbibliothek-berlin.de/SBB00006BAB00000000

2 Translation of Ibn al-Haytham's first text

[B44b] [E16] *Determination of the meridian line by means of one shadow by al-Ḥasan ibn al-Ḥasan ibn al-Haytham.*[2]

In the name of God, the Merciful, the Compassionate.

[1] If you want to determine the meridian line in the plane of the horizon by means of one shadow, you prepare a plane parallel to the horizon in a place where the sun rises, and you place on it a straight gnomon in an erect perpendicular position.

[2] When the sun has risen in the place, and the shadow of the gnomon has appeared on that plane, then it [the sun] has produced the [i.e. a non-zero] altitude.[3] When you have obtained the altitude, you mark at the tip of the shadow a point, and you mark a point in the middle of the width of the shadow, in a place close to the gnomon. Then you place the ruler on the two points, and you draw a straight line joining the two points.

[3] Then you know [i.e., find out] the meridian altitude on that day. If it is equal to the altitude which you have obtained, then the line which you have drawn is the meridian line. If the meridian altitude is not the same as the altitude which you have obtained, then this [meridian altitude] must be greater than it [the altitude which you obtained], so you subtract the lesser from the greater, and you keep the remainder; and you call it the "difference between the two altitudes."

[4] Then you construct a circle divided into degrees in an accurate way, just like the circle on the circumference of the astrolabe or what is similar to that. Then you count on it [an arc containing] degrees in the amount of twice the complement of the altitude which you have obtained. And once you have obtained that number, you use an accurate compass, and you open its two arms, and you put one arm on one of the endpoints on that [amount of] degrees on the divided circle, and you move the other arm forward and backward until it is located on the other endpoint of [the arc containing] those degrees.

[5] Once that position has been precisely obtained, you place the two arms of the compass [keeping the same fixed compass-opening] on the line which you have drawn in the middle of the shadow, and you cut off from it a part of the size of the compass opening; call that line [segment] the "line of the shadow."

[2] The name al-Ḥasan ibn al-Ḥasan ibn al-Haytham (B44b:2) is omitted in E16:1.

[3] We can also read B44b:6 as *akhadhta* instead of *aḥdatha* E16:5 and translate "then you take the altitude".

[6] Then bisect that line and erect on its midpoint a perpendicular, I mean a straight line at a right angle. Then place the arm of the compass on the point on the circumference of the divided[4] circle and count sixty degrees, and move the other arm [E17] of the compass [B45a] forward and backward until it is located on the other end of these degrees. Then the opening of the compass is [equal to] the radius of the divided circle.

[7] Then place one of the arms of the compass on the end of the shadow line, whichever end you want, and move the other arm until it is located on the perpendicular drawn on the midpoint of the shadow line. It is necessary that the other end meets that perpendicular, because the compass opening is the radius of the divided circle and the shadow line is the chord of twice the complement of the altitude, and twice the complement of the altitude is less than a semicircle. Therefore the shadow line is less than the diameter, so half of it is less than the compass opening.

[8] And when the arm of the compass meets the perpendicular, fix the arm of the compass in its position on the perpendicular, and describe by means of the other arm a circle; it will pass through the two endpoints of the shadow line and will produce a circle equal to the divided circle. The shadow line is a chord in that circle, and the smaller of the two arcs which are cut off by the shadow line is equal to twice the complement of the altitude, and the greater arc [of the two] is more than a semicircle.

[9] Then you double the meridian altitude of [the sun at] the beginning of [the sign] Aries at the locality in which you are, and you subtract from it the difference between the two altitudes, and you count the number of degrees on the divided circle equal to the remainder.

[10] When that amount has been precisely obtained,[5] you open the compass until its two arms are located on the two endpoints of these degrees on the divided circle, then you place one of the arms of the compass on the endpoint of the shadow line closer to the gnomon, and you move the other arm until it meets the circumference of the [abovementioned] segment of the circle which is greater than a semicircle. And it meets the circumference of this circle because twice the meridian altitude at the beginning of Aries is in all localities on earth except the equator less than a semicircle, and on the equator it is [equal to] a semicircle, so twice the [meridian] altitude of the beginning of Aries is in all localities on earth not greater than a semicircle. If the difference

[4] Reading *maqsūma* for *mustaqīma* in B44b21 and E16:19.

[5] Reading *taḥarrara* with B45a:14 for *taḥaddada* in E17:12.

between the two altitudes is subtracted, [B45b] the remainder is in all cases less than a semicircle. But the greater segment of the circle in the plane of the horizon is greater than a semicircle. Therefore the arm of the compass meets the circumference of that segment, so you mark on the position where the arm of the compass meets the arc of this segment a point.

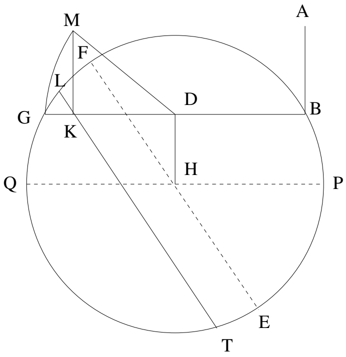
[11] Then you count of the degrees on the divided circle [an amount] equal to the difference between the two altitudes. You place the two arms of the compass on the two endpoints of [the arc equal to] these degrees, then you place one of the arms of the compass on the endpoint of the shadow line which is next to the end of the shadow, and you move the [E18] other arm until it intersects the smaller [arc] segment, which is twice the complement of the altitude. It is clear that the arm of the compass intersects that arc because the difference between the two altitudes is part of that arc. Then you mark the place where the arm of the compass intersects the smaller arc by a point.

[12] Then you place the ruler on that point and on the other point on the greater arc, and you join the two points by a straight line. This line intersects the shadow line because the two assumed [end]points are on both sides of the shadow line.

[13] Then you draw from the intersection point a perpendicular to the shadow line on the side which is in the direction of the altitude, I mean if the altitude is Eastern you draw the perpendicular towards the Eastern side [of the shadow line], and if the altitude is Western you draw the perpendicular towards the Western side. You put one of the two arms of the compass on the point in the middle of the shadow line and you describe an arc of a circle with distance the endpoint of the shadow line [i.e., with radius half the shadow line]. It intersects the perpendicular drawn to the shadow line; then you mark on the place of its intersection a point. Then you place the ruler on that point and on the midpoint of the shadow line, and you draw a straight line. This line is the desired meridian.

[14] This is the figure. [B46a] [E19] The gnomon is the [line] on which is A, and BG is the shadow line, D is the middle of the shadow line, arc BG is twice the complement of the altitude, arc BT is twice the meridian altitude at the beginning of Aries from which the difference between the two altitudes has been subtracted, and arc GL is the difference between the two altitudes. Line TL is the [line] joining the two endpoints of these arcs, point K is [the point] at which this line intersects the shadow line, line KM is the perpendicular drawn from that

line, arc GM is the arc drawn with centre D and distance DG, and line DM is the desired meridian line.[6] Praise to God, Lord of the Worlds, and may God bless His prophet Muḥammad and his entire family.



[6] Explanatory note: The diagram without the gnomon can be seen as an orthogonal projection of the celestial sphere on the meridian plane. Point H, the centre of the sphere, is labelled in the figure but not defined in the text, and the gnomon is assumed to be parallel to the vertical line. Then line GDB is parallel to the horizon; it is the projection of the circle of instantaneous solar altitude above the horizon. We will now show that LKT is the projection of the daily path of the sun (a circle parallel to the equator). We add the intersections QHP of the horizon with the meridian plane and EHF of the meridian plane with the plane of the celestial equator; QHP is parallel to GDB and EHF is parallel to TKL. The dashed lines and labels P, Q, E, F are not found in the manuscript.

Put a the instantaneous altitude, A the meridian altitude, ϕ the geographical latitude and $|\delta|$ the declination, which we will assume to be southern because the figure was drawn for this case. Since arc $GB = 2(90 - a)$ we have arc GQ= arc $BP = a$. But arc $GL = A - a$, therefore arc $LQ = A$ is the solar meridian altitude so point L is on the daily solar path, where point L is below F because the declination is southern (L is above F if the declination is northern). The meridian altitude for the sun in the beginning of Aries (declination 0) is $90 - \phi$. Therefore, according to the text, arc $BT = 2(90-\phi)-(A-a)$. Since $A = 90-\phi-|\delta|$ it follows arc $BT = 90-\phi+|\delta|+a$. By construction arc $BP = a$, arc $PE = 90 - \phi$, so as a consequence of the text we have arc $ET = |\delta|$. Therefore T is also a point on the daily solar path, and LT must be the orthogonal projection of the solar path on the meridian plane.

Thus point K is the perpendicular projection of the sun at the moment of observation. Then the cosine of the azimuth (measured from the north or south point of the horizon) is $KD/DM = KD/DG$ since DG is the radius of the altitude circle.

If we now consider BG as the shadow cast by the gnomon, $\angle GDM$ is equal to the azimuth of the sun, therefore line DM points precisely towards the north, so it is the meridian line.

For a more extensive commentary and additional figures see [3].

3 Translation of Ibn al-Haytham's second text

[B46b] [E20] *Treatise by al-Ḥasan ibn al-Haytham on the determination of the meridian line with utmost precision.*

[1. Introduction]

[1.1] The art of astronomy is based on the motions of the stars, and the motions of the stars were obtained [by our predecessors] and are still obtained [today] by means of observations. Observations are carried out by constructing instruments in a precise way and placing them in a position similar to the position of the universe. But placing the instruments in a position similar to the position of the universe can only be done by determining the meridian line. Therefore the determination of the meridian line is one of the foundations of the art of astronomy, which cannot be complete without it.

[1.2] The experts in this art have always determined the meridian line, but they have determined it by methods based on an approximation, which influences the position of the meridian line. In the determination of the meridian line, they have mostly relied on what they considered as the most correct of the [methods], that is the Indian circle. That is to say: a circle is drawn in the plane of the horizon and at its centre a gnomon is placed vertically. From that [gnomon] one takes two shadows, one Eastern and one Western, in such a way that the two endpoints are on the circumference of the circle. Then they join the two points which are the endpoints of those two shadows by a straight line. They bisect that line and they draw from its midpoint a line at right angles. Then that line is the meridian line.

[1.3] But we have explained in our treatise *Instruction on the positions of errors in the method of observation*[7] that in this method there is an error with a noticeable magnitude, by which the line that is found [by their procedures] differs from the position of the meridian line. [E21] And we have explained this notion in the above-mentioned treatise. Nevertheless, we will detail in this present treatise what we have mentioned there, as follows:

[1.4] At the two moments of time when the two endpoints of the shadow are on the circumference of the circle, the sun is not on the same time circle [parallel to the equator], because the sun moves always with its characteristic [ecliptical] motion, the poles of whose sphere [of the

[7] See [7, p. 258 no. 12].

ecliptic] are not the same as the [north and south celestial] poles of the universe. Therefore it [the sun] passes from any time circle on which it moves to another [time] circle in an extremely small time interval; thus it is not fixed on any time circle, not even for one moment of time. [B47a] But if the sun is at two different moments of time at two different time circles, not on the same circle, then the distances from[8] the meridian of the endpoints of the two shadows which have been found are not equal distances, and the meridian line is not perpendicular to the line joining the two endpoints of the shadows. Thus the meridian line cannot be determined with utmost precision by the two equal shadows.

[1.5] Again, the endpoint of the shadow is not a precise point because in the end of the shadow is a delicate shadow which blurs the exact point at the end of the shadow.

[2. The underlying geometrical theorems][9]

[2.1] Since this is the case, and since the determination of the meridian line is necessary for precise observations, we decided to develop a method for finding the meridian line, by which this line can be obtained with utmost precision. Thus we say:

[2.2] If the line joining any two fixed points on the [celestial] sphere is perpendicular to the plane of the meridian circle, the plane of the meridian circle bisects that perpendicular.

[2.3] Thus if from those two points two lines are drawn to a fixed point on a gnomon which [gnomon] is perpendicular to the horizon, whence that circle is the meridian circle of [the gnomon], and if the two lines are extended rectilinearly, in such a way that they end at the horizon plane, then the two points at which they end in the horizon plane are on different sides of the meridian line, and the line which joins them is perpendicular to the meridian line and is bisected by[10] the meridian line.

[2.4] [Proof:] For since the gnomon is perpendicular to the horizon, the perpendicular dropped from its tip to the plane of the horizon is in the meridian plane, and its tip is in the [plane of the] meridian circle, and [therefore] the foot of its perpendicular is on the meridian line. And we call this point the centre of the base of the gnomon.

[8] Reading *ʿan* B47a:3 for *ʿalā* E21:8.

[9] We have supplied an explanatory Figure 1 at the end of the translation. Ibn al-Haytham's treatise does not contain figures.

[10] Reading *wa-yanqasimu bi-khaṭṭ* for *wa-yaqsimu khaṭṭ* in B47a:15, E31:20.

[2.5] And the point which [E22] bisects the line joining the two above-mentioned points on the sphere is in the plane of the meridian circle. Thus if we imagine a straight line which joins that point to the tip of the gnomon, that [B47b] line is in the plane of the meridian circle. Thus if we imagine that line extended rectilinearly, to end at the horizon [plane], then the point at which that line meets the plane of the horizon is in the plane of the meridian circle. But it is also in the plane of the horizon. Therefore it is on the intersection between the planes of the horizon and the meridian circle, which is the meridian line.

[2.6] Thus the line joining these two points, I mean the centre of the base of the gnomon and the point which is the end of the line joining the tip of the gnomon and the midpoint of the line between the two fixed points [on the celestial sphere], is the meridian line.

[2.7] We imagine two lines drawn from the two fixed points on the sphere to the tip of the gnomon, and extended rectilinearly until they end at the plane of the horizon. Then these two lines are in one and the same plane together with the line drawn to the tip of the gnomon from the midpoint of the line joining the two fixed points on the sphere. So these three lines are also in the same plane after being extended rectilinearly beyond the tip of the gnomon. Therefore the two points at which the two lines end are on the intersection between the plane of the horizon and the plane of the two lines. But the point on the plane of the horizon which is the endpoint of the line extended in the plane of the meridian circle is also on the intersection between the plane of the horizon and the plane of the two lines. Therefore the three points in the plane of the horizon, at which [points] the three lines [through the tip of the gnomon] end, are on one straight line.

[2.8] Therefore let us imagine the straight line passing through these three points [in the horizontal plane]. Since the plane of the triangle on [i.e., next to] the horizon, I mean the plane containing the three lines, is perpendicular to the plane of the meridian circle, and the plane of the horizon is perpendicular to the plane of the meridian circle, therefore the straight line which is the intersection is perpendicular to [E23] the plane of the meridian circle.

[2.9] And since the two angles at the tip of the gnomon in the upper triangle which [B48a] we have described above are equal, the two angles of the triangle corresponding to it, which is next to the horizon, are also equal to one another. Thus the line which is the intersection between the plane of the triangle next to the horizon and[11] the plane of the

[11] Reading *wa-bayna* instead of *bayna* in B 48a:3 and E 23:3, end of line.

horizon is bisected by the line which bisects the angle of the triangle.

[2.10] And since this [bisecting] line is in the meridian plane, because it is the line joining the midpoint of the perpendicular joining the two fixed points on the sphere and the tip of the gnomon, and this line bisects the line joining the two endpoints of the two lines extended from the fixed points on the sphere to the horizon, which intersect one another at the tip of the gnomon, therefore the dividing point is on the meridian line, since the dividing point is in the plane of the meridian circle and it is [also] in the plane of the horizon, so it is on the intersection which is the meridian <line>.[12]

[2.11] This point is on the line joining the two endpoints of the two lines drawn from the fixed points on the surface of the sphere to the plane of the horizon, which [line] is the base of the triangle next to the horizon. So this line which is the base of the triangle is bisected by the meridian line.

[2.12] But it has become clear that this line is perpendicular to the plane of the meridian circle, so it is perpendicular to the meridian line, so the meridian line bisects at right angles that line, which is the base of the triangle whose vertex is the tip of the gnomon.

[2.13] Thus it is clear from this exemplification that if we find on the plane of the horizon two points in such a way that the lines drawn from them to the tip of the gnomon, if extended rectilinearly, end at two fixed points on the [celestial] sphere, in such a way that the line joining them is perpendicular to the plane of the meridian circle, and is bisected <by the plane of the meridian circle>[13] and [if] we join these two points in the plane of the horizon by a straight line, and if we bisect it, and draw through the bisecting point in the plane of the horizon a line perpendicular to that line joining the two points, then that [perpendicular] line is the meridian line with utmost [B48b] precision.

[3. Theoretical construction of the meridian line in the horizontal plane]

[3.1] And since that has been made clear, let us now proceed to find the two points according to the description that we have mentioned.

[12] The scribe left out a word < *khaṭṭ* > between two lines B48a:9 and B48a:10, see E23:9.

[13] Between *niṣf al-nahār* in B48a:18 and E23:17 and *bi-niṣfayni* in B48a:18 and the beginning of E23:18 we have added <*wa-yanqasimu bi-saṭḥ dā'irat niṣf al-nahār* >, as in the similar passage in E35:14.

[3.2] Thus we say: For every time circle in the sphere of the fixed stars, its position does not change, and the [E24] position of none of [the fixed stars] changes. Therefore if the altitude of any point on a time circle above any horizon is a known altitude, then [the point] is fixed and does not change its position.

[3.3] But every time circle is bisected at right angles by any great circle that passes through the two poles of [the time circle], which are the two poles of the universe, and the intersection is a diameter of the time circle.[14]

[3.4] And every assumed point on the surface of the earth has a meridian circle which passes through the two poles of the universe and which is known in position: it does not change and is not moved [anywhere].

[3.5] And [thus,] for every assumed point on the earth, its meridian circle bisects every time circle at two known fixed points on its circumference and in one unchangeable diameter of the time circle.

[3.6] If from any known point on the circumference of a [i.e., any] circle a perpendicular is drawn to a diameter with known position among the diameters[15] of the circle, and if we extend it to the other side [of the diameter], then it will cut off on both sides of the diameter two known equal arcs which are not different and do not change.

[3.7] Thus if a perpendicular is drawn from any known point on the circumference of a time circle to the diameter of that circle, which [diameter] is in the plane of the meridian circle and if [the perpendicular] is extended to the other side of the meridian circle, then it cuts off from the time circle an arc at a known point,[16] whose distance to [any of] the two endpoints of the diameter in the plane of the meridian circle is the same as the distance of the first known point from that endpoint of the diameter.

[3.8] Thus if any two points on a time circle are at equal distances from the endpoint of the diameter of that circle, which [endpoint] is on the circumference of the meridian circle, then the altitudes of these

[14] Reading *quṭran li'l-dā'ira* with B48b:5 instead of *quṭr al-dā'ira* in E24:3. According to a traditional interpretation of Euclid's *Elements* Book 1, a circle is a surface area bounded by the curve that is called the circumference of a circle [1, vol. 1, pp. 183-185]. Thus two circles (surface areas) may intersect in a straight line segment.

[15] Reading *min aqṭār* for *wa-aqṭār* in B48b:11, E24:8.

[16] The Arabic manuscript text is in disorder due to scribal error. In order to make mathematical sense, we have removed the two words *nuqṭa maʿlūma* in E24:11, B48a:14 and placed two corresponding words *bi-nuqṭatin maʿlūmatin* in E24:12, B48a:15 after the word *qawsan*.

[points] are equal. And that is [so] because the two points according to this description are on the circumference of exactly the same circle among the circles parallel to the horizon, which are the almucantars [B49a] of altitude.

[3.9] Every fixed star moves during one night on the circumference of exactly the same time circle, and during one night it does not depart from that circle by an amount which can be perceived by the senses or by any instrument.[17] And the centre of [the fixed star] does not depart from the circumference of the time circle by any amount which has an effect on anything. [E25]

[3.10] And since all that we have mentioned is as we have mentioned, if any fixed star has a known altitude above a known horizon during a known night, then the point of the circumference of the time circle, at which the centre of that star moves during that night, is [a] known [point]. Of any pair of equal altitudes of the fixed star [during that night], one is Eastern and the other is Western.

[3.11] Thus the two points on the circumference of the time circle at which the centre of the star is [located] in the two moments of the two [equal] altitudes, are at equal distances from the endpoint of the diameter of the time circle which [endpoint] is on the circumference of the meridian circle, and the line joining those two points is perpendicular to the plane of the meridian circle, and the plane of the meridian circle bisects it.[18]

[4. Principle of the instrument]

[4.1] Since this has become clear, if the altitude of any fixed star is taken during any night at a known locality on earth, by means of an instrument with a sighting tube, and the star is observed through the sighting hole in the tube, while the star does not exceed the hole of the tube nor is less than it, or the entire body is observed and the excess (part) of the sky is observed all around the star and of equal size, then if the axis of the tube is imagined to be extended rectilinearly until it ends at the body of the star, it ends at the point on the circumference of the time circle at which the centre of the star is [located]; this is the point about which is was shown above that it is [B49b] a known and fixed point which does not change, because its altitude is known.

[17] According to Islamic astromoners, the declination of a star changes due to the precession of the equinoxes but the effect is noticeable only after many years.

[18] Reading *wa-yaqsimuhu* instead of *wa-yaqsimu* in B 49b:12, E 25:8.

[4.2] Thus if the axis of the tube is also imagined to extend towards the horizon, then the point at which this line ends at the horizon is known, fixed and unchangeable. Therefore the entire extended line from the point at the centre of the star to the point in the plane of the horizon is known in position, fixed in the imagination, and its position does not change. Thus every point which is assumed on this line, and which is imagined to be on this line, is fixed and does not change.

[4.3] Then, if the star moves and arrives at the Western side, and its Western altitude is taken, equal to the Eastern altitude [E26], by exactly the same instrument by which the Eastern altitude was taken, and the axis of the tube, if extended rectilinearly, ends at the point on the circumference of the time circle at the centre of the star, and moreover, this line passes through exactly the assumed point on the first line, which passes through the axis of the tube, which [point] is a fixed unchangeable point, and [if] the taking of the Western altitude is according to the above-mentioned description, I mean with utmost precision,

[4.4] then that line also, which passes through the axis of the tube at the moment of the Western altitude, is known in position, and it is fixed and does not change, and its position with respect to the meridian circle is similar to the position of the first line, and it ends at the plane of the horizon, and the distance of the point in the plane of the horizon, at which this line ends, from the plane of the circle of the meridian is the same as the distance of the first point at which the first line ended from the meridian circle, and these two points are the two points which we have defined above.

[4.5] Therefore, if these two points are joined by a straight line, and that line is bisected, and from the midpoint a line is drawn at right angles, and extended in the plane of the horizon, then that line is the meridian with utmost precision.

[5. Construction of the instrument][19]

[5.1] Thus it remains for us to find two points in the plane of the horizon by means of an instrument carrying a tube, in the way we have set out. Let us first describe the instrument.

[5.2] We take a bar of copper [B50a] or flexible wood, very solid, with length four spans of the hand. Let it be square in thickness, I mean that its width is equal to its depth. And let its width and depth be the size

[19] We have supplied an explanatory Figure 2 at the end of the translation. Ibn al-Haytham's treatise does not contain figures.

of two fingers,[20] and its faces are parallel, and [each face] perpendicular to the adjacent face. This can be realized if we determine [i.e., scrape off] the angle[s] of the bar by means of a right angle [carpenter's square] in such a way that its two arms cover the edges of the [adjacent] faces of the bar.

[5.3] Let us mount this bar in the well-known [lathe] and scrape off [the edges] on its end along an interval of two fingers, in such a way that it becomes round and cylindrical and it becomes smaller [near the end].

[5.4] On one of its [rectangular] faces we draw a line[1] which extends in the middle of the face, and which is parallel to the two edges of the bar, which are the two [long] sides of that face.

[5.5] Then we take a copper plate[1], with two plane surfaces, of moderate thickness, whose length is one span of the hand and width one span of the hand. We heat [E27] its edges until they become perfectly straight, and then we draw on one of its two surfaces a line parallel to its edge, such that the distance between [the line] and the edge is half the width of the bar.

[5.6] Then we assume on that line a point such that the distance between it and the end of the line is one finger. Then we make that point the centre, and we draw with distance [i.e. radius] the other end [of the line], I mean the far end [of the line], but before the end of the plate[1], an arc of a circle.

[5.7] Then we draw from the assumed point[21] a line perpendicular to the first line. It is extended to end at the arc that has been drawn on the plate[1], so this arc becomes a quarter circle. Then we divide this arc into ninety degrees.

[5.8] Then we draw behind this arc another arc close to it, and then we remove the remaining part of the plate[1] at that [outer] arc, so that its edge becomes circular. Then we pierce this plate[1] at the assumed point, which is the centre of the arc, by a round hole.

[5.9] Then we take an alidade with two [rectangular] vanes [i.e., sights, diopters], just like the alidade of an astrolabe, and we make on one of its ends a pointer and we make the other end round. We make its length equal to the length of the plate[1]. We pierce the round end of the alidade and we mount this alidade on the plate[1] so that the hole

[20] According to al-Qurashī in (Rebstock 2001, 81), one span of the hand is half a hand ell, and a span of the hand can be nine or twelve fingers, thus a span is roughly 20 cm and a finger roughly 2 cm.

[21] Reading the singular *nuqṭa* instead of the plural *nuqaṭ* in B50a:13, E27:5.

fits on top of the hole. [B50b]

[5.10] We place in the two holes an axis, and we insert into [the axis] a peg[22] to fasten it.

[5.11] We make the length [perpendicular to the alidade] of each of the [rectangular] vanes one and a half finger. We mount one of the vanes near the pointer and the other one before the hole and close to it.

[5.12] Then we draw in the middle of the surface[s] of the two perpendicular vanes, two lines parallel to the ends of the vanes [perpendicular to the alidade], and we determine on each of the two lines a segment such that its length added to the thickness of the alidade is equal to half the width of the bar.

[5.13] Then we pierce at the two points which are the endpoints of the two segments, two holes with diameter the magnitude of one barleycorn.

[5.14] Then we take a hollow round copper tube, such that its thickness is equal to the width of each of the two holes, and its length exceeds the length of the plate[1] by the magnitude of two fingers.

[5.15] Then we take a round funnel with diameter the magnitude of two fingers, and we mount it onto one of the ends of this tube and weld them together. It is necessary that if this tube is entered in the holes of the two sights, it goes by force and does not slide, so that if the tube is [placed] in the two holes, it cannot move and cannot change its position. [E28]

[5.16] If this is ready, we mount the copper plate[1] on the end of the bar, I mean the square end [of the bar], which was not scraped off [as a cylinder], and we make [a strip of] four fingers of the plate[1], on the side of the divided arc, mounted on the face of the bar. We make the edge of the plate[1], I mean the edge parallel to the line in which the hole is, [overlap] with the edge of the bar, and let the surface of the plate[1] on the side of the alidade and the divided [arc], be adjacent to the bar.[23]

[5.17] Then the line drawn on the plate[1], I mean the line through the hole, covers the line[1] drawn in the middle of the face of the bar. We fasten the plate[1] to the bar, and weld them firmly together.

[5.18] Then we take a round base [made] of flexible wood, solid in hardness, such that its diameter is a span of the hand, and let its surface be made even.

[22] Compare the "horse" of an astrolabe.

[23] The text seems to indicate that the plate[1] is mounted vertically on the bar, see Explanatory Figure 2 below, not horizontally.

[5.19] Then we apply on its surface a round copper plate[2].

[5.20] Then we draw on this surface two parallel circles, close to one another, and we remove the excess part of the plate[2] and the base, i.e. what is outside the greater circle. We make the circumference of the base equal to the circumference [B51a] of the greater circle.

[5.21] Then we drill in the middle of the base a hole at the centre of the plate[2]. We make it round such that its width is the size of the thickness of the round cylindrical end of the bar, in such a way that if it is mounted on that base, the round end will fit neatly in that hole, but it is nevertheless possible to turn the bar in that hole.

[5.22] Then we take a copper ruler, such that its length exceeds the radius of the base by a little bit, and its width is the magitude of one finger, and its thickness is small.

[5.23] Then we draw on the middle of the face[2] of the bar which has been described above, which [face] is at the edge of the plate[1], I mean the edge coinciding with the edge of the bar, I mean the face[2] perpendicular to the face[1] of the bar [i.e. the face perpendicular to the face on which line[1] is drawn and to which plate[1] is attached], a line[2] which extends exactly in the middle of the face[2] of the bar, until it ends at the end which is next to the round part [of the bar].

[5.24] Then we remove from the body of the bar a [horizontal] slit on the base of the bar, near the end of the bar which is adjacent to the plate[2], towards the line[2] drawn in the middle of the face[2] of the bar. And the thickness of [the slit] has the same magnitude as the thickness of the thin ruler.

[5.25] Then we enter the thin ruler into this slit, and we make it extend in a direction contrary to the direction of the plate[1], so that the end of the thin ruler and the end of the plate[1] are in opposite directions with respect to [E29] the vertical bar. We fasten the thin ruler to the vertical bar and weld them firmly together.

[5.26] Then we sharpen the tip of this thin ruler until its edge is extremely sharp and thin, and in such a way that this sharp edge is perpendicular to the line[2] drawn on the middle of the face[2] of the vertical bar, and [perpendicular] to the face[2] of the bar [itself].

[5.27] Let the surface of this thin ruler, I mean the surface adjacent to the end of the vertical bar, be extremely even, so that if the vertical bar is mounted on it and its round end enters in the hole of the base, the surface of the smaller ruler covers the surface of the base and its sharp edge intersects the circumference of the circle [B51b] drawn on the surface of the base.

[6. How to use the instrument]

[6.1] Thus when all of this has been done, the instrument is ready to be used for the determination of the meridian line.

[6.2] Thus let a plane parallel to the horizon be flattened and made even by the balance and the plumbline.

[6.3] Then we bury the circular base which we have described above, which is the base of the instrument which we have set out, in this plane, and we immerse it in [the ground] until the surface of the plate[2] coincides with the flattened plane parallel to the horizon.

[6.4] Then we mount the bar on this base and we enter the round end in the hole in the base so that the divided arc on the plate[1] is adjacent to the base.

[6.5] Then we enter the tube in the holes in the two vanes and we place the funnel on the side of the horizon. The tube has to be entered in the two holes in the vanes until the end of the tube is at the end of the alidade, I mean the upper end [of the alidade] in which the hole [for the axis] is. Then the funnel is on the side of the earth, and the excess part of the tube, which is the magnitude of two fingers, is on the side of the funnel. And if we have done this in this way, then the person who looks in the funnel can place his whole eye in the funnel.

[6.6] When the instrument has been mounted according to this description, then one should wait for the night. When the night has become dark and the sky is clear, one should pay attention to a fixed star East of the meridian, and let it be at a moderate altitude above the horizon.[24]

[6.7] Then the bar is rotated, and the observer holds his eye at the edge of the plate[1] and he turns the bar left and right until he sees the star together with the surface of the plate[1].

[6.8] Then he puts his eye near the circumference of the funnel at the end of the tube, and he moves the alidade up and down until he sees the star in a direction parallel to the tube.

[6.9] If he sees the star in a direction parallel to the tube, let him place the part of his face surrounding his eye inside the funnel and look in the tube, and then he sees [E30] the star.

[6.10] If he sees the star, let him move the alidade carefully, with slowness and deliberation, until he sees the whole body of the star, in

[24] Because Ibn al-Haytham prescribes that the star is above the horizon, the plate[1] cannot be mounted horizontally on the bar, as in the reconstructed instrument in [9, vol. 2, p. 146] which can only be used for sighting stars on the horizon.

such a way that he does not see any other part of the sky apart from it, or he sees the entire body of he star and around it the same amount of sky surrounding the star [B52a] equally [on all sides]. It is necessary that the hole of the tube is narrow, of the same magnitude as the hole of this [i.e., an] astrolabe, so that if he sees [some] sky in addition to the magnitude of [i.e., the body of] the star, the excess sky is very small. And if there is excess [sky], he needs to move the alidade until the excess surrounds the star equally [on all sides].

[6.11] So when this position has been obtained precisely, it is necessary that the bar remains as it is, and the observer puts his hand on the thin ruler on the plane of the base, and he takes it and keeps it tight against the base so that its position does not change, and then he uses a sharp knife or scratching [instrument], and he moves it along the edge of the thin ruler, I mean, the edge without markings, and he makes on the plane of the plate a permanent inscription.

[6.12] When he has done this, let him put on the pointer of the alidade some wax and fasten the alidade to the plate[1], so that its position does not change.

[6.13] Then let him take hold of it and observe the star until it is in the Western half, away from the meridian circle. When it is in the Western half, he rotates the vertical bar and turns his eye with the edge of the plate[1], and moves the bar left and right until he sees the star together with the surface of the plate[1].

[6.14] Then he should lower his eye and bring it near to the edge of the funnel, and look at the star, and move the bar carefully until he sees the star in a direction parallel to the tube. And he only sees it in that direction when its altitude has become equal to the first altitude, which is equal to the altitude of the alidade.

[6.15] When he sees the star in a direction parallel to the tube, he places the part of his face around the eye inside the funnel, and looks in the tube until he sees the star in the way we have described above; then its altitude in this situation is equal to the first altitude.

[6.16] And then he puts his hand on the thin ruler and draws along its edge a line by the iron [tool] just like the first line. Then he interrupts his work for the rest of the night.

[6.17] When the morning has arrived, he considers the base, and he will find the two lines which he has drawn on the base, intersecting the circumference of the circle which is in the plane of the base.

[6.18] Then he puts a straight ruler without markings [B52b] on [E31] the two points on the circumference of the circle, which are the points of intersection between the circumference of the circle and the two lines drawn on the plate, and he draws with this ruler a thin line by the iron [tool], passing through the two points, and making a trace on the surface of the plate. It does not pass beyond the two points and does not exceed them.

[6.19] Then he constructs on this line [segment] which joins the two points two equilateral triangles in the plane parallel to the horizon, of which the plane of the base is part, or [he constructs the triangles] on part of that line [segment].[25] I mean if part of this line is cut off, $<\ldots>$[26] the distances of the two endpoints of it, from the two points of the two ends [of the line segment] are equal distances, $<\ldots>$[27] two triangles on both sides of the line, by means of [construction] lines with extreme delicacy.

[6.20] Then he uses a long ruler, and he places its edge on the vertices of the two triangles, and let part of the ruler be on the base and the remainder on the flattened plane parallal to the horizon in which the meridian line has to be drawn. And he draws, by means of the edge of the ruler, a straight line which extends in the flattened plane.

[6.22] Then this line will bisect at right angles the line which joins the two points on the circumference of the circle, so this line is the meridian with utmost precision.

[7. First part of the proof that the instrument produces the meridian line]

[7.1] Thus it remains for us to show that this line, I mean the line found by the instrument which we have set out, is the [meridian] line according to the previous definition.

[7.2] Then we say: The vertical bar on the round base is perpendicular to the plane of the horizon, therefore the round cylindrical part of it which is next to the round basis is perpendicular to the plane of the horizon, $<\ldots>$[28] it is perpendicular to the horizon. If we imagine

[25] Ibn al-Haytham's construction of a perpendicular to a line segment by means of two equilateral triangles was probably inspired by Euclid's *Elements* I:9-10, compare (Heath 1, 264-268).

[26] Probably a passage is missing here, explaining that one should draw two circles with centres the two endpoints of the line segment and radius equal to the line segment. The two circles intersect in the two points which form with the line segment the two equilateral triangles.

[27] Perhaps another word or passage is missing here, meaning: thus he constructs.

[28] A missing passage in B52b:16 must have contained the definition of the line which

that line extended along the length of the bar until its upper edge, and extending beyond it, it is perpendicular to the horizon. Let us call that line the axis of the bar.

[7.3] The line[1] drawn in the middle of the face of the bar, which [line] overlaps with the line drawn in the plate[1], is parallel to this axis.

[7.4] Thus if a perpendicular is drawn from any point on this line[1] to the axis [B53a] of the bar, it is equal to half the width of the bar.

[7.5] And the line drawn on the plate[1], coinciding with the line[1] on the face of the bar, is also parallel to the axis [E32] of the bar. If a perpendicular is drawn from any point imagined on that line, I mean, [the line] on the plate[1], to the axis of the bar, it is equal to half the width of the bar, since the plate overlaps with the face of the bar.

[7.6] Thus if a perpendicular is drawn from the point at the centre of the circle of the hole in the surface of the plate[1], I mean the hole in which the rotation axis of the alidade is [located], to the axis of the bar, if the axis of the bar is extended in a straight line, then it [the perpendicular] is equal to half the width of the bar.

[7.7] But this perpendicular is perpendicular to the rectilinear extension of the line[1] drawn in the middle of the face of the bar, which [extension] overlaps with the line in the plate[1], because the line[1] in the middle of the face of the bar is parallel to the axis of the bar. And these two lines, I mean the axis of the bar and the line[1] drawn on the face of the bar are in one plane, and [that plane] is parallel to the face[2] of the bar, I mean the face adjacent to the edge of the plate[1]. Therefore the plane which contains these two parallel lines is perpendicular to the face[1] of the bar, I mean the face coinciding with the plate[1]. Therefore the plane containing the two parallel lines is perpendicular to the surface of the plate[1]. Therefore the perpendicular drawn from the centre of the hole in the plate to the axis of the bar is perpendicular to the surface of the plate[1].

[7.8] And the line drawn in the middle of the vane of the alidade near the centre of the hole which is in the vane [of the alidade], I mean the hole in which the tube was entered, if one imagines it [the line] extended in a straight line to end at the surface of the plate[1], is equal to half the width of the bar because it had been assumed that way.

[7.9] And this line is perpendicular to the surface of the plate[1], because the surface of the vane [of the alidade] is perpendicular to the surface of the plate[1].

is called "that line" in the next passage. E noted the inconsistency of the text but decided to remove the previous passage "is perpendicular to the plane of the horizon".

[7.10] The axis of the tube, which passes through the middle of the hole of the tube, passes through the centre of the hole of the vane [of the alidade]. And if we imagine a straight line [B53b] extended in the middle of the surface of the alidade, I mean the surface coinciding with the plate[1], then it is parallel to the axis of the tube.

[7.11] And if a perpendicular is drawn from any point on the axis of the tube to the imagined line in the middle of the surface of the alidade, then it is perpendicular to the surface of the plate[1], because it is parallel to the line drawn in the middle of the surface of the vane [of the alidade]. Each of these perpendiculars is equal to half the width of the bar. And the line imagined in the middle of the surface of the alidade passes through the centre of the hole [E33] in the plate[1]. Thus the perpendicular drawn from the centre of the hole in the plate[1] to the axis of the tube, is perpendicular to the surface of the plate, and is equal to half the width of the bar.

[7.12] And it has become clear that the perpendicular drawn from the centre of this hole to the face[1] of the bar is perpendicular to the surface of the plate[1], and it is equal to half of the width of the bar. But from one point only one perpendicular can be drawn to exactly the same plane. Therefore the imagined perpendicular drawn from the centre of the hole of the plate[1], perpendicular to the surface of the plate[1], is perpendicular to the axis of the tube, and perpendicular to the axis of the bar. And [thus] the part of this perpendicular between the centre of the hole and the axis of the tube is equal to half the width of the bar.[29]

[7.13] Therefore the point of this perpendicular which is at the axis of the tube is at the axis of the bar, so the axis of the tube and the axis of the bar meet at a single point on that perpendicular. This point is outside the body of the bar because the hole in the plate[1] is elevated above the [upper] end of the bar.

[7.14] But this notion is necessary for these two axes in [all] different case[s] of the position of the alidade with respect to the tube, because if the alidade is moved on the surface of the plate[1], the distance between the axis of the tube and the surface of the plate does not change.

[7.15] Again, the axis of the tube and the line in the middle of the surface of the alidade are parallel, thus they are in one plane. And this plane [B54a] passes through the perpendicular to the surface of the plate[1] drawn from the centre of the hole in the plate[1]. I mean that

[29] The sentence "And [thus] ... bar" in B53b:14-15 is repeated by typographical error in E 33:8-9.

this perpendicular is in this plane.

[7.16] Thus if the alidade moves, then the plane of the two parallel lines moves by the motion of the alidade, while the perpendicular drawn from the centre of the hole is fixed and its position does not change.

[7.17] Thus this perpendicular is like the rotation axis of the plane of the two parallel lines. If the plane of the two parallel lines moves, then each point on the rotation axis does not change. Therefore the point on the axis of the tube which is at the end of the rotation axis does not change, and is not moved by the motion of the alidade.

[7.18] But this point is the point of intersection between the axis of the tube and the axis of the bar. Thus the point on the axis of the bar at which the axis of the tube meets the axis of the bar, is one fixed point which does not change.

[7.19] And if the alidade turns around its rotation axis, and similarly also, if the axis of the bar turns around its base by a circular motion [E34], its axis does not move and does not depart from its position. And every point on the axis of the bar does not change and is not moved, if the bar moves on its base by the circular motion. Therefore the point on the axis of the bar at which the axis of the bar meets the axis of the tube, is exactly one single point which does not change and is not moved, neither by the motion of the bar, nor by the motion of the alidade.

[7.20] Thus at the moment at which the altitude of the star is taken on the Eastern side, if the rotation axis of the tube is extended in a straight line to end at the centre of the star and in the other direction to meet the plane of the horizon, then it passes through the fixed point on the axis of the bar.

[7.21] Again, at the time of taking the altitude of the star on the Western side, if the axis of the tube is extended in a straight line to end at the centre of the star, and in the other direction to meet the plane of the horizon, it passes through that same point on the axis of the bar.

[7.22] But the two points at which the centre of the star is [located] at the two moments of the two equal altitudes, are two fixed points on the [celestial] sphere. And the line joining them is perpendicular to the plane of the meridian circle, [B54b] and the plane of the meridian circle bisects it.[30]

[7.23] Thus if the two points which have been obtained in the plane of the horizon, which are at the ends of the two [positions of the] axis of the tube at the two moments of [taking] the altitudes, are joined by a straight line, and [if the line is] bisected, and if from its midpoint a line

[30] Reading *wa-yaqsimuhu* with B54b:1, where E34:11 has *wa-yaqsimu*.

is drawn at right angles, then that line is the meridian line with utmost precision, as has been shown above.[31]

[8. Effect of refracted rays.]

[8.1] But perhaps one may object to this proof by saying: none of the lights of the star, extended from the heavens to the earth, and none of the visual rays, extended from the eye to the star, extends in a straight line, but they are refracted at the concavity of the sphere.

[8.2] And if they are refracted, then the axis of the tube, in the rectilinear extension of which the eye perceives the star, is not rectilinearly extended to the centre of the star but it is refracted.

[8.3] And therefore the two lines extended from the plane of the horizon to the point of intersection at the axis of the bar, do not end at the two points on the circumference of the time circle at which the centre of the star was found at the two moments of the two [equal] altitudes.

[8.4] Then we say in answering this objection: The lights of the stars [E35] and the visual rays will necessarily be refracted at the concavity of the sphere. But points with similar positions on the sphere with respect to the meridian circle and with respect to the zenith will be refracted in similar ways, and from [refraction] points with similar positions with respect to the meridian circle and similar positions with respect to the point at the zenith.

[8.5] And the two points at which the centre of the star is at the two moments of the two [equal] altitudes are at equal distance from the point at the zenith, and their distances to any [assumed] point on the meridian circle will also be equal.

[8.6] Therefore the points at which the visual rays of the eye refract to the centre of the star, at the two moments of [equal] altitudes, are at equal distance of the zenith and at equal distance of any [assumed] point on the meridian circle.

[8.7] Thus the line which joins these two points is bisected by the meridian circle, and [B55a] the line joining the midpoint of this line to the zenith is perpendicular to this line joining the two points of refraction.[32]

[31] Ibn al-Haytham stops here in order to refute a possible objection. The proof continues in [9.1].

[32] Our translation follows E35:10, where the superfluous word *mutasāwiyatayn* in B55a:2 is removed.

[8.8] And similarly if any point is assumed at the meridian circle, then the two lines drawn from it to the two refraction points are equal, and the line drawn from it to the midpoint of the line joining the two refraction points is perpendicular to the line joining the two refraction points.

[8.9] Thus the line joining the two refraction points is perpendicular to the plane of the meridian circle, and the plane of the meridian circle bisects it.

[8.10] Since this is the case, the two points from which the two lines were drawn which were extended in a straight line along the axis of the tube to the fixed point of the axis of the vertical bar are the two refraction points.

[8.11] They are two fixed points, because their position with respect to the two points at which the centres of the stars are located in the two moments of [taking] the [equal] altitudes is a similar position, and they are on the two sides of the meridian circle, and the line joining them is perpendicular to the meridian circle and the plane of the meridian circle bisects it.[33] The rest of the proof is as before.

[9. Second part of the proof that the instrument produces the meridian line.]

[9.1] And since the axis of the tube is parallel to the imagined line in the middle of the surface of the alidade, covering the surface [E36] of the plate[1], the plane in which the axis of the tube always rotates is parallel to the surface of the plate[1], therefore each perpendicular drawn from it to the surface of the plate[1] is equal to half the width of the [vertical] bar.

[9.2] The axis of the [vertical] bar is parallel to the surface of the plate[1], and the perpendicular drawn from any point on it to the surface of the plate[1] is equal to half the width of the [vertical] bar. Thus the axis of the [vertical] bar is in the plane in which the axis of the tube always rotates, which plane is parallel to the surface of the plate[1].

[9.3] And the line[2] drawn in the middle of the face[2] of the bar, I mean the face[2] to which the edge of the thin ruler is perpendicular, is parallel to the edge of the plate[1].

[9.4] Thus if a perpendicular is drawn from any point on this line[2] [B55b] to the edge of the plate[1], then it is equal to half the width of the [vertical] bar, and it is perpendicular to the surface of the plate[1]

[33] Reading with B55a:12 *yaqsimuhu saṭḥ dā'ira* where E35:19 has *yaqsimu dā'ira*.

because the face[2] of the [vertical] bar is perpendicular to the surface of the plate[1].

[9.5] Therefore this line, I mean the line[2] in the middle of the face[2] of the bar, I mean the face[2] perpendicular to the surface of the plate[1], is in the plane parallel to the surface of the plate[1], I mean the plane containing the axis of the tube and the axis of the bar.

[9.6] And since the face[2] of the bar, which [face] contains this line[2], is perpendicular to the surface of the plate[1], this face[2] is perpendicular to the plane containing the axis of the tube and the axis of the bar.

[9.7] Therefore the plane containing the axis of the tube and the axis of the bar is perpendicular to the face[2] of the bar, and the line[2] drawn in the middle of the face[2] of the bar is the intersection between these two planes.

[9.8] But the sharp edge of the thin ruler, which covers the surface of the base, is perpendicular to the face[2] of the bar, and to the line[2] drawn in the middle of the face[2] of the bar. Thus this line, I mean the edge of the thin ruler, is in the plane containing the axis of the bar and the axis of the tube.

[9.9] And this line, if extended in a straight line inside the body of the bar, meets the axis of the bar, and it meets it at the point which is the centre of the hole in the surface of the base, and this line is perpendicular to the axis of the bar because it is perpendicular to the line[2] in the middle of the face[2] of the bar which is parallel to the axis[34] of the bar.

[9.10] Thus the line which is the edge of the thin ruler, if extended in a straight line in the direction opposite the thin ruler, contains with the axis of the vertical bar a right angle.

[9.11] But the axis of the tube, at the moment of taking the altitude of the star, contains an acute angle with the axis of the vertical bar, because the complement[s] of the Eastern and Western altitude are each less than a quarter of a circle,[35] and the axis [E37] of the tube and the end of the thin ruler are in one plane.

[9.12] Therefore these two lines will meet if extended in a straight line.[36]

[9.13] But the edge of the thin ruler is in the plane parallel to the

[34] B55b:15 and E36:19 have incorrectly "face."

[35] Here Ibn al-Haytham assumes again that the star is not exactly on the horizon.

[36] This is a consequence of the parallel postulate in Book 1 of Euclid's *Elements*, see (Heath 1, 202).

horizon. Thus the point at which [B56a] these two lines meet is in the plane parallel to the horizon. Therefore this point is the determined point at which the axis of the tube and the plane of the horizon meet at the moment of [taking] the altitude.

[9.14] Thus the line which is the edge of the thin ruler at the moment of [taking] the altitude ends at the determined point which is in the plane of the horizon. And the same is the situation of this line at the moment of [taking] the second altitude. Thus in the two moments of [taking] the altitudes, the line which is the edge of the thin ruler ends at the two determined fixed points in the plane of the horizon.

[9.15] And these two lines, I mean the edge of the thin ruler in the two moments of [taking] the altitudes, pass through the centre of the hole in the base.

[9.16] And since the two altitudes are equal, and their complements are equal, the two angles which are contained by the axis of the tube and the axis of the bar are equal.

[9.17] But the two angles which are contained by the axis of the bar and the edge of the thin ruler, if extended in a straight line, are right angles, and the axis of the bar is common to the two triangles which are formed at the two moments of [taking] the two altitudes by [a] the axis of the tube, [b] the axis of the bar and [c] the imagined line on the rectilinear extension of the edge of the thin ruler. Therefore these two triangles are equal.

[9.18] Thus the two lines which are on the rectilinear extension of the edge of the thin ruler at the two moments of [taking] the altitudes, and which end at the two fixed points in the plane of the horizon, are equal, and the two beginning points of them are the centre of the circle of the base.

[9.19] Thus the line drawn from the centre of the circle of the base to the midpoint of the line joining the two points in the plane of the horizon, is perpendicular to the line joining the two points.

[9.20] And this line, I mean the [line] drawn from the centre of the circle of the base to the midpoint of the line joining the two points, is in the plane of the horizon.

[9.21] And it has become clear from what has been explained before that the line drawn from the midpoint of the line joining these two points, extended in the plane of the horizon, perpendicular to the line joining the two points, is the meridian line with utmost precision. [E38]

[9.22] Therefore the line drawn from the centre of the hole of the base to the midpoint of the line joining [B56b] the two fixed points is the meridian line with utmost precision, since it is perpendicular to the line joining the two fixed points and passing through its midpoint.

[9.23] And the edge of the thin ruler at the moment of taking the altitude cuts the circumference of the circle of the base in the direction opposite the direction of the plate[1] and the alidade and the tube.

[9.24] And the two points in the plane of the horizon, at which the axis of the tube ends, are in the direction of the plate[1] and the alidade and the tube.

[9.25] Therefore the edge of the thin ruler is at the two moments of taking the altitudes in the direction contrary to the direction of the two lines drawn from the centre of the base to the two fixed points, while they are on the rectilinear extension of these two lines, I mean that the edge of the thin ruler at the moments of the two equal altitudes is at the rectilinear extension of the two lines drawn from the centre of the base to the two fixed points.

[9.26] Therefore the two lines drawn from the centre of the base to the two fixed points, if extended in a straight line from the point of the centre, end at the two points at which the line of the thin ruler intersects the circumference of the circle of the base, and they are equal.

[9.27] Thus the line joining these two points, I mean the points at the circumference of the circle of the base, is parallel to the line joining the two fixed points which are in the plane of the horizon.

[9.28] And the line drawn from the centre of the circle of the base to the midpoint of the line joining the two fixed points, if extended in a straight line from the centre, ends at the midpoint of the line joining the two points at the circumference of the circle of the base, and it is perpendicular to it.

[9.29] And it has been shown that this line is the meridian with utmost precision.

[9.30] Thus if the line joining the two points at the circumference of the circle, which [points] were produced by the edge of the thin ruler intersecting the circumference of this circle, is bisected and a line is drawn from the midpoint at right angles [and] extended in the plane of the horizon, then that line is the meridian line [B57a] with utmost precision.

[9.31] Thus it has been shown by [geometrical] proof that the line which was found by the instrument which we have described, and by means of the two altitudes of the star, [E39] is exactly the same line of which it had been proved that it is the meridian line with utmost precision.

[10. Repeated measurements during successive nights]

[10.1] Even though a person is skilled and righteous, inattentiveness and error may sometimes occur. When he has determined the meridian line in the way we have explained, and after he has obtained the line in the plane parallel to the horizon, it is necessary that he returns the following night or later, and observes the star whose altitude he took [on the first night], or another fixed star, and that he takes its equal Eastern and Western altitudes which are different from the altitudes which he obtained in the first night, and determines by means of them the meridian line in the way we have mentioned.

[10.2] If it coincides with what he found the first night, he has been successful. If there is a difference, he is in error.

[10.3] And [even] if it coincides with the first line, he returns the third night and observes that star or another one and takes two equal altitudes of it, different from the first altitudes and different from the second altitudes, and determines by means of them the meridian line in the way we have explained before.

[10.4] And if the three lines agree, then the work has been done with utmost correctness. And if one of them differs [from the other two], then the work is resumed and carried out accurately, until it agrees in three nights, so that the three lines which have been determined agree with one another.

[10.5] Thus in this way is the determination of the meridian line with utmost precision, and [about] this we wanted to inform [the reader] in this treatise.

[11. Motivation of choices made in constructing the instrument]

[11.1] It remains for us to mention the reasons why we have arranged the instrument according to the description which we have set out.

[11.2] We have made the vertical bar long, so that a sitting person can place his eye inside the funnel at the end of the plate[1] at the moment of taking the altitude.

[11.3] We have made the alidade on the plate[1], not on the bar, so that one can draw on it [i.e., the plate] a quarter circle.

[11.4] We have made on [B57b] the plate[1] a quarter circle and we have divided it into ninety [equal] degrees so that the magnitude of the [i.e., any] altitude can be obtained with this instrument. Thus its magnitude cannot be confused at the moment of taking the second altitude.

[11.5] The magnitude of the altitude can be obtained by means of this instrument because the arc which is cut off by the pointer at the time of taking [E40] the altitude, which [arc] is between the pointer and the end of the arc, I mean the end which overlaps with the line[1] in the middle of the face[1] of the bar, which [line] is perpendicular to the plane of the horizon, is similar to the imagined arc which is cut off by the axis of the tube and the axis of the vertical bar.

[11.6] For the axis of the tube is parallel to the line which is extended through the middle of the surface of the alidade and which passes through the pointer. And the axis of the bar is parallel to the line[1] extending in the middle of the face[1] of the bar, which [line[1]] concides with the line drawn on the plate[1] which [line on the plate] is the end of the quadrant. Thus the angle which is cut off by the first pair of lines is equal to the angle cut off by the second pair of lines. Therefore the two arcs subtended by them are similar.

[11.7] But the arc cut off between the axis of the tube and the axis of the bar[37] is similar to the arc of the azimuthal circle between the zenith and the centre of the star.

[11.8] Therefore the arc which is cut off by the pointer, adjacent to the vertical bar, is similar to the arc between the zenith and the centre of the star. Therefore the remaining [complement] arc of the divided quadrant is similar to the arc of the altitude, which is the altitude of the star at that moment.

[11.9] And we have made the thin ruler in the direction opposite to the direction of the plate[1] because if a person places his eye inside the funnel at the moment of taking the altitude, he is sitting against the vertical bar, while he is in the direction of the plate[1]. Then he cannot move the thin ruler [if it were] in the direction of the plate[1], because the person who takes the altitude would be an obstacle to it. If he does not [i.e. cannot] turn the thin ruler he does not [i.e., cannot] turn the vertical bar because they are welded together.

[11.10] So because of these reasons we have arranged the instrument according to the description which we have composed.

[37] We have emended the word *saṭḥ* in B57b:11, E40:8 to *masṭara* for mathematical sense.

[12. Alternative description of the instrument and procedure]

[12.1] And since we have explained all of this, let us summarize the procedure in order that the precise determination of the meridian line [B58a] is [done] by an easy and simple way, and the work is not difficult for anybody. And this is as we [now] describe:[38]

[12.2] We take a quadrangular plate[1] of copper, exactly a square, with right angles, such that its length is a quarter of an ell, and we draw on its sides a square. We make one of its angles the centre. Then we draw, with radius the two lines containing that angle, a quadrant of a circle, and divide it into ninety [equal] degrees.

[12.3] We make at the end of one of the two lines a round cylindrical rotation axis, protruding from the plate, and such that the axis [of the cylinder] is in the rectilinear extension of the line whose end is at the centre [E41].

[12.4] We sharpen the [edges of the] angle of the plate[1] facing the centre until the end of the angle becomes a point.

[12.5] Then we make another round copper plate[2] whose radius is equal to the width of the square plate, and we make the base of the plate[2] squared, and on the edge of this plate[2] a circle is drawn and divided into three hundred and sixty [equal] degrees.

[12.6] At the centre a hole is drilled in the magnitude of the rotation axis of the squared plate[1]. Thus if the axis is entered in the hole, the squared plate[1] is perpendicular to the round base[2].

[12.7] At the end of the axis we drill a hole and we insert a peg which holds the axis in such a way that it does not go out the hole.

[12.8] We make for the round plate[2] a circumference in such a way that under it is a circle perpendicular to its lower surface. Thus if it [the plate] is placed on top of the earth, it is possible to rotate the axis on the base.

[12.9] We make for this circumference on its down side two rulers intersecting at right angles, welded together so that they become like a cross, and the length of each of them is the magnitude of the diameter of the base, which is half an ell. Then we weld the two rulers on the circumference of the base so that it is possible to mount this base on top of a square [even] if it [the square] is smaller than it [the base].

[38] The idea of this simplified instrument is the same as the instrument which Ibn al-Haytham described above. The bar is omitted and the plate[1] rotates directly on top of the plate[2], which is placed on a supporting structure in the form of a ring on top of a cross. This combination can be mounted on some sort of pedestal.

[12.10] Then we take for the quadrant an alidade with two vanes, just like the alidade of the astrolabe, and we pierce the two vanes with two equal holes facing one another. Then we take a round funnel, such that the diameter of its width is two fingers, and its depth is two fingers, and we mount [it] [B58b] on one of the sides of the tube.

[12.11] Then we make in the middle of the [upper] edges of the vanes two notches in which the tube can be entered, or we widen the two holes in the vanes until the tube can be entered in them, or we mount on the tube two hollow bodies in which the vanes can be entered if it [the tube] is mounted on the vanes.

[12.12] Then we take a square of the shape of a lamp [or lampstand], such that its height is three spans of a hand, and the width of its top and its base are less than the width of the base of the square which is one span.

[12.13] If this has all been finished, the instrument is ready.

[12.14] If one needs to determine the meridian line, let a plane parallel to the horizon be prepared, and let the square be put on a place in it, and let the base of the square [i.e., the lampstand] be pinned to the ground with pins such that it is fixed in its position and cannot move.

[12.15] Then we place the instrument on top of this square, and we join the base of the instrument to the top of the square by means of wax. This can be done by filling the top of the square with wax, heating the wax with fire and then putting the base of the instrument on the wax and submerging it until [E42] the two rulers are immersed in the wax. When this has been finished, the setting up of the instrument is finished.

[12.16] Then one attends the night. Once the stars have appeared, a fixed star is chosen, and let is be one of the brightest, and let it be on the Eastern side of the meridian.

[12.17] Let its altitude be taken by the quadrant and the tube. When its altitude has been determined, a point is marked on the circumference of the base, at the angle of the square plate, I mean the sharpened angle [i.e. the angle with sharpened edges].

[12.18] Then we glue the alidade to the quadrant with wax, so that it cannot be moved.

[12.19] Then we wait until that star is on the Western side, and then we move the quadrant and look in the tube until we see that star in the hole of the tube, while the alidade is all the time glued to the quadrant in its first position. Once that star has been seen in the tube, we mark a point on the circumference of the base at the angle of the square plate.

[12.20] And if the two points have been obtained on the circumference of the base of the instrument, $< \ldots >$[39] on a plumb-line with a thin thread and a conical weight such that its vertex is the point.

[12.21] We attach the thread of the plumb-line to one of the two points [B59a] which have been obtained on the circumference of the base, and we suspend the plumb-line, and we raise and lower it until the point of the vertex of the weight is on the plane of the earth.

[12.22] In that situation we mark a black point on the plane of the earth at the vertex of the plumb-line.

[12.23] Then we raise the plumb-line and attach it to the other point at the circumference of the base, and we suspend the plumb-line until the point of the vertex of the weight is on the plane of the earth, and then we mark another point in black at the vertex of the [weight of the] plumb-line.

[12.24] When the two points have been obtained on the plane of the earth, the instrument is removed and then a straight ruler is placed on the two points. We draw a straight line from one of the points to the other in black, because it is clearer.

[12.25] We bisect this line, and from the midpoint we draw a line at right angles, and it is extended in the plane of the earth parallel to the horizon. It is the meridian line.

[12.26] The proof that this line is the meridian line is the preceding proof: The fixed point[40] to which the line[s] is [i.e., are] drawn from [a] the midpoint of the [line] perpendicular to the plane of the circle of the meridian, [E43] joining the two points [i.e. positions] of the centre of the star, to [b] the point at which the two axes of the tube intersect during the moments of the two [equal] altitudes, has a similar position [as in the previous proof]. Thus the two [axes] intersect at a point of the line drawn between the midpoint of the perpendicular to the meridian [plane] to the midpoint of the line joining the two points which were marked on the circumference of the base.

[12.27] And this line is a fixed line, whose position does not change, because the base of the instrument is fixed and does not change its position, and therefore any point on it is fixed and does not change its position. Thus the point of intersection on this line, [i.e., the intersection] between the two axes of the tube at moments of the two [equal] altitudes, is a fixed point, whose position does not change. And that is what we wanted to explain.

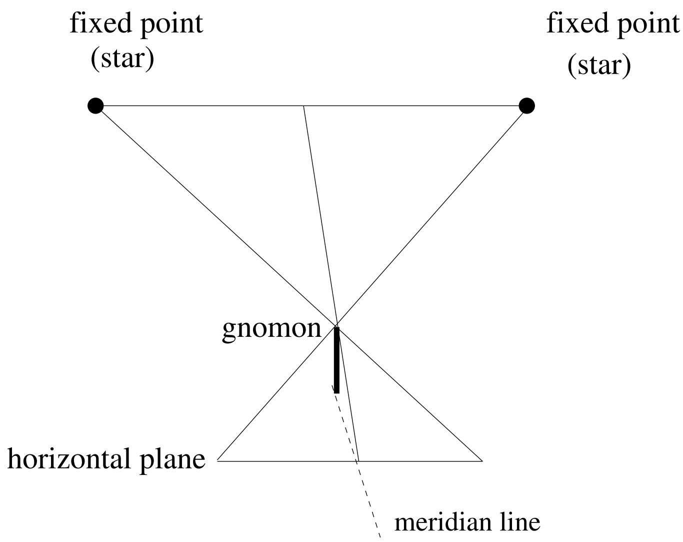
[39] One or more words must be missing in B58b:20, E42:9.

[40] The translation follows Sezgin's emendation of *al-nuqṭa* to *li'l-nuqṭa*, compare footnote 22 in E43.

[12.28] This is the time to end this treatise. (end of [B59a] and E43])

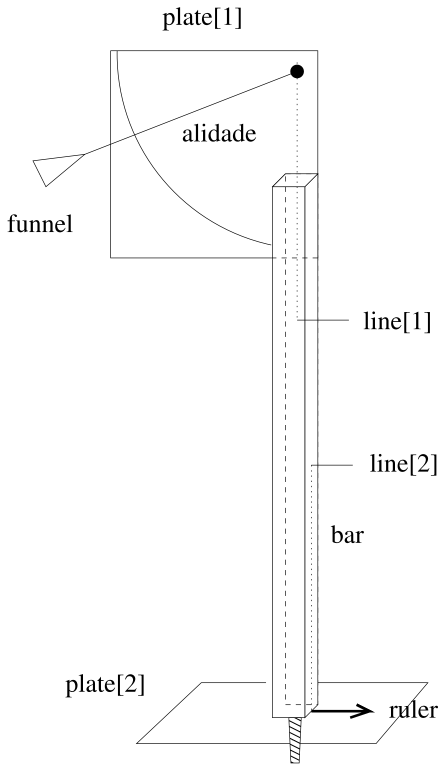
4 Explanatory figures

The following two figures do not occur in ibn al-Haytham's text.



Explanatory Figure 1

Explanatory Figure 1 displays the gnomon and the positions of the same fixed star at the same altitude East and West of the meridian. Assume that the gnomon is a vertical line in the meridian plane. Then the meridian plane bisects [a] the segment between the two positions of the star, and [b] the angle formed by the two lines from the tip ot the gnomon to the two positions of the star. Ibn al-Haytham extends the two lines to meet the horizon plane and he draws the line segment between the two points of intersection. Then the meridian line (dashed line) joins the foot of the gnomon with the midpoint of the line segment between the two points of intersection.



Explanatory Figure 2

Explanatory Figure 2 shows Ibn al-Haytham's meridian instrument in a simplified way, in order to illustrate the steps in the proof in the text concerning plate[1], plate[2], line[1] and line[2]. In the figure, line[1] is drawn on the back face of the bar (called face[1] in the proof), and line[2] on the right face (called face[2] in the proof). Note that the plates, lines and faces are not distinguished in the Arabic text and the numbers are my additions and therefore conjectural. The alidade is indicated symbolically by a straight line and the vanes are not shown. We have assumed that the plate[1] is mounted vertically along the bar, in agreement with the text, and in order that stars above the horizon can be sighted.

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Abstract

This paper contains an annotated English translation of the two surviving treatises by Ibn al-Haytham (ca. 965-1041) on the determination of the meridian line. These very technical treatises survive in an Arabic manuscript in Berlin, and they were published by Fuat Sezgin in an excellent Arabic edition which appeared in this journal in 1986. The edition has not been translated before, but it has served as a source for a reconstruction of Ibn al-Haythams instrument for the determination of the meridian line. A study of Sezgins editions, which are translated in this paper, shows that the reconstructed instrument gives an incorrect representation of Ibn al-Haytham's ideas.