Rashed, Roshdi. **Les mathématiques infinitésimales du IXe au XIe siècle. Vol. I. Fondateurs et commentateurs: Banū Mūsā, Ibn Qurra, Ibn Sinān, al-Khāzin, al-Qūhī, Ibn al-Samḥ, Ibn Hūd.**

Al-Furqān Islamic Heritage Foundation, London, 1996. xiv+1106+VI pp. ISBN 1-873992-18-1.

This book contains critical editions in Arabic with French translations and commentaries of the following medieval mathematical works:

(1) Banū Mūsā (Iraq, early ninth century), *Book on the measurement of plane and spherical figures.* This work is known in a 12th century Latin translation by Gerard of Cremona (ca. 1114-1187), and in an Arabic revision by Naṣīr al-Dīn al-Ṭūsī (1201-1274), which Rashed has edited on the basis of 26 manuscripts. The work is about the area of the circle and the triangle, the surface area and volume of the sphere, the trisection of the angle and the construction of two mean proportionals between two given lines. The original version of the Banū Mūsā themselves has not yet been found.

(2) The treatise by Thābit ibn Qurra (Syria and Iraq, 836-901) on the measurement (i.e. geometrical determination of the area) of a segment of a parabola. Thābit knew that Archimedes had solved the problem but he did not know Archimedes's solution itself.

(3) The work by Thābit ibn Qurra on the measurement of the solid of rotation of a segment of a parabola about its axis or diameter.

(4) A long treatise by Thābit ibn Qurra on sections of a cylinder and its surface area. In this text, Thābit derives the fundamental properties of the ellipse as a plane section of a cylinder in a way similar to the method by which Apollonius of Perga (ca. 200 B.C.) deduced the fundamental properties of the ellipse, parabola and hyperbola as plane sections of a cone. Thābit also finds the area of an ellipse and he expresses the surface area of an oblique cylinder in terms of the perimeter of a certain ellipse.

(5) A short text by Ibrāhīm ibn Sinān (Iraq, 907-946) on the measurement of a segment of a parabola. This text survives in two slightly different versions, both of which are edited and translated in the book under review.

(6) The extant part of the commentary by Abū Jaᶜfar al-Khāzin (Iran, middle of the 10th century) on Book 1 of the *Almagest* of Ptolemy (ca. 150 A.D.). In this text, al-Khāzin proves several theorems related to isoperimetric figures: if a (convex) polygon has the same perimeter as a regular polygon with the same number of sides, the regular polygon has a greater area (the proof is correct for a triangle but incomplete if the polygon has more than

three sides); if a regular polygon and a circle have the same perimeter, the circle has a greater area; if a solid of rotation of a regular polygon about the axis through one of its angular points and its centre has the same surface area as a sphere, the volume of the sphere is greater than the volume of the solid. Similar theorems were studied in Greek antiquity by Zenodorus in the second century B.C.

(7) A treatise by Abū Sahl al-Kūhī (Iran, late tenth century) on the measurement of the solid of rotation of a segment of a parabola about the axis or a diameter. This treatise also survives in two slightly different versions, both of which are edited and translated in this book.

(8) A text by Ibn al-Samḥ (Spain, 979-1035) on the ellipse as a section of the cylinder. This text only survives in a medieval Hebrew translation, which has been translated into French by Tony Levy. This treatise is more elementary than (4).

(9) Rashed added Propositions 313.16., 313.19 and 432.18-21 of the book *Istikmāl* (Perfection) of al-Mu'taman ibn Hūd (Spain, died 1085), on isoperimetric properties, the quadrature of the parabola, and properties of conic sections. Rashed did not include the (false but related) Proposition 432.22, on the areas of segments of the hyperbola and ellipse, and Proposition 432.23 on the ratio of the perimeters of similar segments of conic sections.

With the exception of (4), (8) and (9), the contents of the texts are well known in the modern historical literature. The texts (1)-(5) and (7)-(9) had not yet been critically edited, and thus their editions and translations are a welcome contribution to the literature. Text number (6) had recently been published by Richard Lorch in a critical Arabic edition with an English translation [*Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 3 (1986), 150-229, Arabic paging 49]. On page 740, Rashed justifies his reedition of the text by his project to include all the texts on the same theme in the volume under review, and he says that he made around 20 "improvements" to Lorch's edition. The reviewer has compared the two editions and he has found only 8 slight differences which concern the meaning of the text, and around 50 emendations which Rashed took over from Lorch's edition without giving due credit. Lorch had made these emendations to restore al-Khāzin's text from corrupted passages in the unique medieval Arabic manuscript.

The title of the book, "Infinitesimal mathematics", should be taken with a grain of salt. In the texts (1)-(9), the medieval authors did not use infinitesimal arguments in the way of the 17th-century mathematicians, but

the Greek "exhaustion method", which Archimedes also used in his work *On the sphere and cylinder.* Surprisingly, Rashed's book contains no edition, translation, or even reference to a text by the late 10th century Islamic author al-Karābīsī on the determination of the volume of a torus. For a critical edition with a German translation of this text, see [E. Bessel-Hagen and O. Spies, Das Buch über die Ausmessung der Kreisringe des Aḥmad ibn ᶜUmar al-Karābīsī, *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abteilung B (Studien)*, 1 (1931), 502-540].