

Rashed, Roshdi. **Les mathématiques infinitésimales du IXe au XIe siècle. Vol. 2. Ibn al-Haytham.** *Al-Furqān Islamic Heritage Foundation, London, 1993. xii+581+v pp. ISBN 2-87723-119-4.*

Ibn al-Haytham was a prominent scientist in the medieval Islamic tradition. He was born in Basra in present-day Iraq around 965 and he died in Cairo shortly after 1040. His most important work is in optics, but he also wrote more than 70 mathematical treatises. The book under review contains editions of the Arabic texts with French translations of the following:

(1) A short treatise on the quadrature of lunes. A lune is a crescent-like figure bounded by two arcs of circles.

(2) The *Quadrature of the circle*. The treatise boils down to a philosophical argument for the existence of a square equal to a given circle. Ibn al-Haytham promises to construct the square in a later treatise, which is not extant and which may not have been written.

(3) A long treatise on lunes with many baroque theorems, mostly to the effect that a certain lune is equal to the sum or difference of a certain circle and a certain triangle. In the commentary, Rashed relates some of Ibn al-Haytham's propositions to the behaviour of the function $f(x) = (\sin^2 x)/x$ which was studied by Euler in connection with lunes. Euler used this function in order to construct, by means of ruler and compass, certain lunes equal in area to squares which can also be constructed by means of ruler and compass. Three such lunes had been found by Hippocrates of Chios in the fifth century B.C. Treatise (3) does not contain new contributions to this famous problem.

(4) The quadrature of the two solids of rotation of a parabolic segment about its diameter and about an ordinate respectively. The volume of these solids is determined by Archimedean methods. This treatise is one of the high points of Arabic-Islamic mathematics.

(5) A treatise on the measurement of the sphere by means of the Archimedean "exhaustion method". Ibn al-Haytham considers a hemisphere and the circumscribed cylinder, and he approximates the volume of the cylinder minus the hemisphere by circumscribed and inscribed solids consisting of n thin parallel rings of equal height. The volumes of these solids can be computed from the formula for the sum of the first n squares.

(6) A treatise on isoperimetric properties. Ibn al-Haytham proves that the circle has a greater area than any regular polygon with the same perimeter, that the sphere has a greater volume than any regular polyhedron with the same surface area, and that of two regular polygons with the same perimeter,

the one with the greater number of sides has a greater area. He states that if two regular polyhedrons have the same surface area, the one with the greater number of faces has the greater volume. He then seems to have lost control of himself, because he proves this theorem for the tetrahedron, octahedron and icosahedron only, and the enormously long proof is only a partial success. The same problem was studied by Pappus of Alexandria (ca. A.D. 300) in Book V of his *Mathematical collection*. This treatise may be an unfinished work, which will certainly repay further study by historians.

(7) A brief commentary on Proposition 1 of Book X of Euclid's *Elements*.

(8) A treatise on the justification of the procedure for root extraction in Hindu-Arabic numbers.

(9) A similar treatise on the extraction of cube roots.

Treatises (2) and (4) are well known in the historical literature, but the others either were available in Russian translation only, or had not been previously studied. The Arabic editions and translations are carefully done. The mathematical commentary follows Ibn al-Haytham's reasonings closely, but not always insightfully, and the reader of this book has to invest a considerable amount of time to understand Ibn al-Haytham's complicated treatises (3) and (6).

In the introduction, Rashed discusses problems in the biography of Ibn al-Haytham and the lists of works attributed to him by medieval sources. Rashed suggests that there were two Ibn al-Haythams (brothers?): a natural philosopher, Muḥammad ibn al-Ḥasan ibn al-Haytham, author of the well-known intellectual autobiography, and a mathematician and scientist, al-Ḥasan ibn al-Ḥasan ibn al-Haytham, who is the author of almost all extant works attributed to "Ibn al-Haytham". This suggestion has already caused a debate [see A. I. Sabra, One Ibn al-Haytham or Two? An exercise in reading the bio-bibliographical sources, *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 12 (1998), 1-50].

Rashed has called the book under review "Infinitesimal mathematics" because treatises (1)-(6) relate to the determination of areas and volumes of curved figures. However, in the treatises on lunes and on isoperimetry, Ibn al-Haytham does not give arguments which are equivalent to modern integrations of functions or determinations of limits. The reader who cannot get hold of this book can consult Rashed's brief summaries of most of the above-mentioned treatises by Ibn al-Haytham in a chapter entitled "Infinitesimal determinations, quadrature of lunules and isoperimetric problems" in *Encyclopedia of the history of Arabic science*, Vol. 3, 418-446, Routledge, London,

1996..