Régis Morelon (ed.), Thābit ibn Qurra. *Œuvres d'astronomie*. Paris: Les Belles Lettres. 1987, CXLII + 321 (French) + 168 + XV (Arabic) pp. (Collection sciences et philosophie arabes. Textes et études). ISBN 2-251-35561-8.

Thābit ibn Qurra (221 H./A. D. 836–288 H./A. D. 901, see GAS VI, pp. 163–170) played a pivotal role in the assimilation of Ptolemaic astronomy in the Arabic-Islamic tradition, and he made significant contributions of his own to theoretical astronomy, in particular to the theory of lunar crescent visibility. The book under review contains critical editions of nine highly interesting Arabic texts attributed to him, together with French translations and commentaries. Of these texts, only two had been previously edited in Arabic (nos. 7, 9 in the list below), and (defective) medieval Latin translations of two others had been published (nos. 1, 3); three texts had been translated or abstracted (nos. 4, 6, 8) and two were completely unknown (nos. 2, 5). When Morelon's book was already in press, Russian translations of the treatises nos. 1–6 and 8–9 below appeared in [2] (texts no. 31, 29, 26, 23, 27, 24, 21 and 22). The publication of [2] does not diminish the value of Morelon's book, which is indispensable for anyone who wishes to make a serious study of Thābit's astronomical works.

The nine texts edited by Morelon are as follows.

1. *Simplifying the Almagest* (GAS VI, p. 90 no. 5a), a very concise summary of the Ptolemaic system.

2. *Description of the orbs and their natures and the number of their motions and the magnitude of their motions* (GAS VI, p. 166 no. 3). This is a more detailed description of the motions of the celestial bodies according to the Ptolemaic system, slightly modified in the light of new observations.

3. *On the solar year* (GAS VI, p. 166 no. 6). In this treatise the solar apogee is shown to be sidereally fixed, not tropically fixed as Ptolemy had assumed. The treatise had been studied by several historians on the basis of a defective medieval Latin translation. The Arabic text is much clearer, so that numerous passages which were incomprehensible in the Latin version can now be understood (see Morelon's commentary). Morelon shows (pp. XLVIII–LIII) that this work was not by Thābit ibn Qurra, but probably by the Banū Mūsā (compare GAS VI, p. 166).

4. *On the retardation of the motion (of a celestial body) in the ecliptic and its acceleration, according to the position (of the body) on the eccentre* (GAS VI, p. 166 no. 1). This very elegant treatise is concerned with the following facts (in modern terms): a celestial body moving with uniform velocity in a circular orbit with centre outside the earth has minimal angular velocity (as seen from the earth) at the apogee, maximal velocity at the perigee, and mean velocity at the points in the ecliptic exactly 90° away from the perigee and apogee. My modern summary is misleading because Thābit did not have a rigorous concept

of instantaneous velocity. He stated his theorems in terms of the time needed by the celestial body to traverse finite arcs of the ecliptic.

5. *Explanation of the method, by means of which, according to Ptolemy, his (i. e. Ptolemy's) predecessors derived the (periods of the) uniform circular motions of the moon* (GAS VI, p. 90 no. 5b; p. 166 no. 2, p. 167 no. 7). This is essentially an explanation by Thābit of a difficult passage in *Almagest* IV, 2. Morelon admits (p. 227) that he neither understands the passage in the *Almagest* nor the analysis in [3], pp. 71–72. We will return to these matters below.

6. *On the computation of the visibility of the lunar crescent* (GAS VI, p. 166 no. 5). This is perhaps Thābit's most important treatise, containing a full explanation of his lunar visibility theory. Morelon has shown in an earlier publication that the treatise was partially inspired by a lost work of Ptolemy, the *Phaseis.*

7. An extract from the *Sanjarī Zīj* of ʿAbdu l-Raḥmān al-Khāzinī (mid-6th/12th century) on the method of Thābit for the prediction of lunar crescent visibility by means of tables.

8. *On the figures described by the shadow of the extremity of a gnomon on a horizontal plane, for all days and all localities* (GAS VI, p. 168 no. 10 = GAS V, p. 270 no. 15). These figures are shown to be straight lines, circles and conic sections.

9. *On the timekeeping instruments called sundials* (GAS VI, 168,9). Thābit solves all cases of the problem of how to construct a sundial on an arbitrary (horizontal or inclined) plane.

Morelon's editions and translations are very good. In treatise no. 1, the text on p. 5, line 12 *wa-fīhi falak al-burūj* (translation on p. 5, line 20, "et dans lequel se trouve l'écliptique") must be corrupt, because Thābit was, like Ptolemy, aware of the precession, so that the ecliptic is not located in the sphere of the fixed stars (cf. p. 8, lines 13–14). Perhaps *wa-fīhi* should be emended to *wa-thumma.* Morelon's commentaries are on the whole detailed, and his evaluations of Thābit's work are optimistic. I now present some supplements and modifications to Morelon's commentary to Treatise 5.

1. In Treatise 5, Thābit discusses how Hipparchus derived the period of the epicyclic anomaly of the moon, according to Ptolemy's account in *Almagest* IV, 2 ([4], pp. 174–179). The underlying lunar model must be the simple model of Hipparchus, in which the moon revolves uniformly on an epicycle, the centre of which rotates uniformly around the centre of the earth. This is the model which Morelon calls "le premier modèle" (p. LXXXV, the terminology is due to O. Pedersen; see Figure 1 below). It is unnecessary to introduce in the discussion refinements made by Ptolemy to explain the behaviour of the moon at the quadratures and octants, such as the "deuxième modèle", which was unknown to Hipparchus. Also note that Ptolemy himself used the model of Hipparchus for the syzygies (compare p. XCIX and *Almagest* V, 10; [4], pp. 239–243). Therefore Figures 5–2 and the corresponding discussion on pp. LXXXV–VI have to be deleted, and Figure 5–5 on p. XC is misleading.

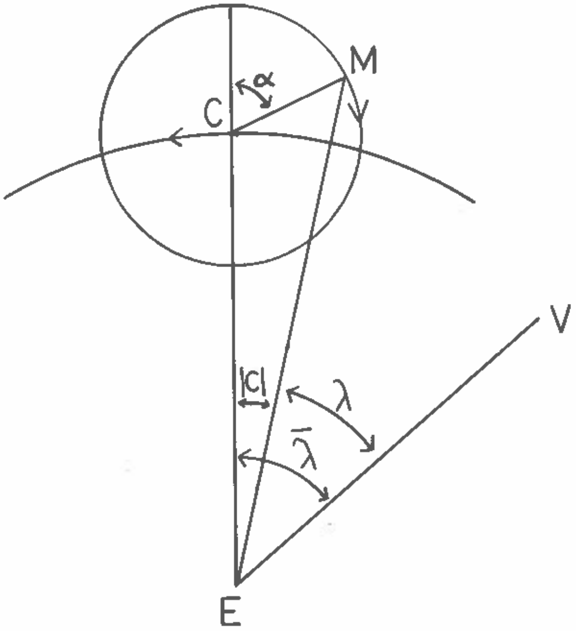

Figure 1

2. Morelon assumes that there is an intimate connection between Thābit's Treatises 4 and 5, and he therefore relates the verbal discussion in Treatise 5, Theorem 1 to the eccentric solar model of *Almagest* III, 3 ([4], pp. 141, 144), to which Thābit refers in Treatise 4 (p. 73). However, one can also relate Theorem 1 to the epicyclic solar model, which Ptolemy describes in *Almagest* III: 3 ([4], pp. 144–150). To do this, in Figures 5–3 (p. LXXXVI) and 5–4 (p. 222) delete the ecliptic, define Z on EO extended such that $EO \cdot EZ = EP^2$, and let Z be the earth, and ABC the solar epicycle with centre E (compare [5]). Then in Figure 5–3 $\angle M_1ZE = \angle OM_1E = q_1$ etc., and in both figures ZB and ZC are tangent to the epicycle. As Thābit says (p. 85), his Theorem 1 is valid for the sun and for the (i. e. for Hipparchus' model of the) moon, and it is not correct to say that the case of the moon is "beaucoup plus complexe" (p. 223) than that of the sun.

3. In order to derive the period of the lunar epicyclic anomaly, Hipparchus made a long list of observations of lunar eclipses. From this list he selected two pairs of lunar eclipses in a way which we will describe below. We first introduce some notation: let the eclipses take place at t_1, t_2 (first pair) and t_3, t_4 (second pair); the t_i $(i = 1, 2, 3, 4)$

are the moments at the middle of each eclipse, when the sun is exactly opposite the moon (as seen from the centre of the earth). For i = 1, 2, 3, 4 we write q_i for the solar equation, $\bar{\lambda}_i$ for the mean longitude of the moon, λ_i for the true longitude of the moon, $c_i = \lambda_i - \bar{\lambda}_i$ for the lunar (epicyclic) equation and α_i for the lunar anomaly at t_i. Compare Figure 1, in which E is the earth, C the centre of the epicycle, EV is the direction of the vernal point and M is the moon (in Figure 1 the indices i have been omitted). The quantity c_i may be positive or negative (in Figure 1, $\lambda < \bar{\lambda}$, so that c is negative).

According to Ptolemy, Hipparchus selected his pairs of eclipses such that

$$t_4-t_3 = t_2-t_1 \qquad (1)$$

$$q_4-q_3 = q_2-q_1 \qquad (2)$$

and such that an extra condition (3a) or (3b) is satisfied, to be described below. From (1) and (2) one can deduce, in a way which does not concern us here (see [3], p. 72), that

$$\alpha_4 - \alpha_3 = \alpha_2 - \alpha_1 \ (\mathrm{mod}\ 360^0) \qquad (4)$$

and

$$c_4 - c_3 = c_2 - c_1 \qquad (5).$$

Hipparchus wanted to find pairs of eclipses such that

$$\alpha_1 = \alpha_2 \text{ and } \alpha_3 = \alpha_4 \qquad (6).$$

Unfortunately, (4) and (5) are necessary conditions but not sufficient conditions for (6). For there are situations (which Thābit enumerates in his Theorem 1, p. 84, namely his second, third and fourth case) where (4) and (5) hold but (6) does not. Thābit says on pp. 88–89 in a verbose way that these situations (i. e. his second, third and fourth case) have to be excluded somehow. This is, says Thābit, the reason why according to Ptolemy one should select pairs of eclipses which satisfy not only (1) and (2) but also an additional third condition. This third condition can be chosen in different ways, and Ptolemy mentions two possibilities, which Thābit quotes verbatim from the Almagest. The two intervals between the eclipses (i. e. $[t_1, t_2]$ and $[t_3, t_4]$) should be such that

(3a) "in one interval it (the moon) starts from its least speed and does not end at its greatest speed, while in the other it starts from its greatest speed and does not end at its least speed"

or such that

(3b) "it (the moon) starts in both intervals from mean speed, not, however, from the same mean speed, but from the mean speed in the arc of increasing speed in one interval, and from the (mean speed) in the arc of decreasing speed in the other" (my translation of Thābit's quotation from the *Almagest* on p. 89, lines 15–19, inspired by [4], p. 178 lines 7–17).

Figure 2 displays the positions of the moon on the epicycle at t_1 and t_3 for (3a) and (3b).

For (3a), we have $c_1 = c_3 = 0$, so that by (5) $c_2 = c_4$. Since M_1 and

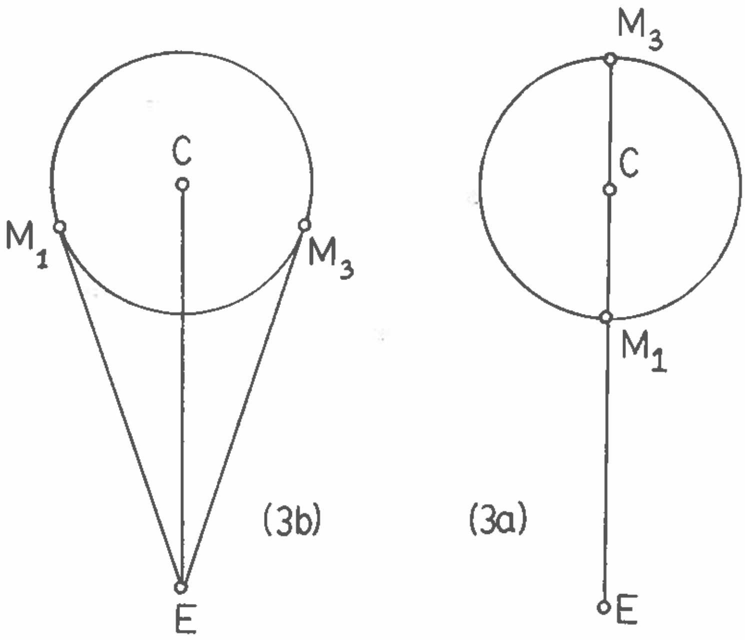

Figure 2

M_3 are in diametrically opposite positions on the epicycle, M_2 and M_4 are also diametrically opposite (because of (4)), so that c_2 and c_4 must both be zero or have opposite sign. We conclude $c_2 = c_4 = 0$. So if we know that M_2 is not diametrically opposite M_1 and M_4 is not diametrically opposite M_3 (as stated in (3a)), then we must conclude $M_1 = M_2$ and $M_3 = M_4$, so that $\alpha_1 = \alpha_2$ and $\alpha_3 = \alpha_4$ as desired.

For (3b), c_1 and c_3 are the maximal and minimal possible values of the lunar equation, so that $c_1 \geq c_2$ and $c_3 \leq c_4$. Therefore by (5) $c_1 = c_2$ and $c_3 = c_4$, and thus $M_1 = M_2$ and $M_3 = M_4$.

Thābit did not give a detailed description involving figures such as Figure 2, but his motivation of the conditions (3a) and (3b) is essentially correct and sheds light on a difficult passage in the *Almagest*.

References

1. GAS = F. Sezgin. *Geschichte des arabischen Schrifttums*, Band V, Mathematik, Leiden 1974. Band VI, Astronomie, Leiden 1978.

2. Sabit ibn Korra. *Matematicheskie Traktaty*, collected by B. A. Rozenfeld (in Russian). Moscow 1984.

3. O. Neugebauer. *A history of ancient mathematical astronomy.* New York, 1975. 3 vols.

4. G. J. Toomer. *Ptolemy's Almagest.* London 1984.

5. O. Neugebauer. The equivalence of eccentric and epicyclic motion according to Apollonius. *Scripta Mathematica* 24 (1959), 5–21. Reprinted in: O. Neugebauer. *Astronomy and History. Selected essays.* New York 1983, pp. 335–351.

JAN P. HOGENDIJK*

R. Rashed (Ed., Transl.), Sharaf al-Dīn al-Ṭūsī; *Œuvres mathématiques. Algèbre et géométrie au XII^e siècle.* Paris: Société d'édition „Les Belles Lettres", 1986. Vol. 1: 470 pp., ISBN 2-251-35562-6. Vol. 2: 459 pp., ISBN 2-251-35563-4.

These two volumes contain editions and French translations of the three extant mathematical works of Sharaf al-Dīn al-Ṭūsī (who flourished in the end of the 6th century H./12th century A. D.): a brief, hitherto unpublished, treatise about the asymptotes of the hyperbola, a short treatise on elementary geometry that had been studied previously by Suter,[1] and a very long hitherto unpublished text whose title has been lost, but to which we will refer as the *Algebra*. This text is one of the most important mathematical works from the Arabic-Islamic tradition, because it contains the most profound medieval discussion of cubic equations that is known to be extant.

The Arabic-Islamic mathematicians appear not to have known the algebraical solution of the cubic equation. However, in the fourth/tenth century they constructed the roots of various cubic equations geometrically by means of conic sections. In the 11th century, 'Umar al-Khayyām wrote his famous *Algebra*, in which he gave geometrical constructions by means of conic sections of the roots of all types of cubic equations. (Because the medieval Arabic-Islamic mathematics only worked with positive coefficients, they had to distinguish 17 types of cubic equations.) 'Umar al-Khayyām's treatment was incomplete in various respects. Some types of equations, such as $x^3 + c = ax^2 + bx$, do not have roots for all possible choices of the coefficients. In such

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* Department of Mathematics, State University of Utrecht, Budapestlaan 6, P.O. Box 80.010, 3508 TA Utrecht, Netherlands.

[1] H. Suter, Einige geometrische Aufgaben bei arabischen Mathematikern. *Bibliotheca Mathematica*, 3. Folge, 8 (1907–8), pp. 23–36, reprinted in: H. Suter, *Beiträge zur Geschichte der Mathematik und Astronomie der Araber*, ed. F. Sezgin, Frankfurt (Institut für Geschichte der Arabisch-Islamischen Wissenschaften) 1986, vol. 2, pp. 217–230.